

The background of the slide is a light blue-grey color with a faint, semi-transparent image. On the right side, there is a large, slightly out-of-focus clock face. On the left side, there is a silhouette of an airplane in flight, moving from left to right.

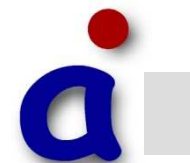
Model-based Code Generation: Esterel Scade

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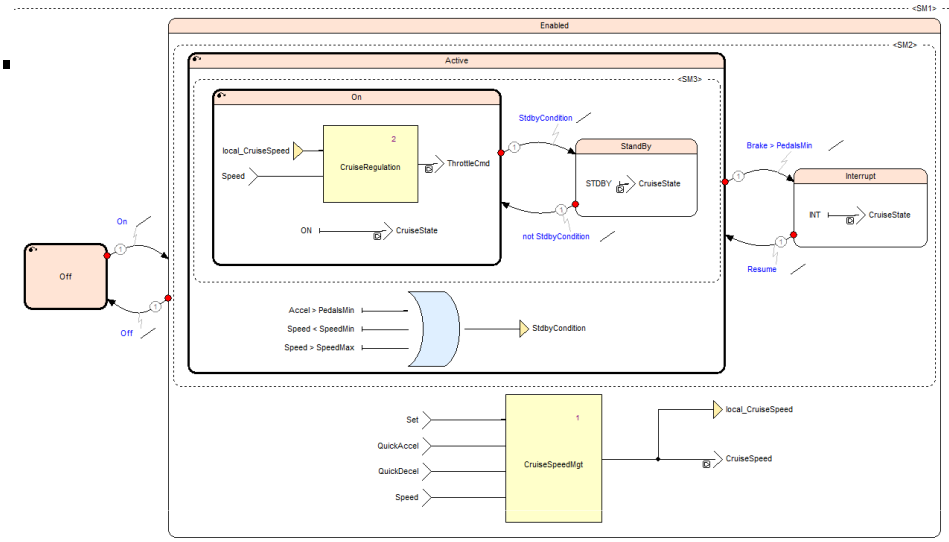
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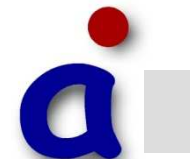
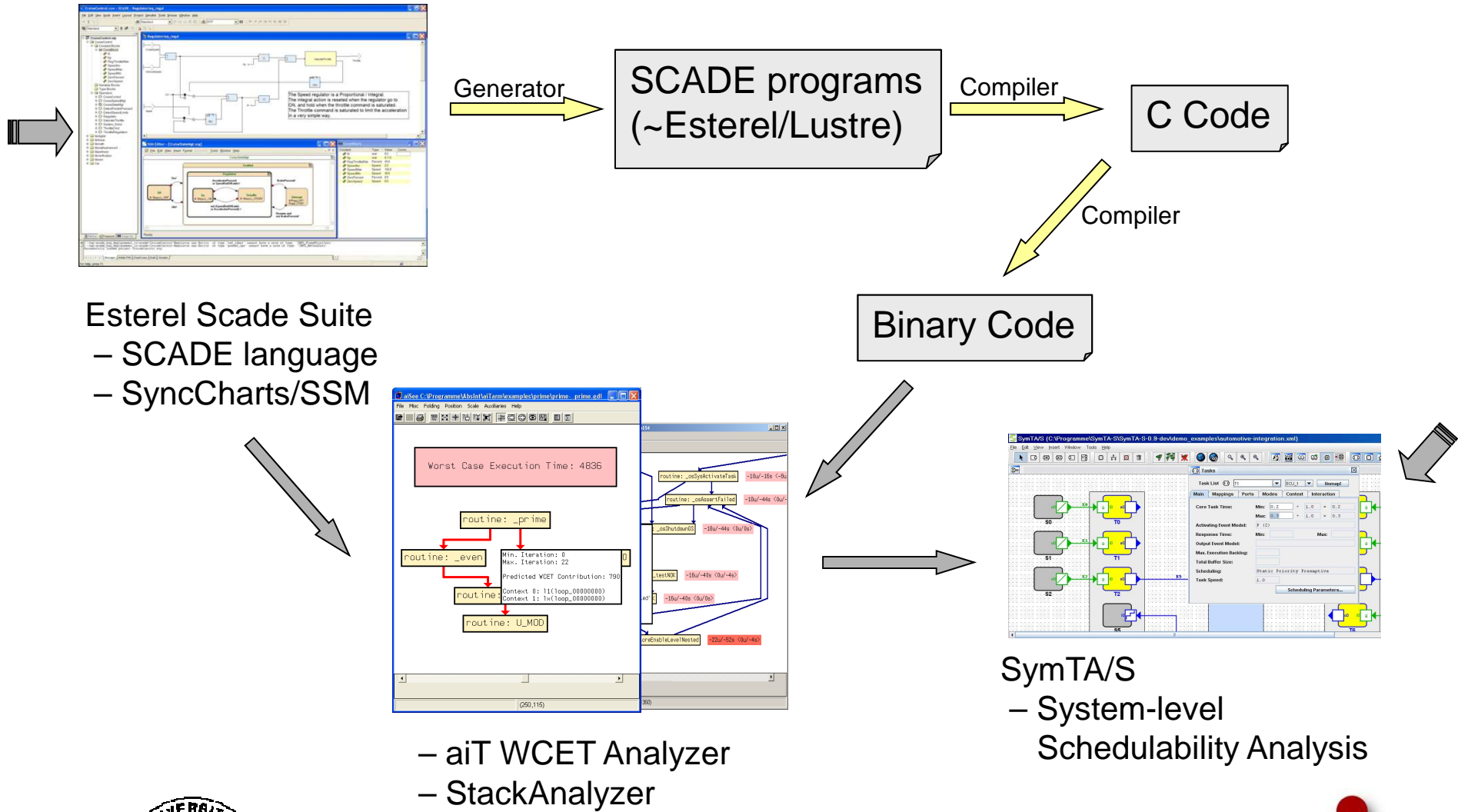


Model-based Software Development

- Model is software specification.
- Hardware/Software codesign.
- Prototyping.
- Formal verification.
- **Automated** & integrated development tools:
 - Simulation.
 - Documentation.
 - Automatic code generation.
- **Automated** & integrated verification and test methods
 - Model checking
 - Static system analysis
 - Synthesis of test suites



Model-based Software Development



Embedded Systems

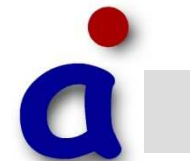
- Typically, embedded systems are reactive systems:

„A reactive system is one which is in continual interaction with its environment and executes at a pace determined by that environment“

[Bergé, 1995]

Behavior depends on input and current state.

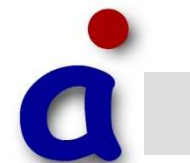
- automata model appropriate



Finite Automata

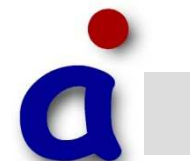
- **Non-deterministic** finite automaton (NFA):
 $M = (\Sigma, Q, \Delta, q_0, F)$ where
 - Σ : finite alphabet
 - Q : finite set of states
 - $q_0 \in Q$: initial state
 - $F \subseteq Q$: final states
 - $\Delta \subseteq Q \times (\Sigma \cup \{\varepsilon\}) \times Q$
- M is called a **deterministic** finite automaton, if Δ is a partial function

$$\delta: Q \times \Sigma \rightarrow Q$$



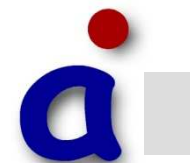
Mealy Automata

- Mealy automata are finite-state machines that act as **transducers**, or **translators**, taking a string on an input alphabet and producing a string of equal length on an output alphabet.
- A machine in state q_j , after reading symbol σ_k writes symbol λ_k ; the output **symbol depends on the state** just reached **and** the corresponding **input symbol**.
- A Mealy automaton is a six-tuple $M_E = (Q, \Sigma, \Gamma, \delta, \lambda, q_0)$ where
 - Q is a finite set of states
 - Σ is a finite input alphabet
 - Γ is a finite output alphabet
 - $\delta: Q \times \Sigma \rightarrow Q$ is the transition function
 - $\lambda: Q \times \Sigma \rightarrow \Gamma$ is the output function
 - q_0 is the initial state



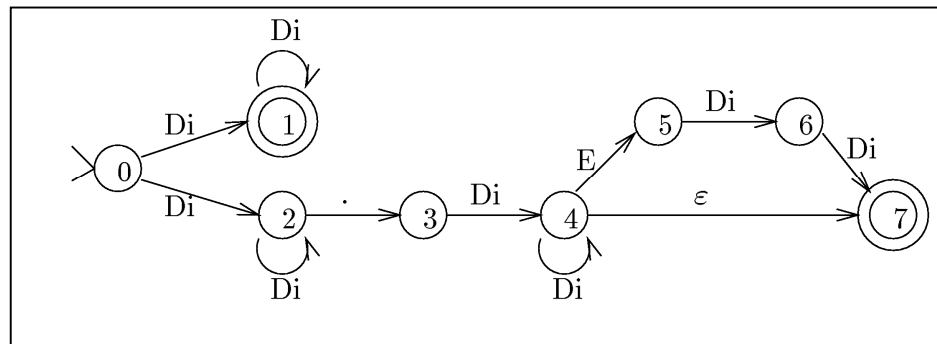
Moore Automata

- Moore automata are finite-state machines that act as **transducers**, or **translators**, taking a string on an input alphabet and producing a string of equal length on an output alphabet.
- Symbols are output after the transition to a new state is completed; output **symbol depends only on the state** just reached.
- A Moore automaton is a six-tuple $M_0 = (Q, \Sigma, \Gamma, \delta, \lambda, q_0)$ where
 - Q is a finite set of states
 - Σ is a finite input alphabet
 - Γ is a finite output alphabet
 - $\delta: Q \times \Sigma \rightarrow Q$ is the transition function
 - $\lambda: Q \rightarrow \Gamma$ is the output function
 - q_0 is the initial state

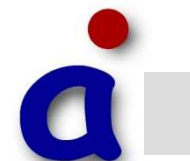


Simple State Transition Diagram

- Used to represent a finite automaton
- Nodes: states
- q_0 has special entry mark
- Final states are doubly circled
- An edge from p to q is labelled by a if $(p, a, q) \in \Delta$
- Example: integer and real constants:

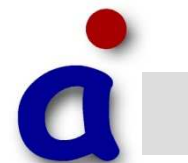


- Problem: all combinations of states have to be represented explicitly, leading to **exponential blow-up**.



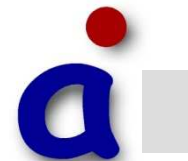
SyncCharts

- Visual formalism for describing **states** and **transitions** of a system in a modular fashion.
- Extension of state-transition diagrams (Mealy/Moore automata):
 - **Hierarchy**
 - **Modularity**
 - **Parallelism**
- Is fully **deterministic**.
- Tailored to control-oriented applications (drivers, protocols).
- Implements synchronous principle.



Synchronous Programming

- Program typically implements an **automaton**:
 - **state**: valuations of memory
 - **transition**: reaction, possibly involving many computations
- **Synchronous paradigm**:
 - **Reactions** are considered **atomic**, ie they take no time. (Computational steps execute like combinatorial circuits.)
 - Synchronous **broadcast**: instantaneous communication, ie each automaton in the system considers the outputs of others as being part of its own inputs.

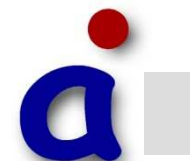


Synchronous Programming

- Important requirement: guaranteeing **deterministic** behavior.
- Time is divided into discrete ticks (also called cycles, steps, instants).
- Simple implementation: **sampling / cyclic executive**:

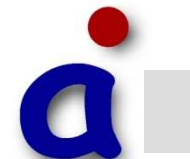
```
<Initialize Memory>
Foreach period do
    <Read Inputs>
    <Compute Outputs>
    <Update Memory>
```

- Verification of timing behavior: prove that the **worst-case execution time** (WCET) of any reaction fits between two iterations of the cyclic executive.
- Implicit assumption: presence of a **global clock**. This makes application in **distributed** environments difficult.



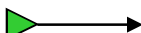


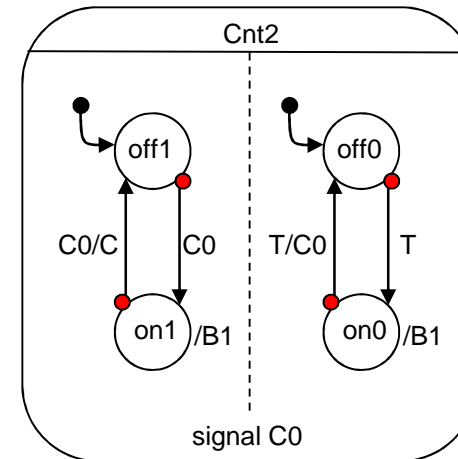
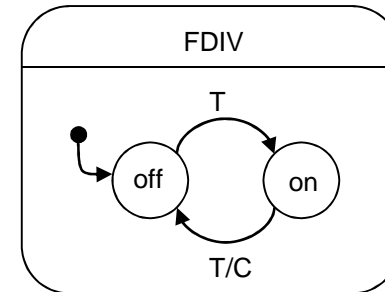
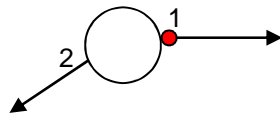
Overview

- **StateCharts:**
 - First, and probably most popular formal language for the design of reactive systems.
 - Focus on specification and design, not designed as a programming language.
 - Determinism is not ensured.
 - No standardized semantics.
- Programming languages for designing reactive systems:
 - **ESTEREL** [Berry]: textual imperative language.
 - **LUSTRE** [Caspi, Halbwachs]: textual declarative language. Tailored to data-flow oriented systems (e.g. regulation systems).
 - **SCADE** [Esterel Inc.]. Enhanced LUSTRE, graphical and textual formalism.
 - **SyncCharts / SSM**: Graphical formalism corresponding to ESTEREL.



