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Embedded Systems 2010/2011 – Assignment Sheet 9

Due: Tuesday, 25th January 2011, *before* the lecture (i.e., 10:10)
Please indicate your **name**, **matr. number**, **email address**, and which **tutorial** you are planning to attend on your submission. We encourage you to collaborate in **groups** of up to **three** students. Only one submission per group is necessary. However, in the tutorials every group member must be capable to present each solution.

Exercise 1: Periodic Scheduling (30 pts.)

For each of the following tasks sets, (1) determine whether an EDF-schedule and/or an RM-schedule exists, and (2) formally prove your answer.

$\Gamma = \{\tau_1, \tau_2, \tau_3\}$	$T_1 = D_1 = 3$	$C_1 = 1$
	$T_2 = D_2 = 4$	$C_2 = 2$
	$T_3 = D_3 = 8$	$C_3 = 1$
$\Delta = \{\tau_1, \tau_2, \tau_3\}$	$T_1 = D_1 = 2$	$C_1 = 1$
	$T_2 = D_2 = 3$	$C_2 = 1$
	$T_3 = D_3 = 4$	$C_3 = 1$
$\Pi = \{\tau_1, \tau_2, \tau_3, \tau_4\}$	$T_1 = D_1 = 2$	$C_1 = 1$
	$T_2 = D_2 = 5$	$C_2 = 1$
	$T_3 = D_3 = 8$	$C_3 = 2$
	$T_4 = D_4 = 20$	$C_4 = 1$

Exercise 2: Aperiodic Scheduling (50 pts.)

Consider the following scheduling problem $1 \mid sync \mid T_w$:

> Using a uniprocessor machine, find a schedule for a set $\mathcal{J} = \{J_1, \ldots, J_n\}$ of n synchronous tasks with computation times $C_1, \ldots, C_n$ that minimizes the weighted sum of the completion times
>
> $$T_w = \sum_{i=1}^{n} (w_i f_i)\,,$$
>
> where $w_i > 0$ is a weight, and f_i is the time at which task i finishes its execution. (*Note:* The schedule is not required to respect the deadlines. We are only interested in minimizing T_w.)

(a) Let $\mathcal{J}$ be a task set, and let σ be a schedule for $\mathcal{J}$ that is optimal with respect to the problem $1 \mid sync \mid T_w$. Formally prove that there exists a nonpreemptive schedule σ^* for $\mathcal{J}$ with the same T_w of σ.

(b) Devise a polynomial-time algorithm that, given a task set $\mathcal{J} = \{J_1, \ldots, J_n\}$, computes a schedule σ for $\mathcal{J}$ that is optimal with respect to the scheduling problem $1 \mid sync \mid T_w$.

(c) Formally prove that your algorithm computes an optimal schedule.

Exercise 3: Priority Ceiling Protocol (20 pts.)

Consider the Priority Ceiling Protocol. Using this protocol, give a picture describing a run of three tasks on one processor:

- Task 1 has the highest priority. Task 1 arrives at time t=6. Task 1 consists of normal computation for 2 time units, followed by critical section 1 for 2 time units, followed by normal computation for 1 time unit.
- Task 2 has lower priority than Task 1. Task 2 arrives at time t=2. Task 2 consists of normal computation for 1 time unit, followed by critical section 2 for 3 time units, followed by normal computation for 1 time unit, followed by critical section 3 for 1 time unit, followed by normal computation for 1 time unit.
- Task 3 has the lowest priority. Task 3 arrives at time t=0. Task 3 consists of normal computation for 1 time unit, followed by critical section 3 for 2 time units, followed by normal computation for 1 time unit.

Your picture should depict which task is executed (and the type of computation: either normal or the critical section the task is in) in the processor at which point in time, covering the interval from time t=0 until all work is done. You should also explicitly give any changes in the priority of the Tasks.