SyncCharts

- States (circles and rectangles):
 - can be named
 - two types:
 - simple state (circle)
 - macrostate (rounded rectangle): contain a hierarchy of other states
 - are optionally labelled*: /<effect>
- Transitions (arrows):
 - are labelled*: <trigger>/<effect>
 - All components are optional.
 - three types:
 - strong abort 
 - weak abort 
 - normal termination 
 - can have priorities (-> determinism)




*Triggers and effects are signals, or combinations of signals using boolean operations or, and and not.



States & State Transition Graphs

- Special states:

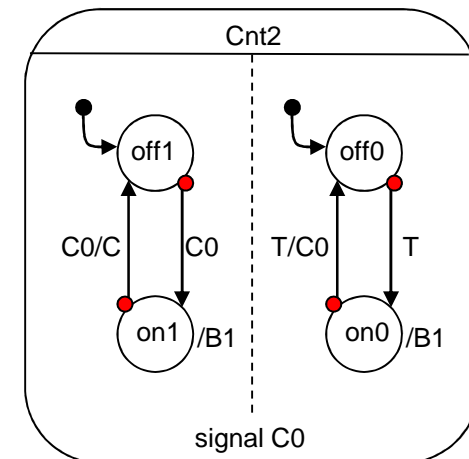
- Initial state:  (alternative notation: )

- Terminal state: 

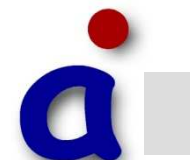
- State Transition Graph: connected labeled graph made of states connected by transitions, with an initial state.

- Two types of states:

- Simple state: just carries a label.
 - Macrostate: contains at least one state transition graph.



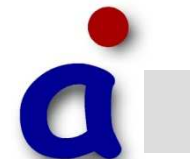
- At each instant there is one and only one **active** state.
- An active state waits for the satisfaction of the trigger of one of its outgoing transitions, at an instant **strictly posterior** to its entering (activation).



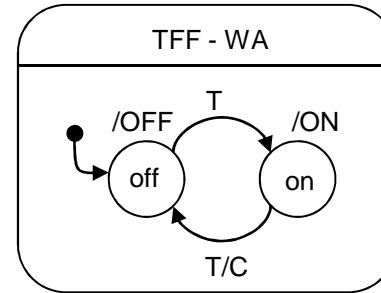
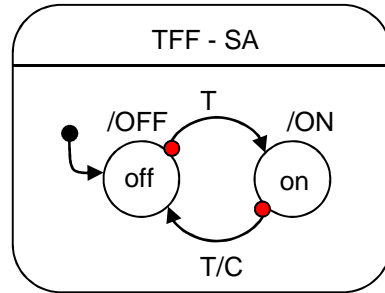
State and Transition Labels

- Signals are characterized by their **presence status** (+, -, \perp).
 - Valued signal: signals conveys a value of a given type.
 - Pure signal: no value conveyed.
- **tick**: implicit signal present at every instant.
- A trigger is **satisfied** \Leftrightarrow associated signal is present.
- Transition labels:
 - When the trigger is satisfied, the transition is said to be enabled.
 - The transition is **immediately** taken and emits the associated signals.
 - The firing of a transition is fully **deterministic** and takes **no time**.
- Node labels:
 - Signal emission depends on transition type (strong/weak abort)
 - Signals are emitted when...

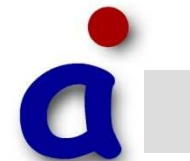
	...entering	...in	...exiting
Weak abort	Yes	Yes	Yes
Strong abort	Yes	Yes	No



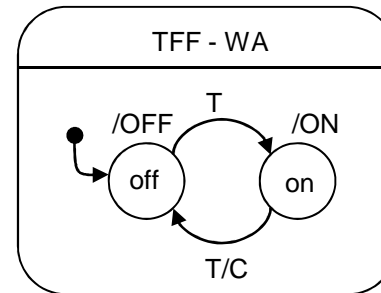
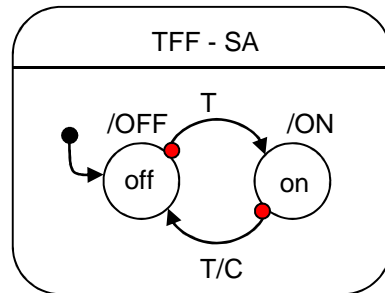
Example: Strong vs. Weak Abort



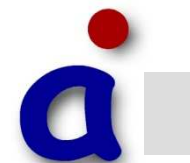
Instant	Input	TFF-SA Output	TFF-WA Output
1			
2	T		
3			
4	T		
5			
6	T		
7	T		
8	T		
9			



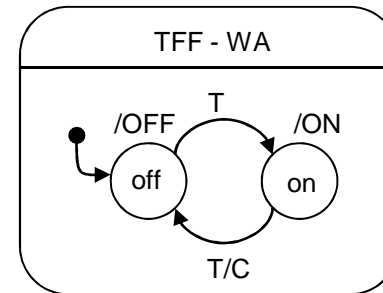
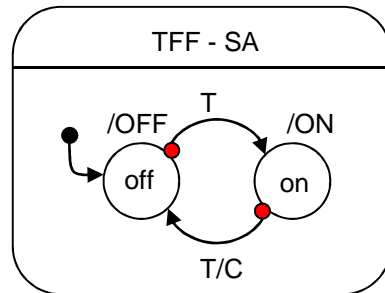
Example: Strong vs. Weak Abort



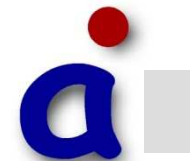
Instant	Input	TFF-SA Output	TFF-WA Output
1		OFF	
2	T	ON	
3		ON	
4	T	C, OFF	
5		OFF	
6	T	ON	
7	T	C,OFF	
8	T	ON	
9		ON	



Example: Strong vs. Weak Abort

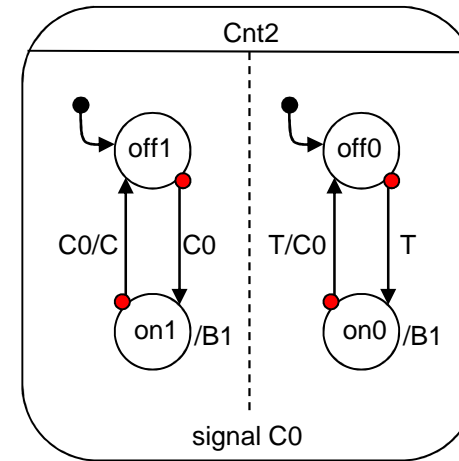


Instant	Input	TFF-SA Output	TFF-WA Output
1		OFF	OFF
2	T	ON	OFF,ON
3		ON	ON
4	T	C, OFF	ON,C,OFF
5		OFF	OFF
6	T	ON	OFF,ON
7	T	C,OFF	ON,C,OFF
8	T	ON	OFF,ON
9		ON	ON





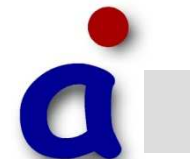
Concurrency

- A macrostate can contain a **parallel** composition of separate concurrent STGs. Graphical notation: dashed separation line.
- STGs are coupled by **shared signals**.
- A **local** signal is declared by the keyword `signal` and its scope is the containing macrostate.
- A set of (concurrent) active states is called a **configuration**.

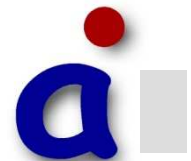
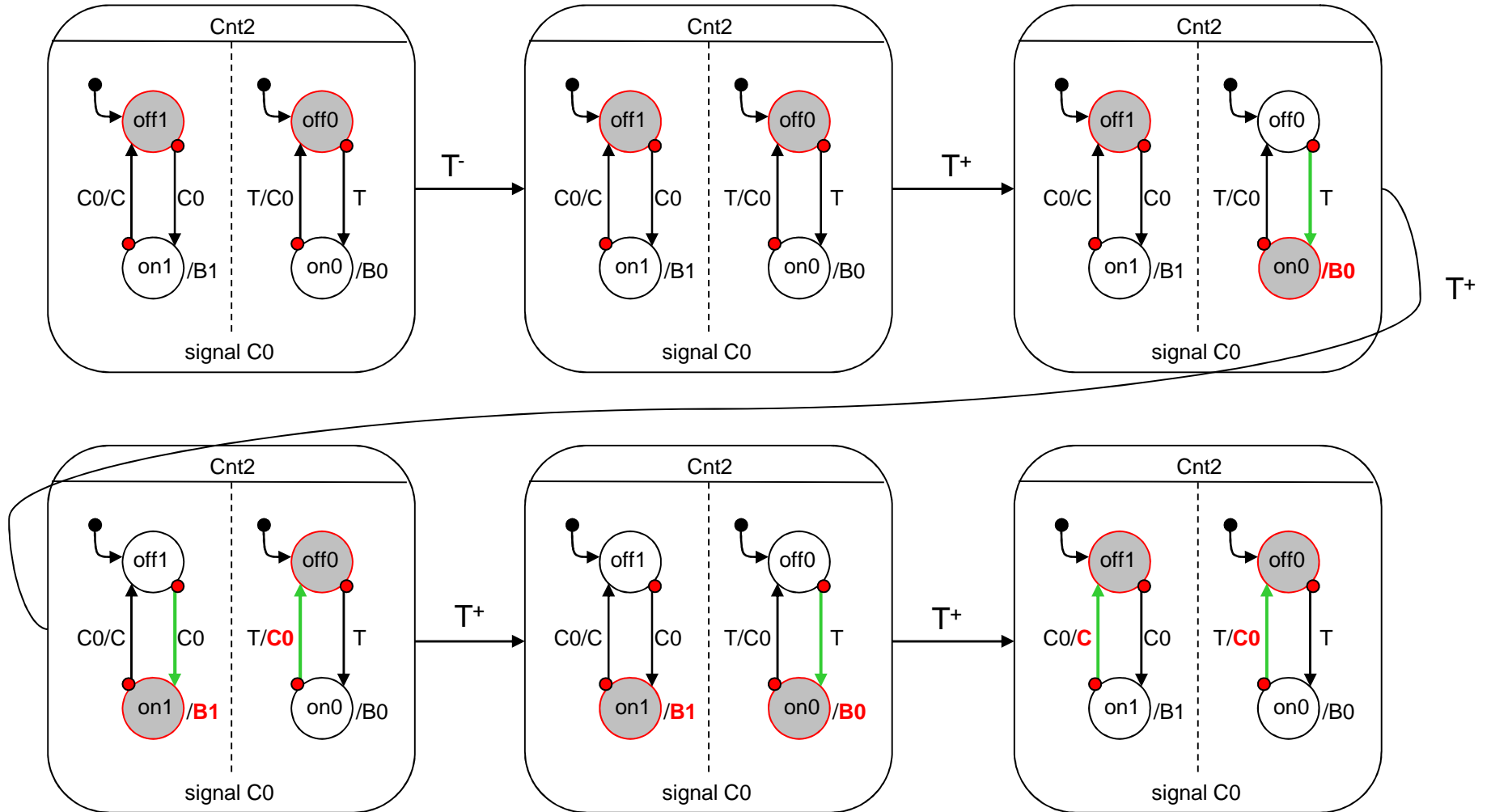


- Notation:

- Active state 
- Taken transition 
- Emitted signal **S**
- S^+ : presence of signal S
- S^- : absence of signal S

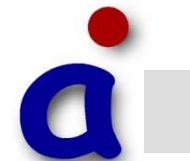
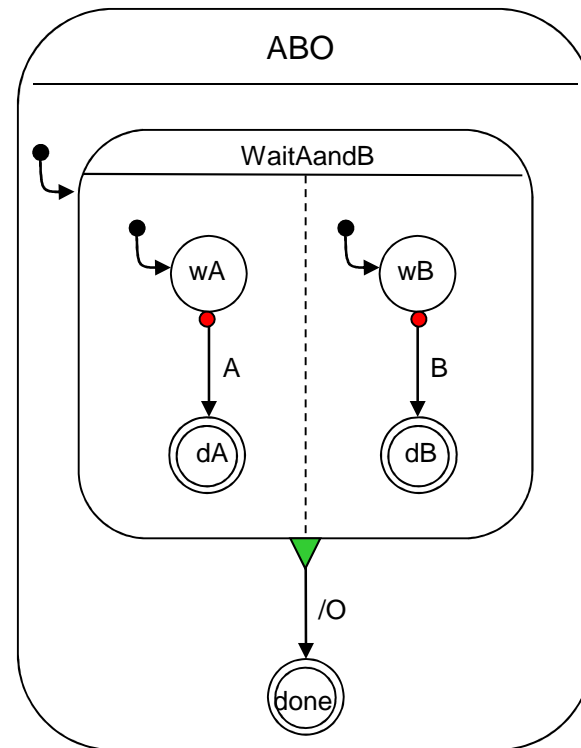


Example Reaction

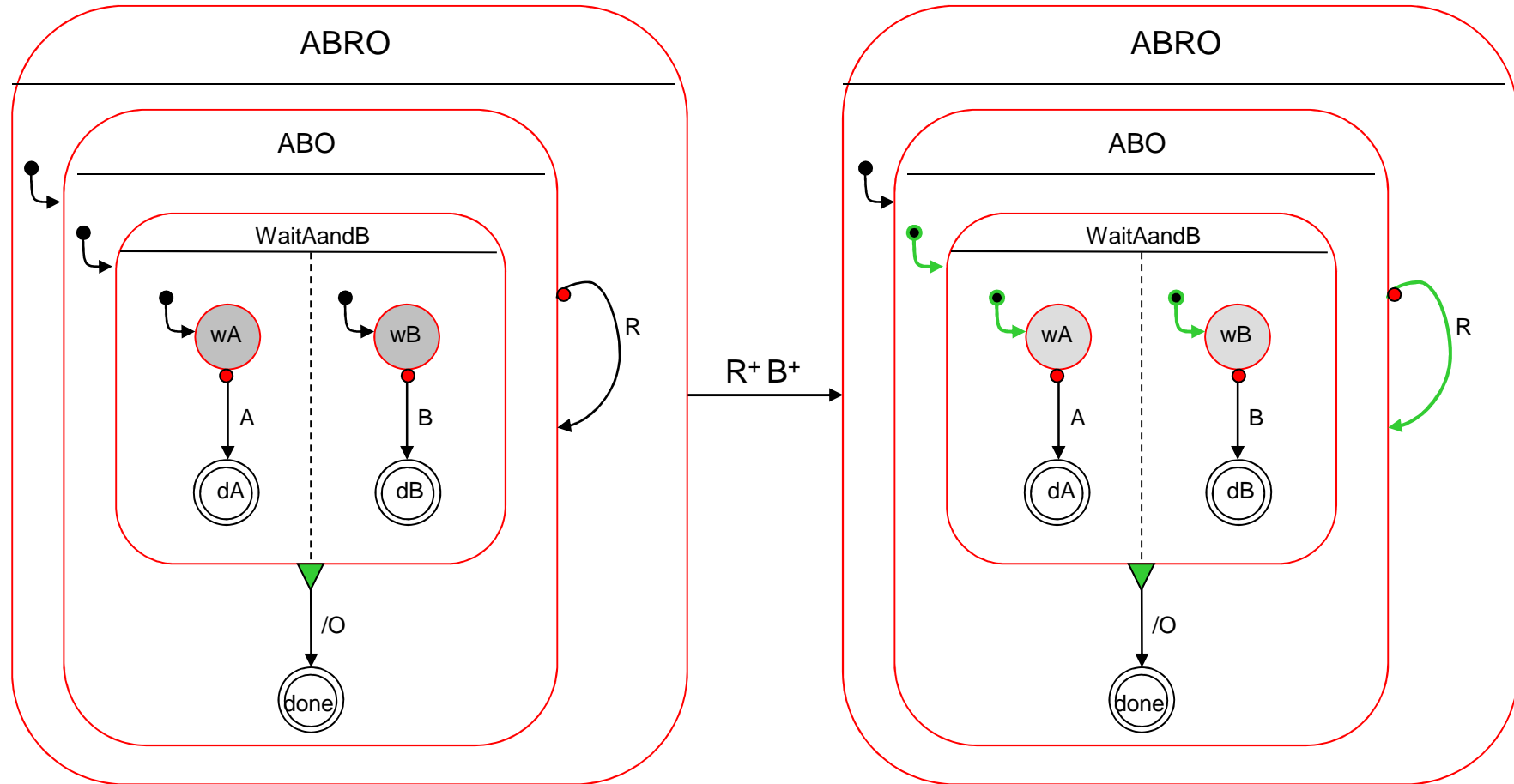


Concurrency and Normal Termination

- When **each** concurrent STG in a macrostate reaches a final state, then the macrostate is **immediately** exited by its **normal termination** transition.

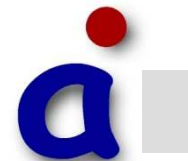


Concurrency and Abort



Transitions

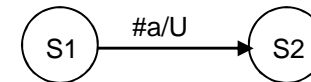
- A strong abort prevents any execution in the preempted state.
- For any state
 - every outgoing transition has a different **priority**
 - any **strong abort** transition has priority over any **weak abort** transition
 - any **weak abort** transition has priority over a **normal termination** transition
- There are no inter-level transitions.



SyncCharts: Advanced Constructs

- Immediate transition

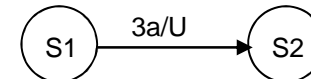
- Syntax: **#**<trigger>/<effect>



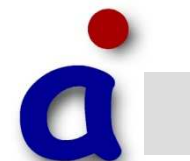
- The trigger may be satisfied as soon as the state is entered: An active state waits for the satisfaction of the trigger of one of its outgoing transitions, at an instant strictly posterior to its entering, **or immediately in case of an immediate transition.**

- Count delays for transitions

- Syntax: <factor><trigger>/<effect>

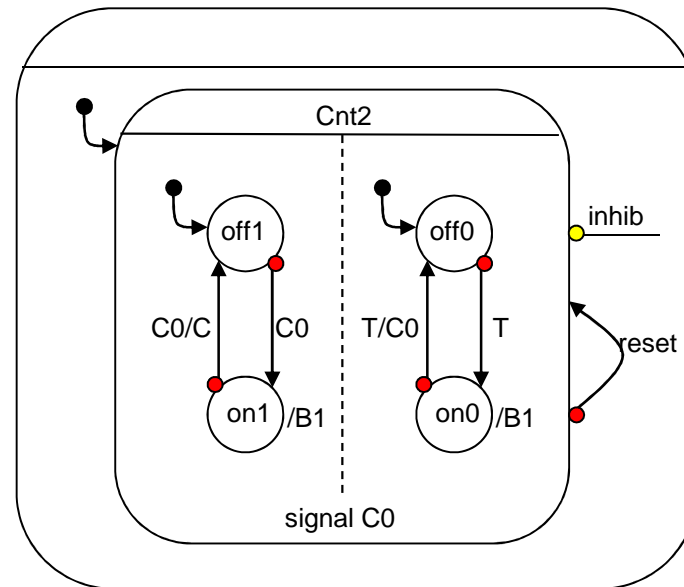


- <factor> is the natural number of instants a transition must be active before it is executed. These active instants need not be consecutive, but the source state (S1) must be active all the time.



Suspension

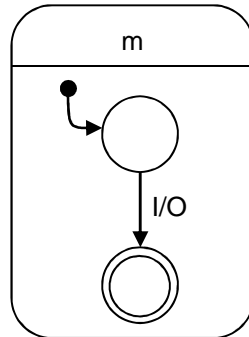
- A suspension is associated with a **trigger**. If the trigger is satisfied the reaction is suspended in the target state: the execution of the preempted state is frozen.
- Notation: $\bullet \xrightarrow{T}$,or: $\textcircled{S} \xrightarrow{T}$
- Note: aborts take priority over suspensions.



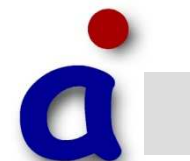
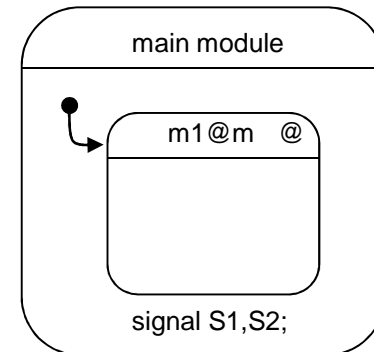
Run Modules

- Macrostates are states that have their own behavior (also called processes). They can be abstracted as **modules**, similarly to procedures in programming languages.
- Modules are instantiated by using **run modules** (corresponding to procedure calls).
- A signal interface renaming has to be defined for run modules.

Module M with interface
input I;
output O;

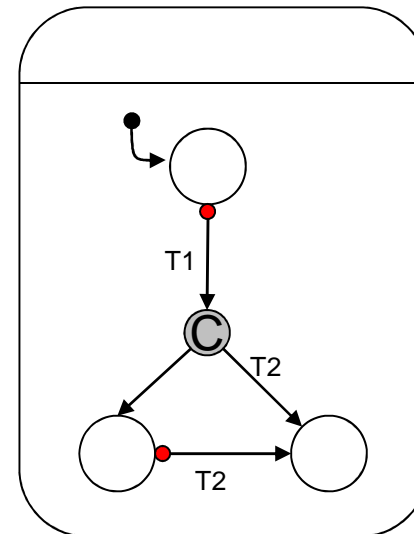
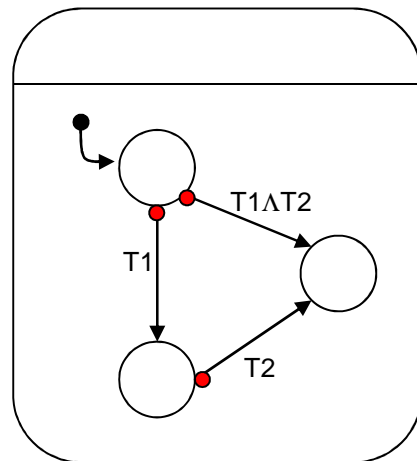


... used as a run module
 with the following signal
 binding:
signal S1 / I;
signal S2 / O;



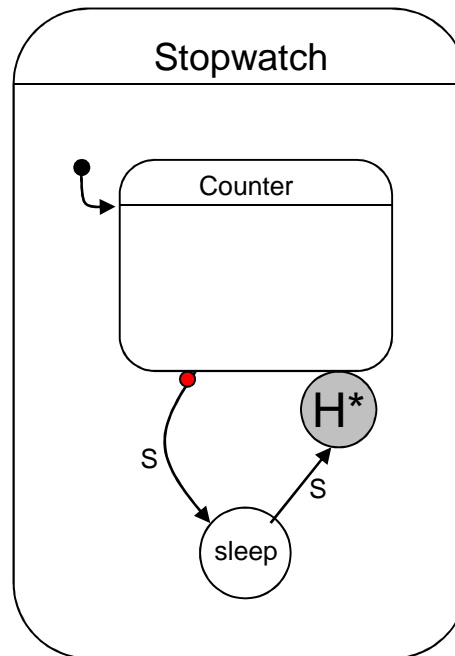
Conditional Connector

- Introduction of conditional pseudostate \odot
- Concise representation of scenarios where a common trigger is shared by several outgoing transitions.
- All departing transitions are immediate.
- One departing "default" transition without condition must be present.

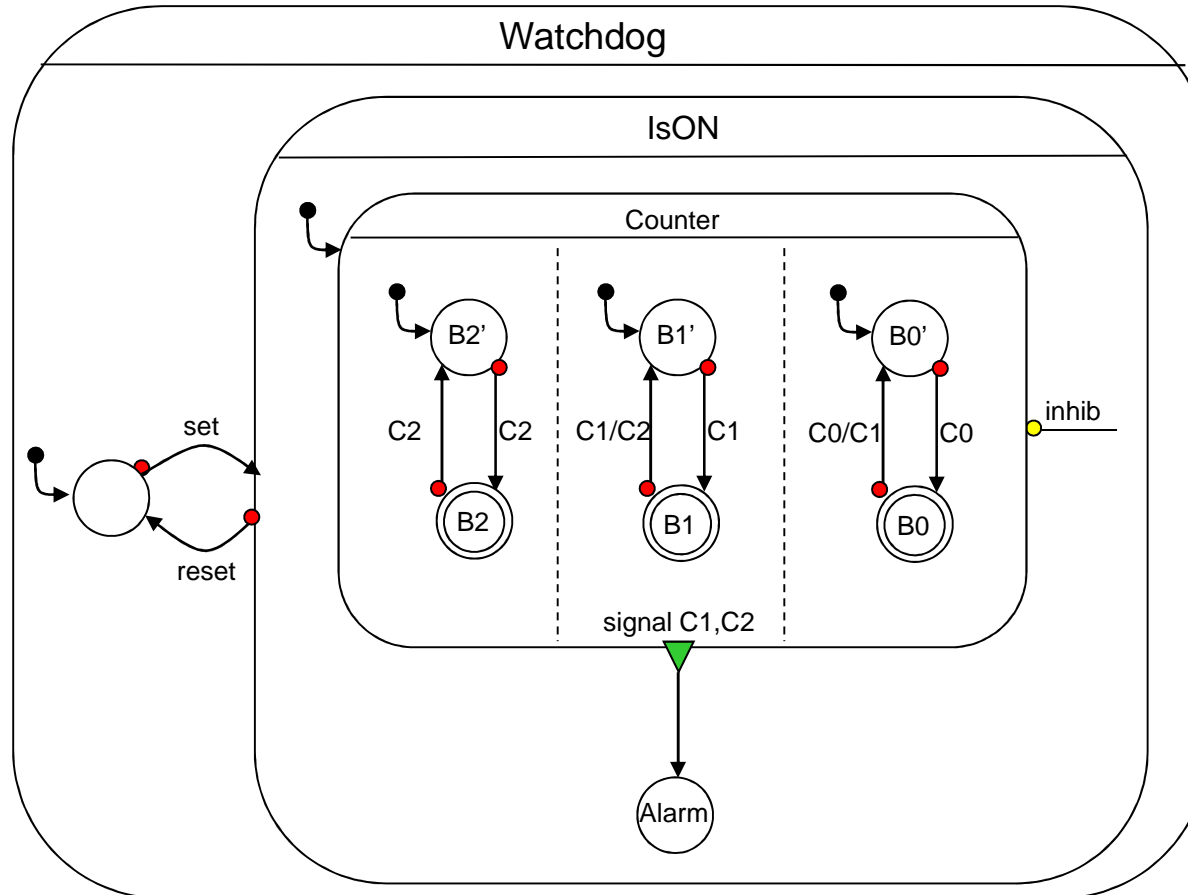


History Connector

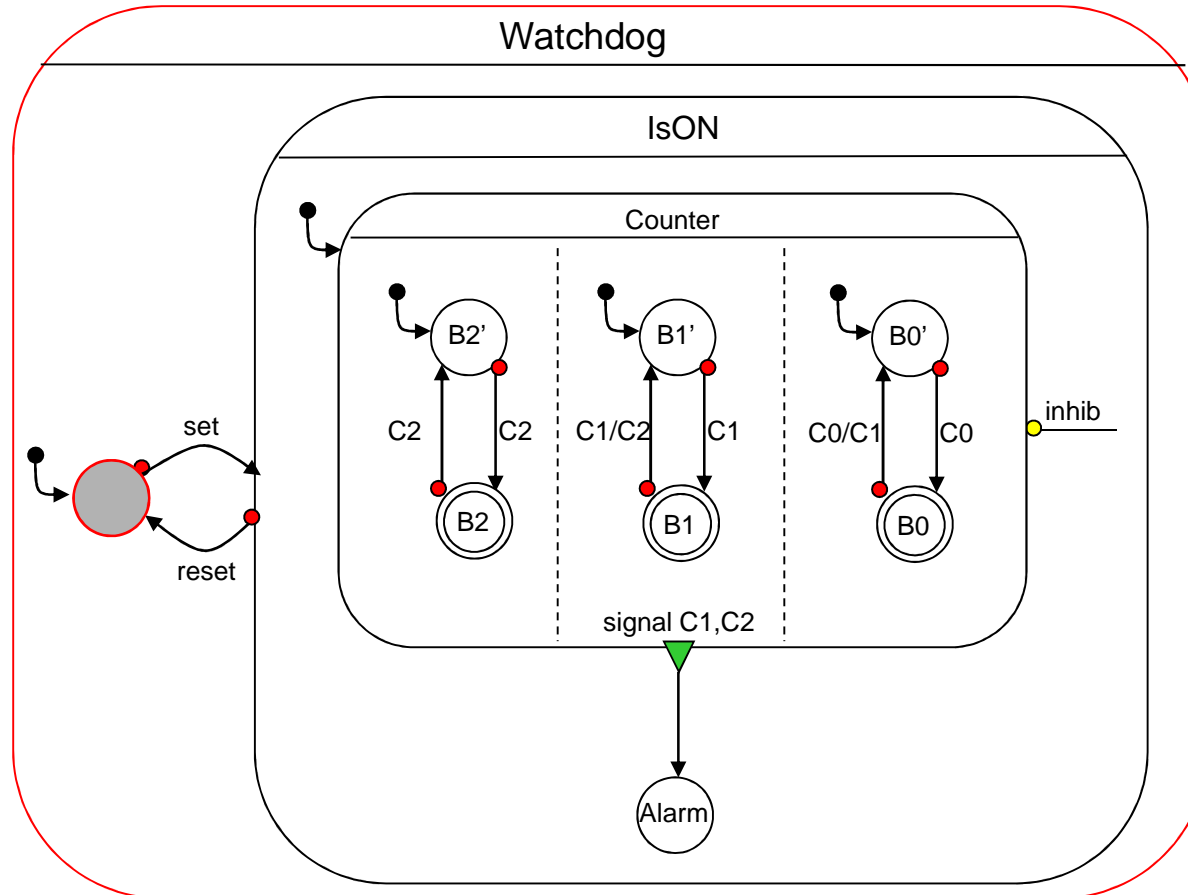
- Directly attached to macrostates
- Only incoming transitions can connect
- The previous state of the macrostate is restored when it is entered through a history connector.



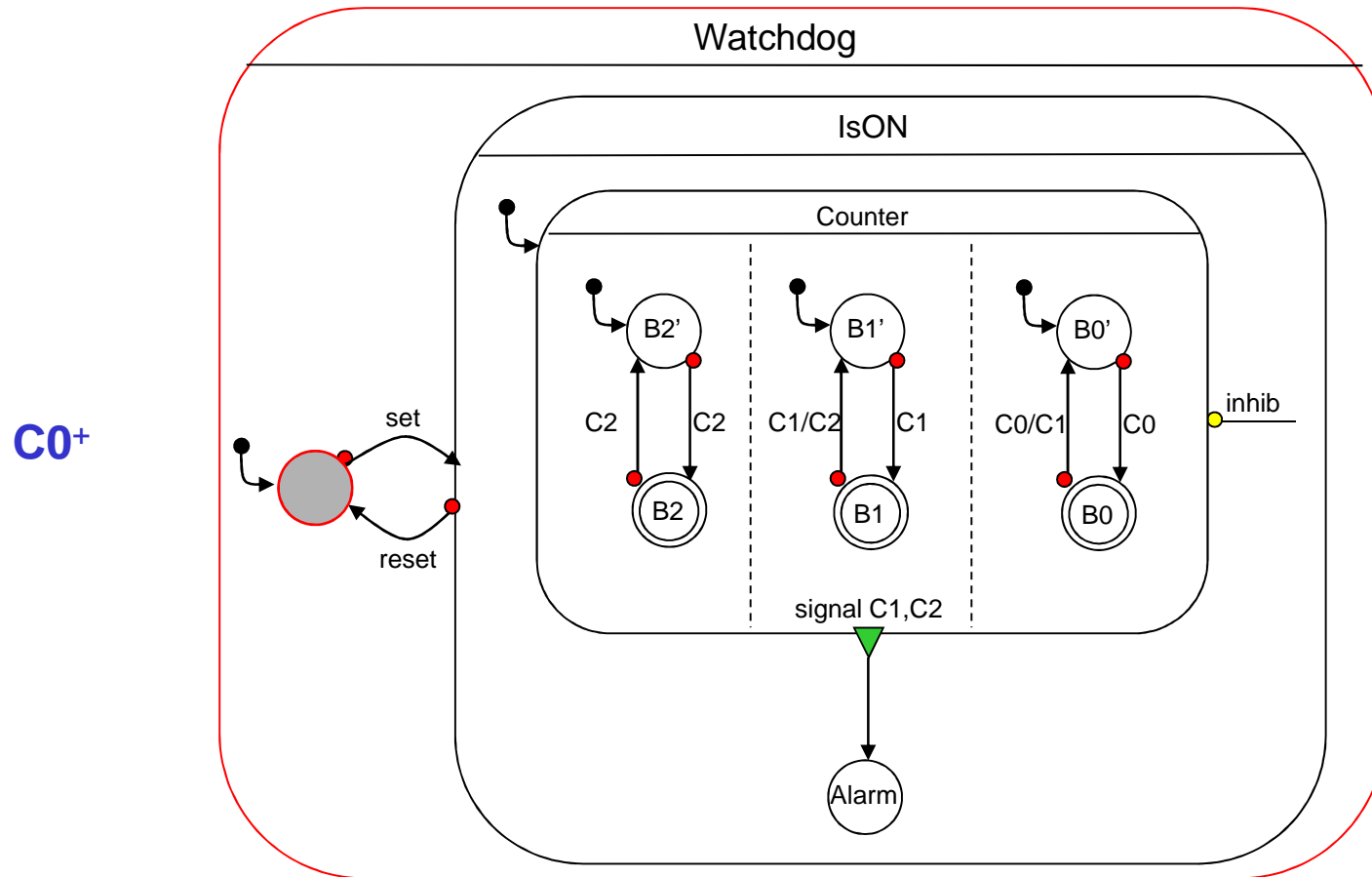
Example: Watchdog



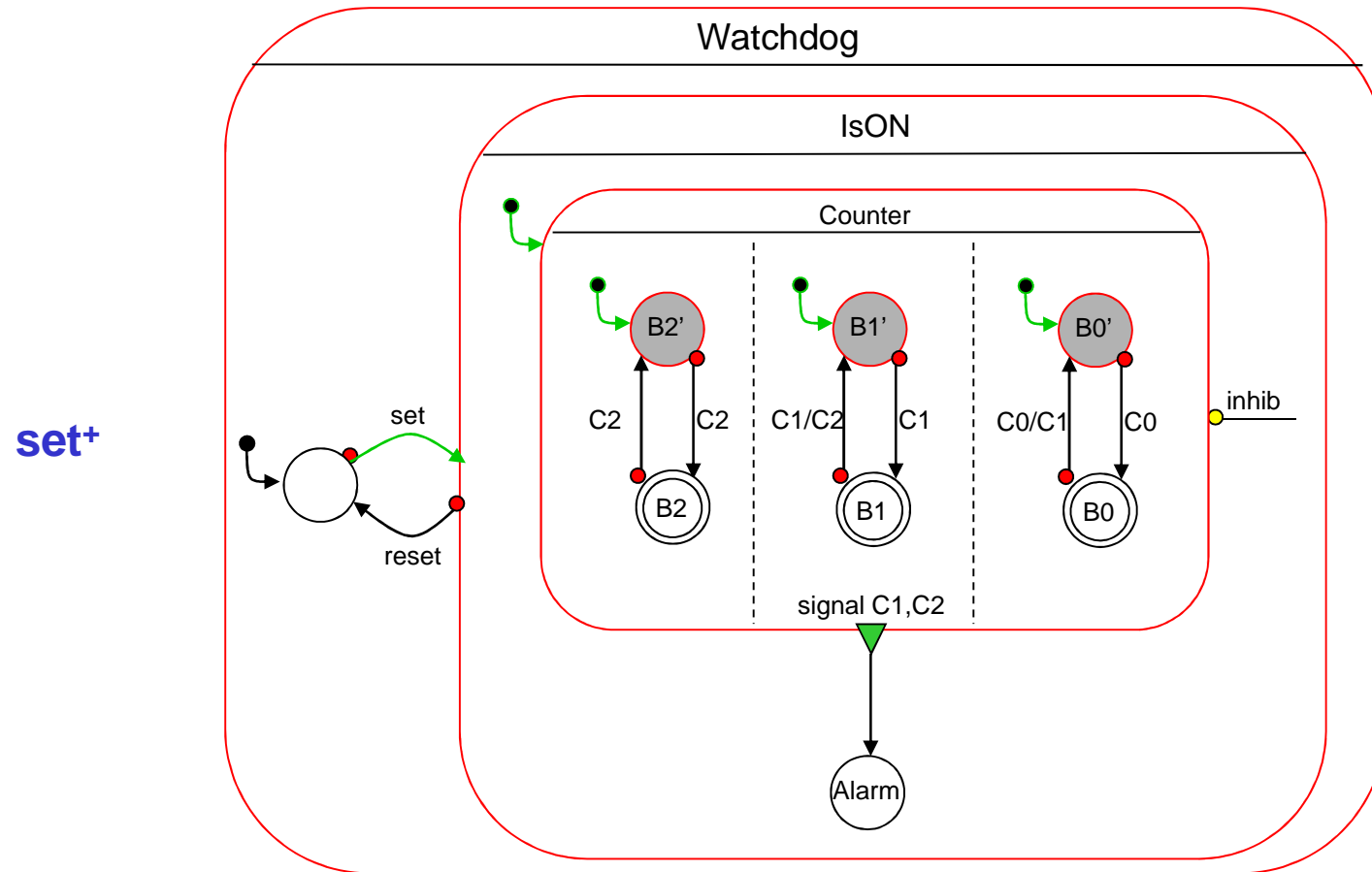
Example: Watchdog



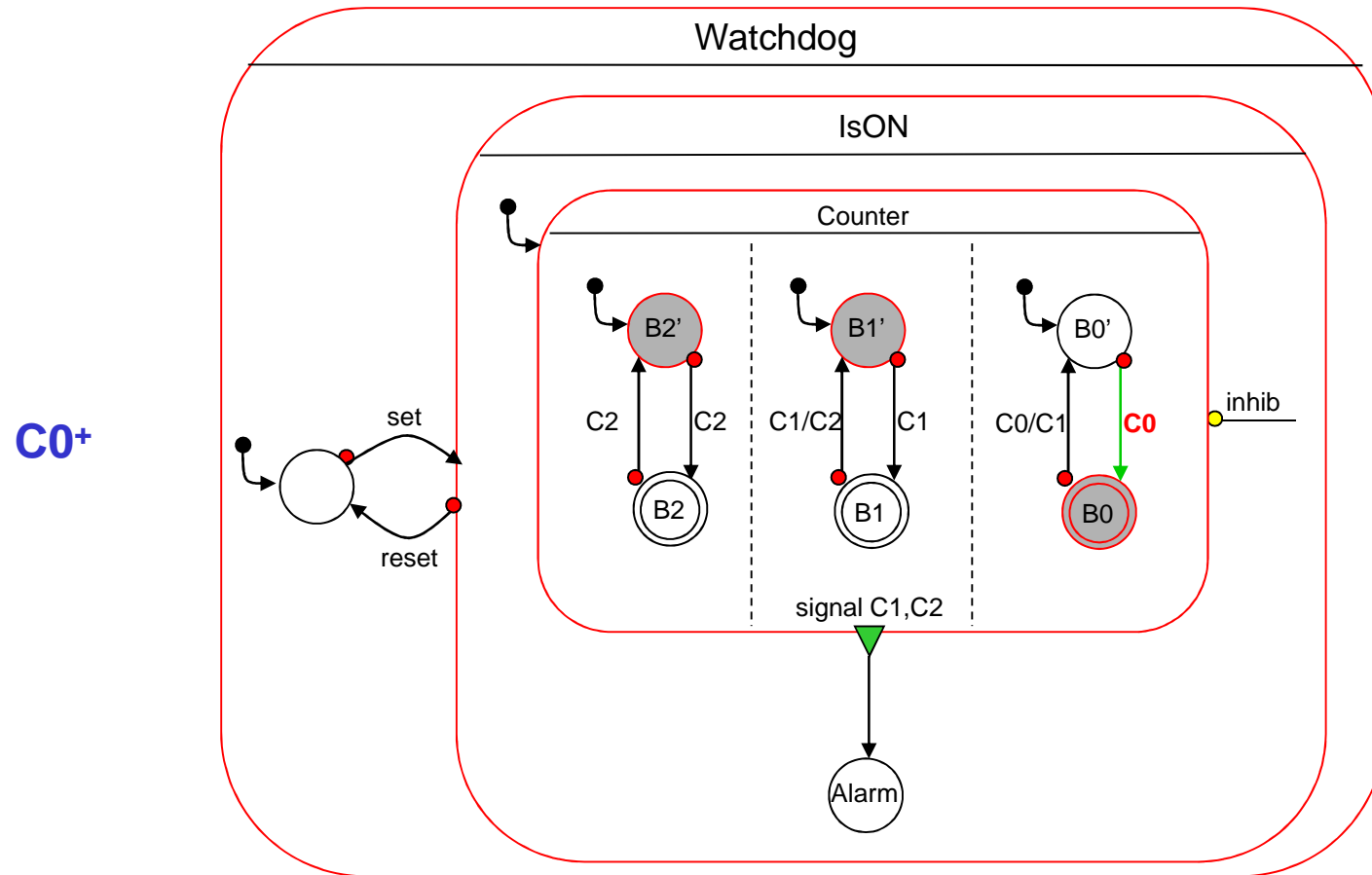
Example: Watchdog



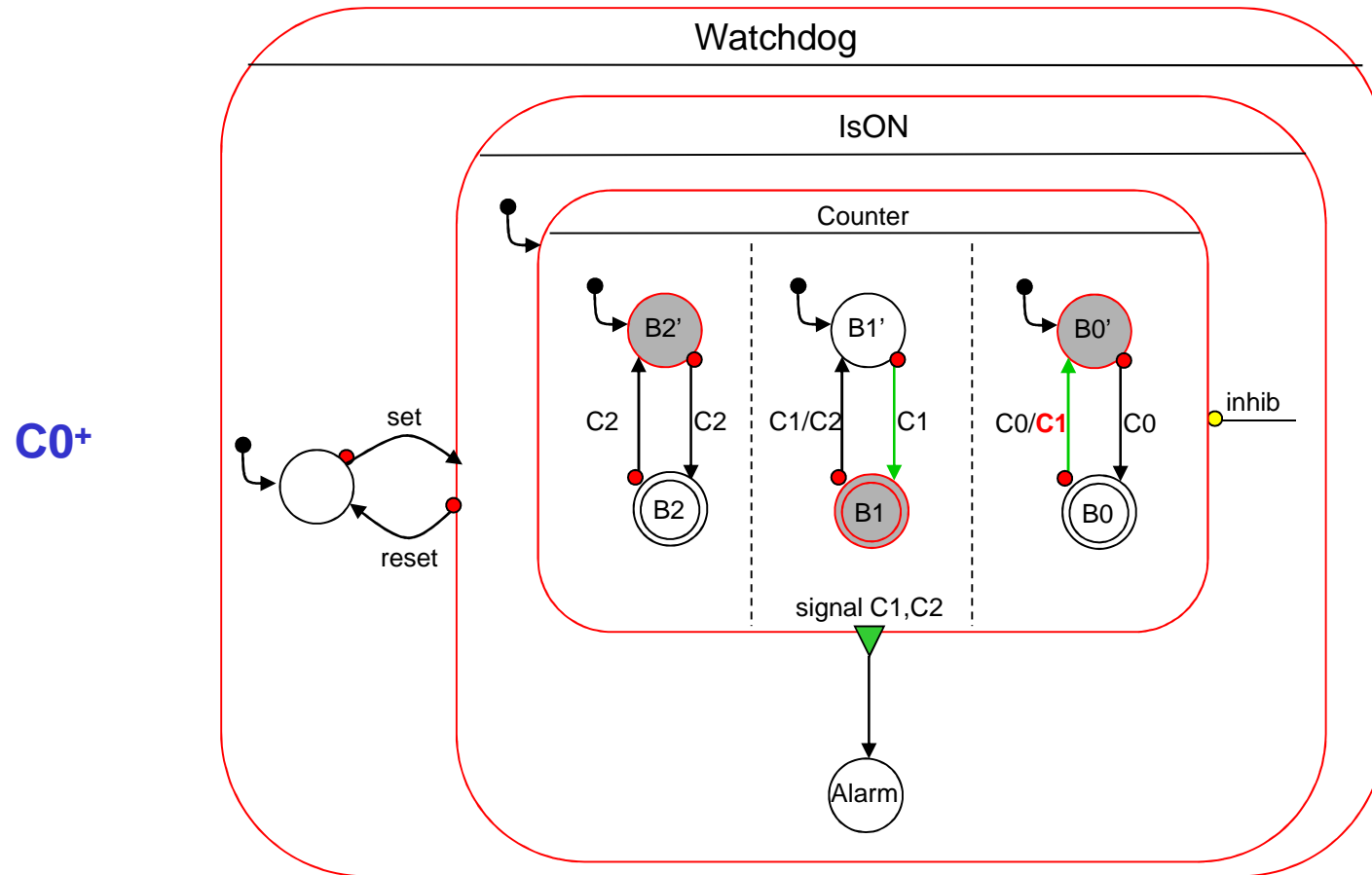
Example: Watchdog



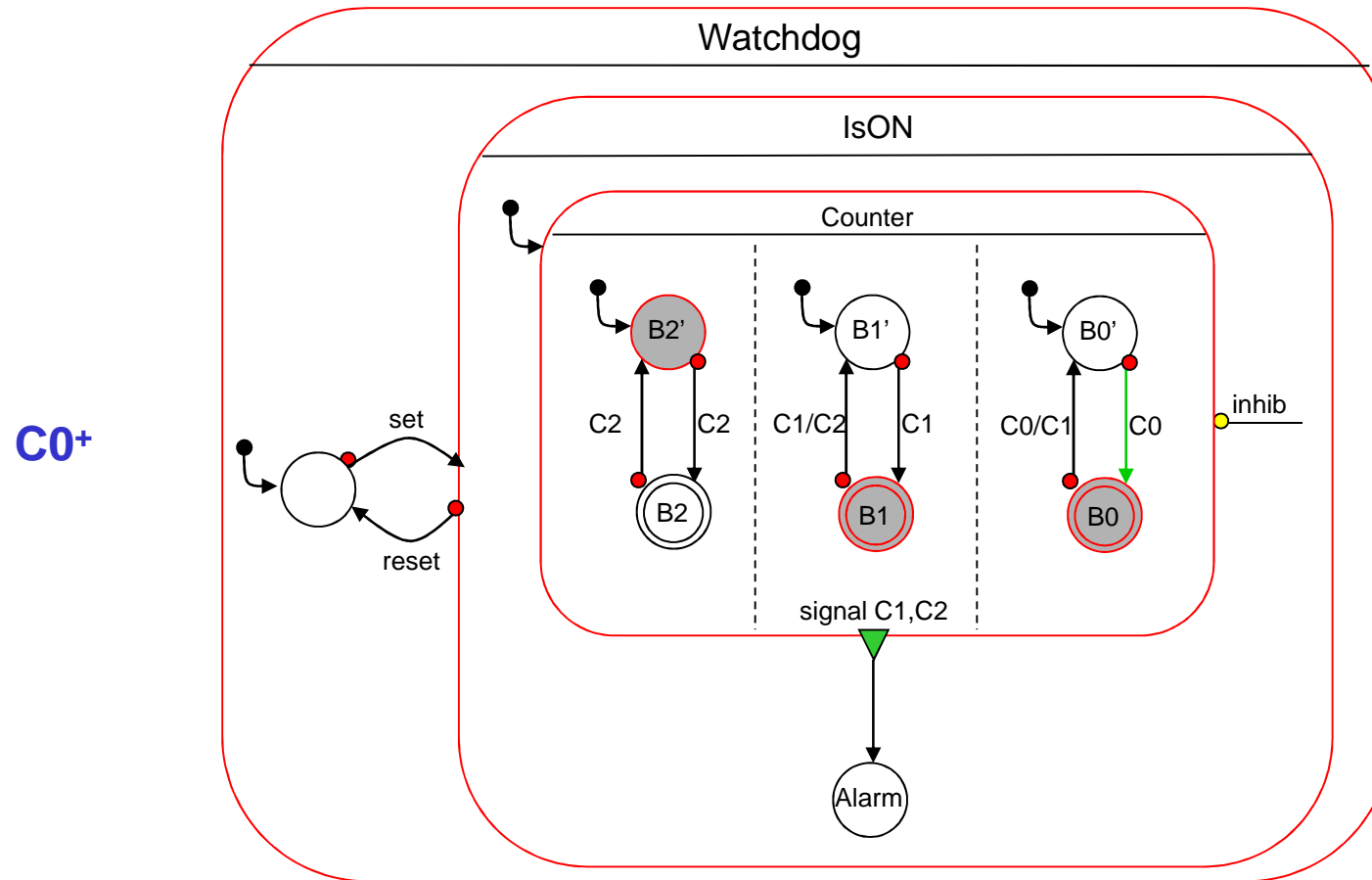
Example: Watchdog



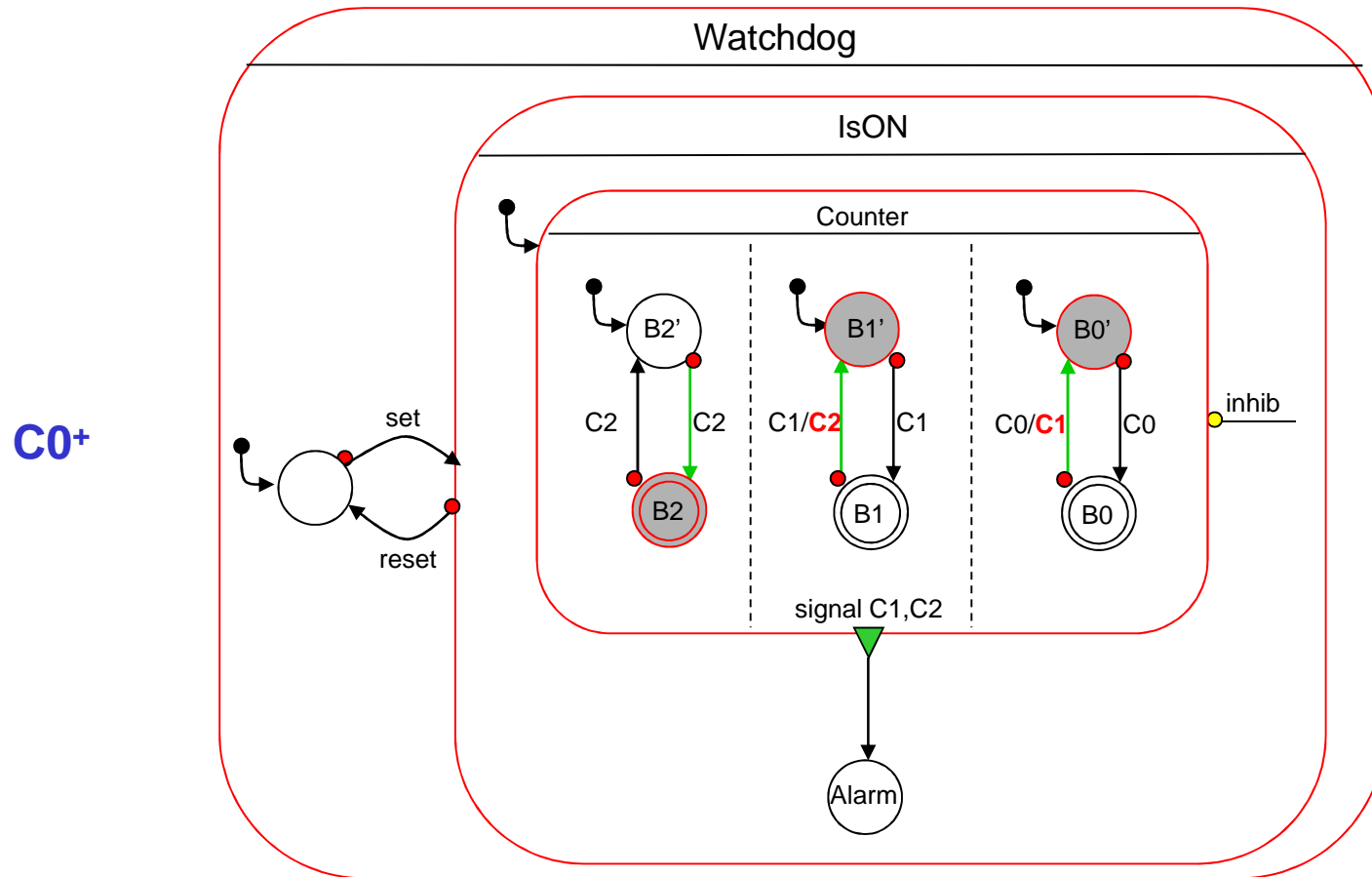
Example: Watchdog



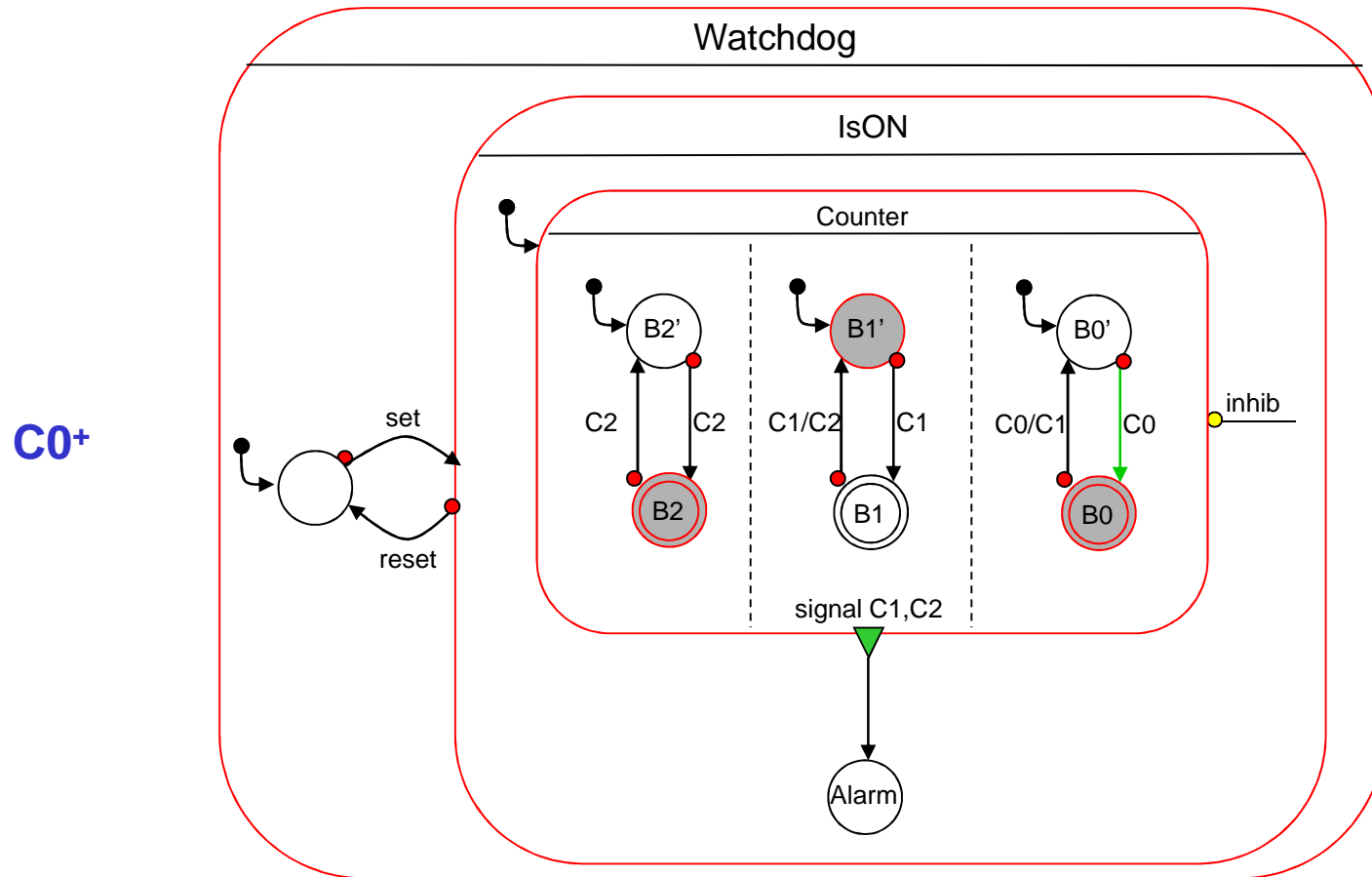
Example: Watchdog



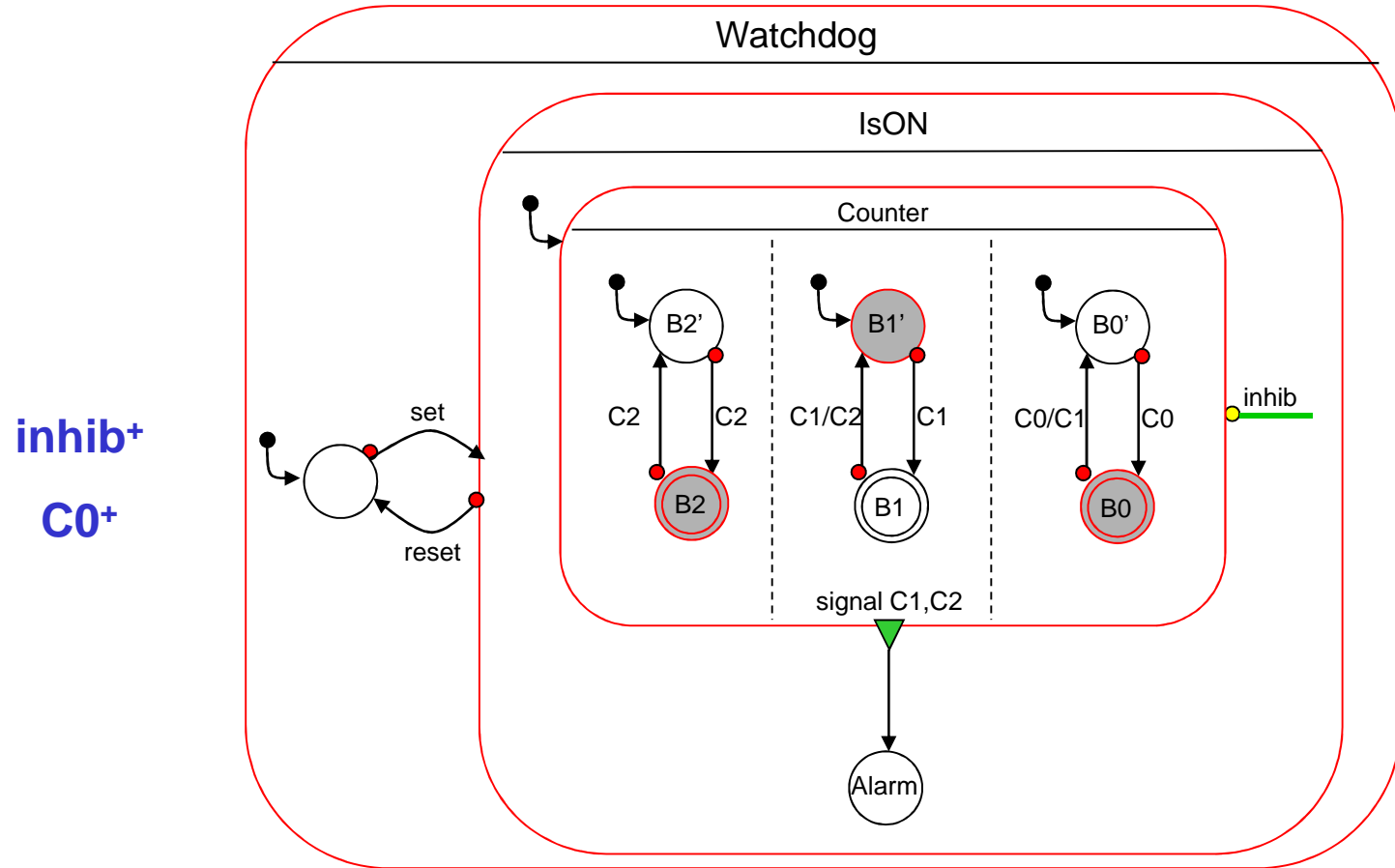
Example: Watchdog



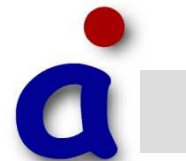
Example: Watchdog



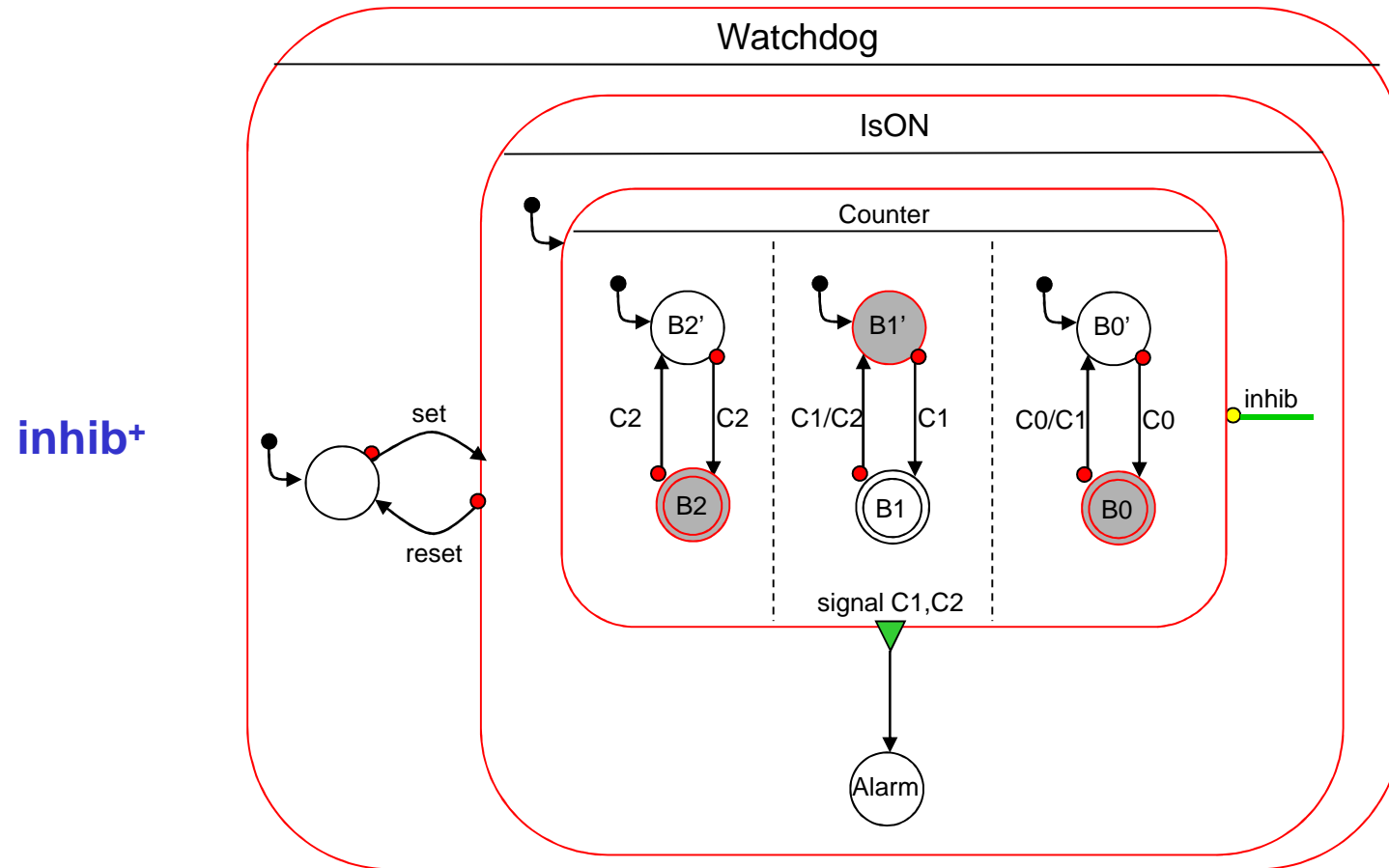
Example: Watchdog



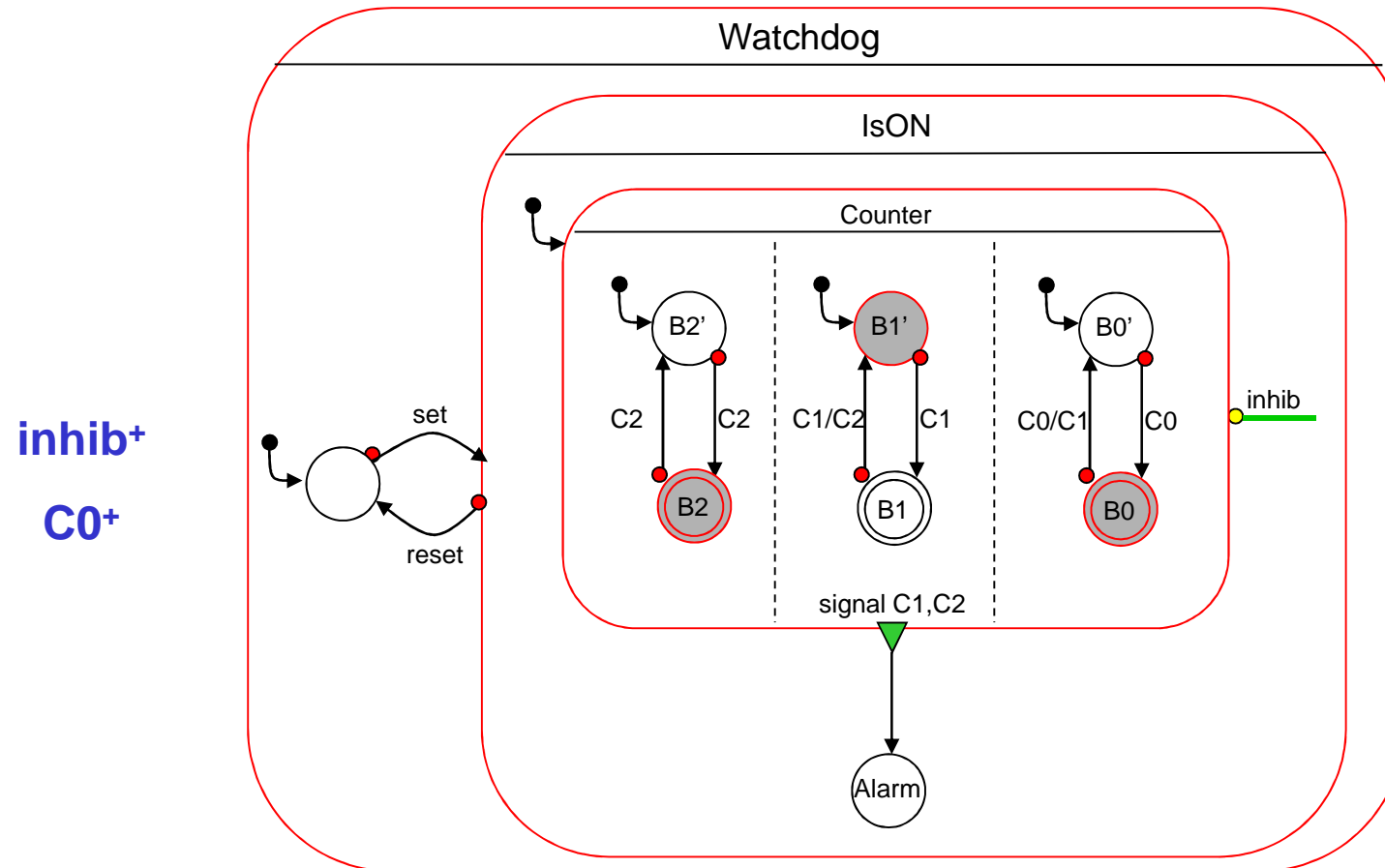
inhib+
C0+



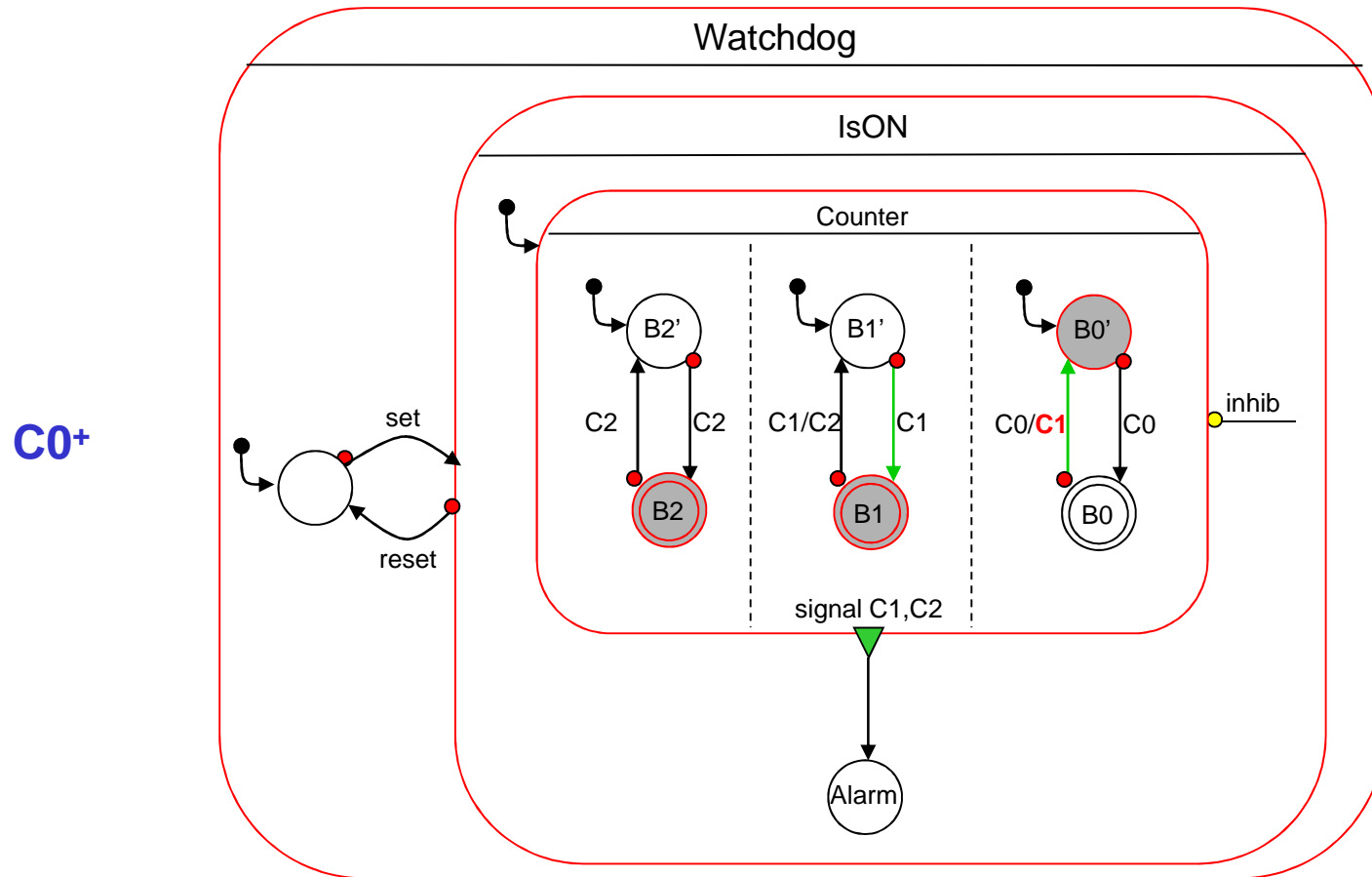
Example: Watchdog



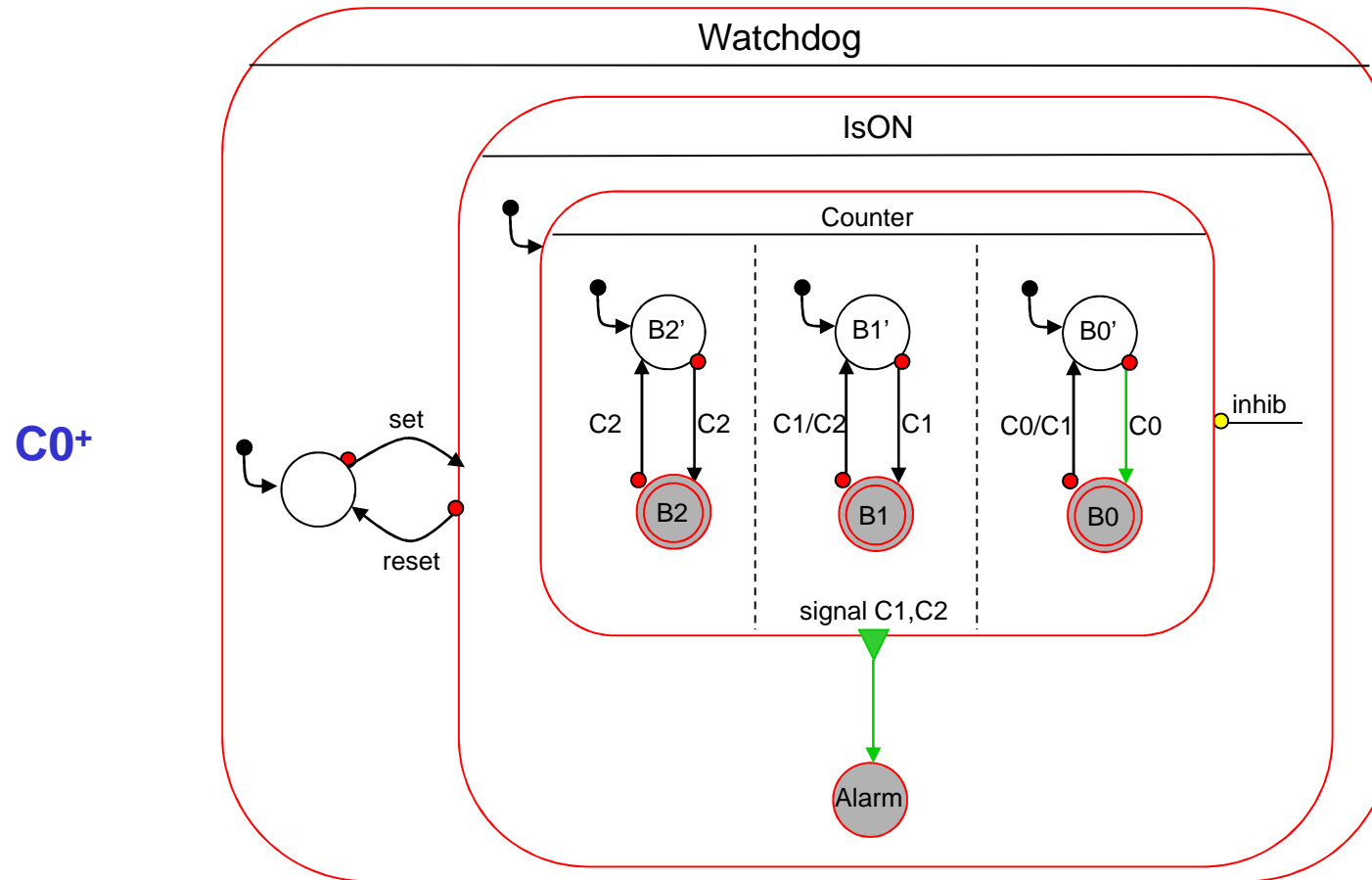
Example: Watchdog



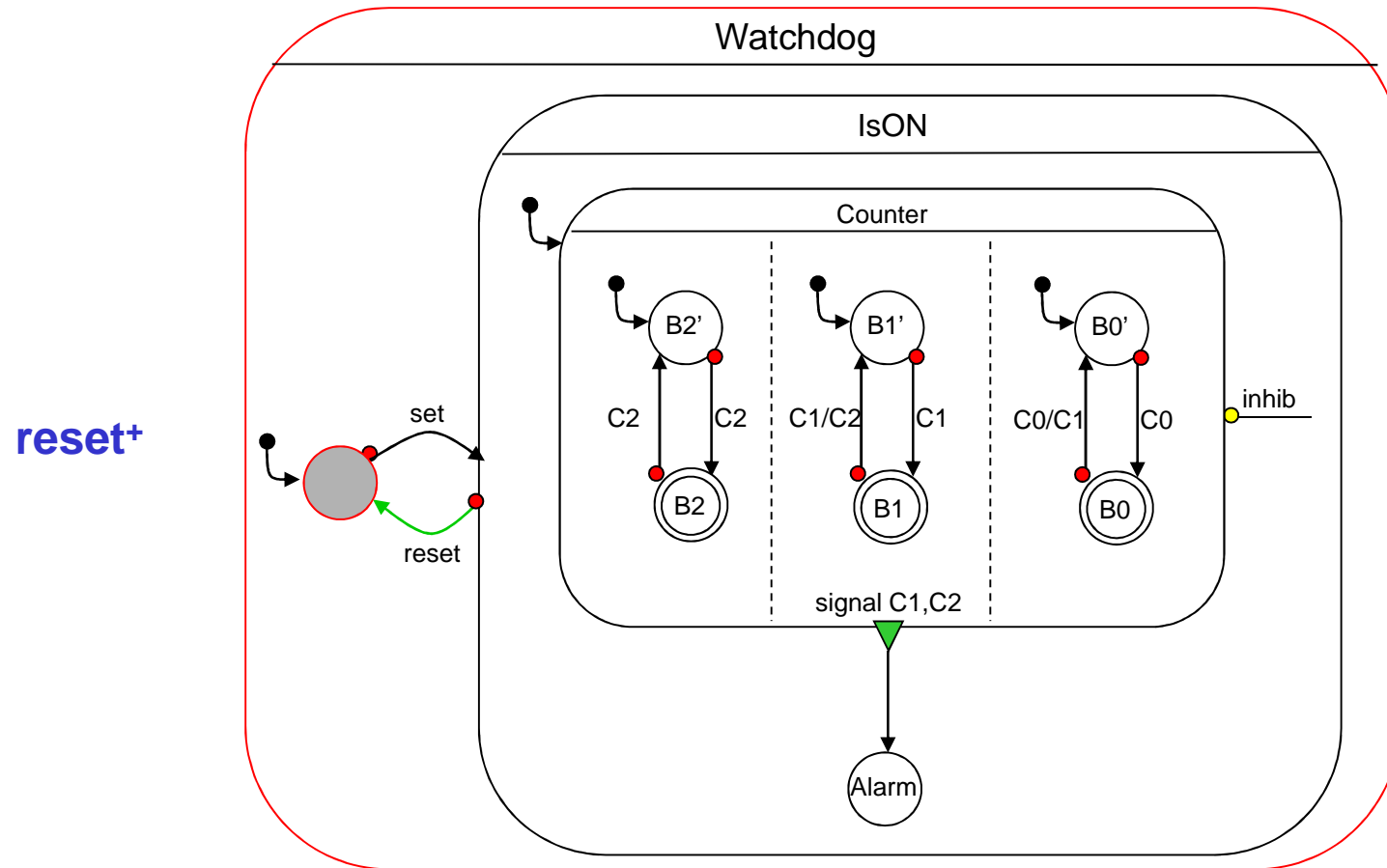
Example: Watchdog



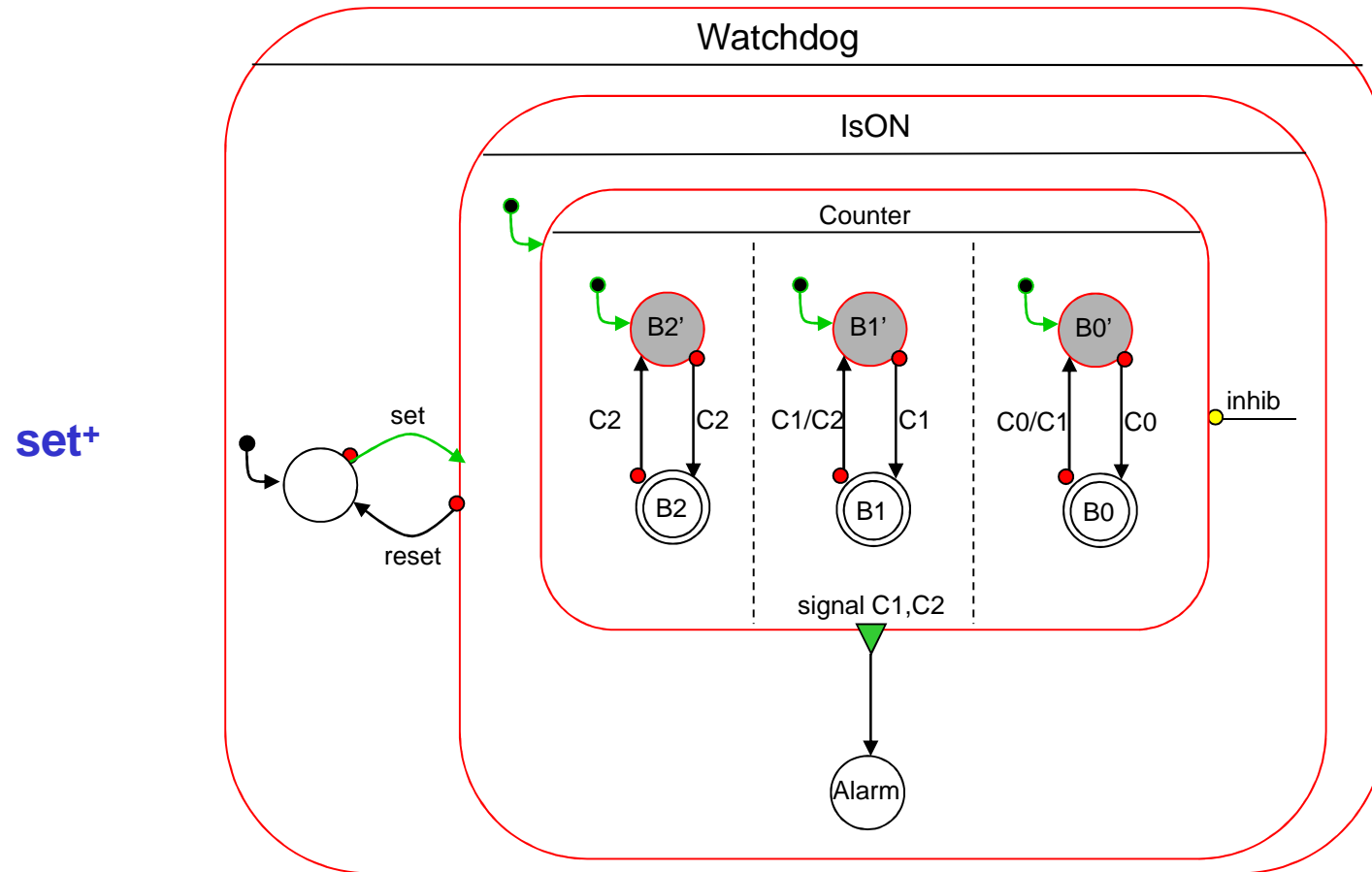
Example: Watchdog



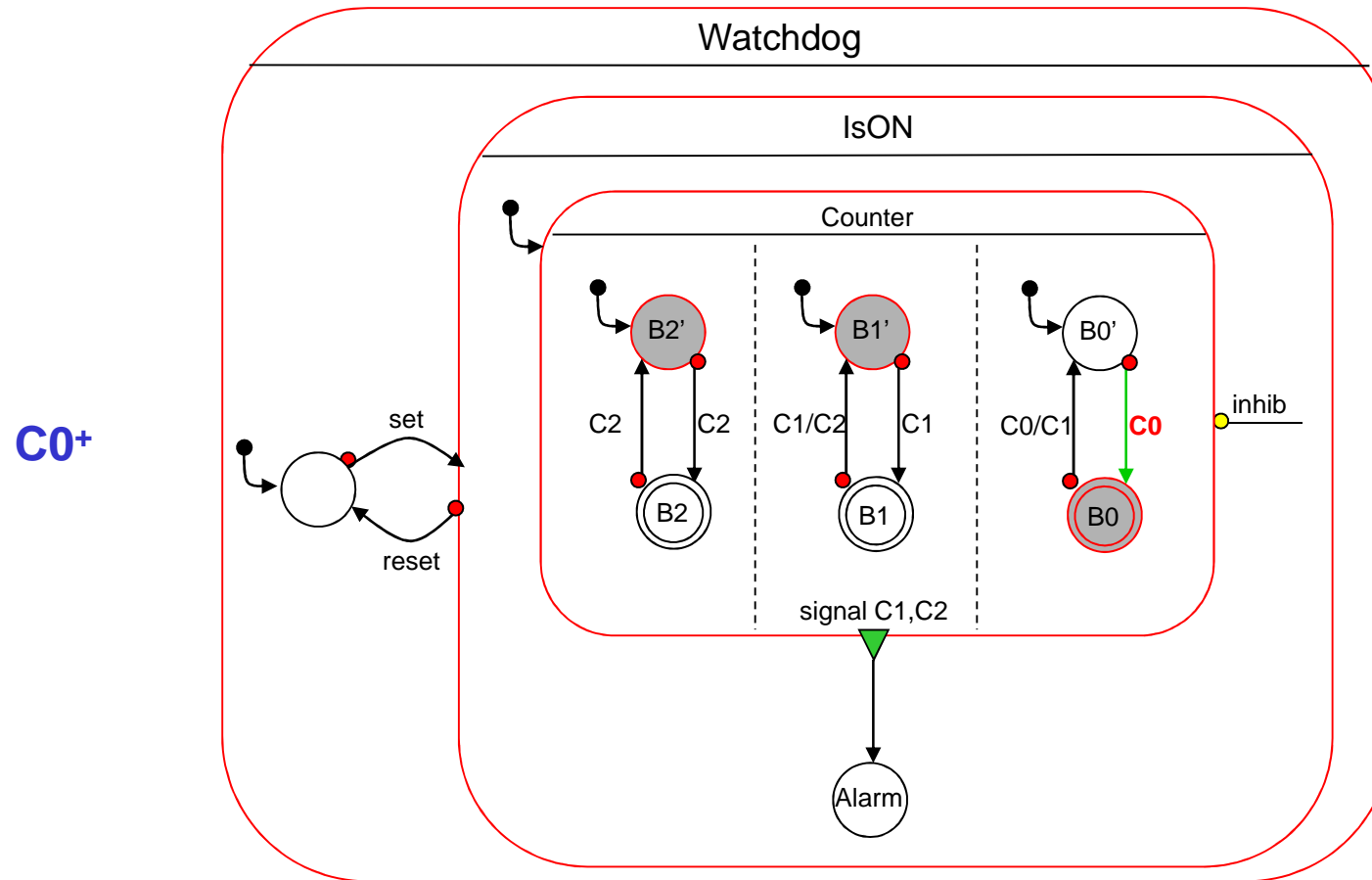
Example: Watchdog



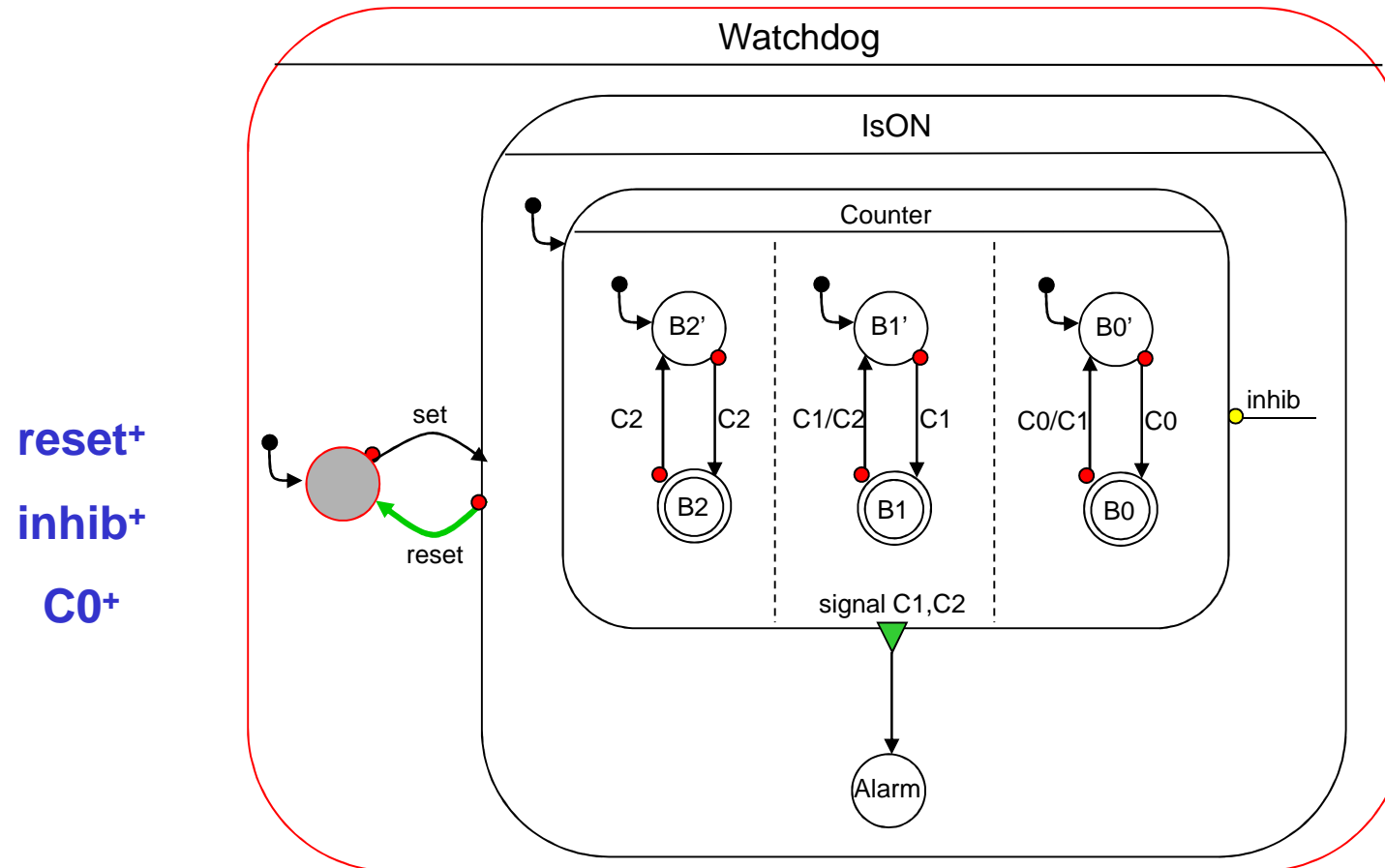
Example: Watchdog



Example: Watchdog

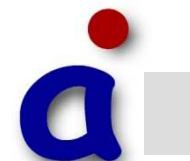


Example: Watchdog

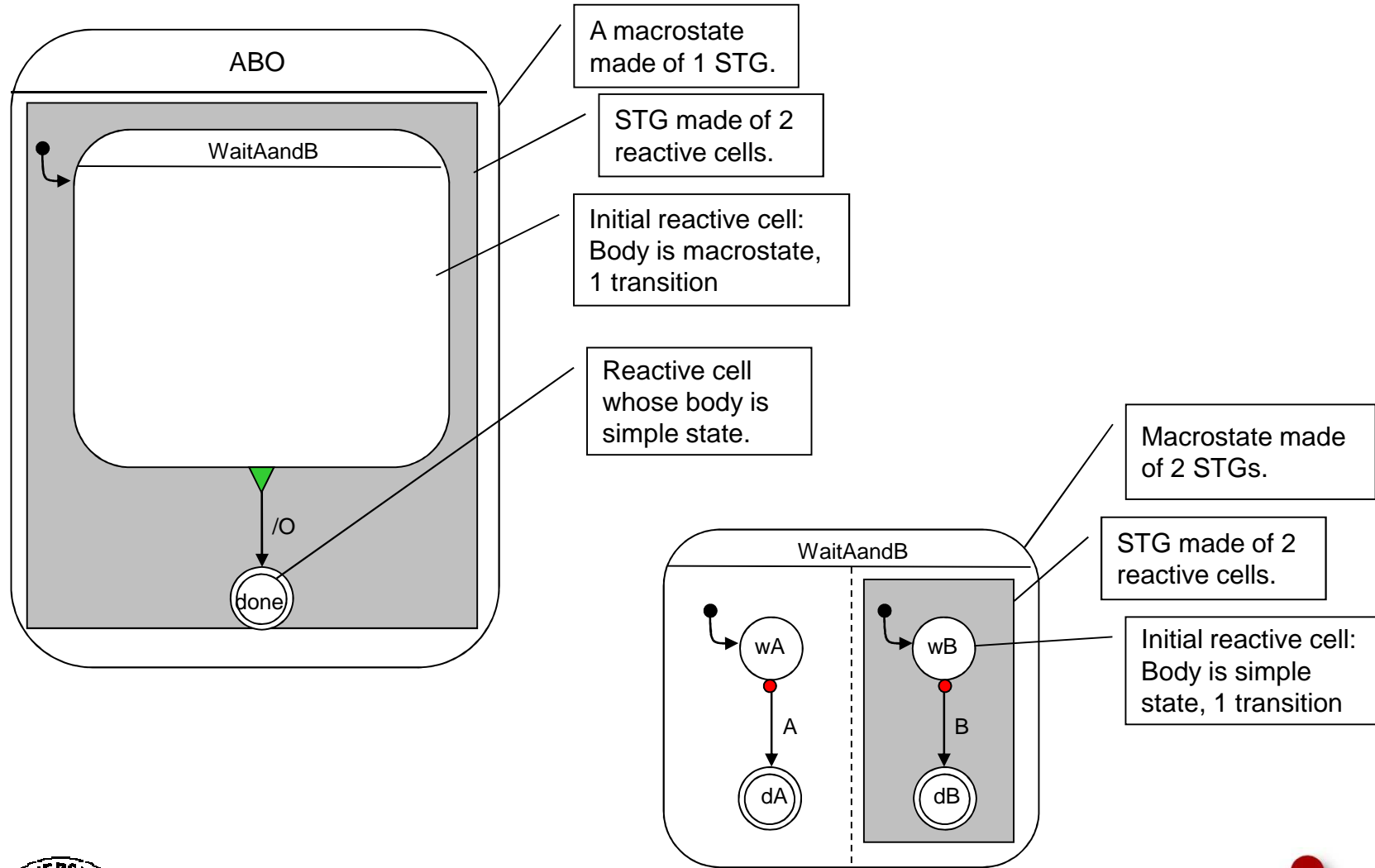


Computing a Reaction – Definitions

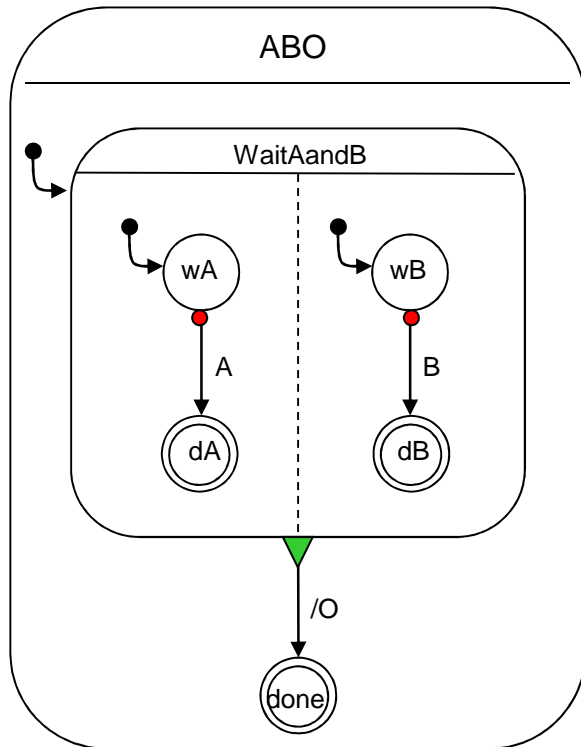
- Each SSM can be represented by unique macrostate, called *Top*, which designates the root of the state containment hierarchy.
- A **state-transition graph** (STG) G is a tuple $G = (S, ini)$ where S is a non-empty set of reactive cells, and ini is the initial reactive cell.
- A **reactive cell** is a tuple $C = (B, R, S)$ such that its body B is a state (simple state or macrostate) and R the set of all its outgoing transitions. The status S of a reactive cell is either *IDLE* or *ACTIVE*.
- A **macrostate** M is a quadruple $M = (G, I, O, L)$ composed from a non-empty set G of STGs, and three possibly empty sets of signals: input signals (I), output signals (O), local signals (L).
- A **transition** has a destination and a label. The destination is a reactive cell, the label is composed of three optional fields: a type, a trigger, an effect. A transition is denoted as a quadruple $R = \langle type, trigger, effect, targetID \rangle$. Feasible types are *sA* (strong abort), *wA* (weak abort), *nT* (normal termination).



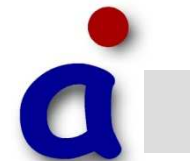
Illustration



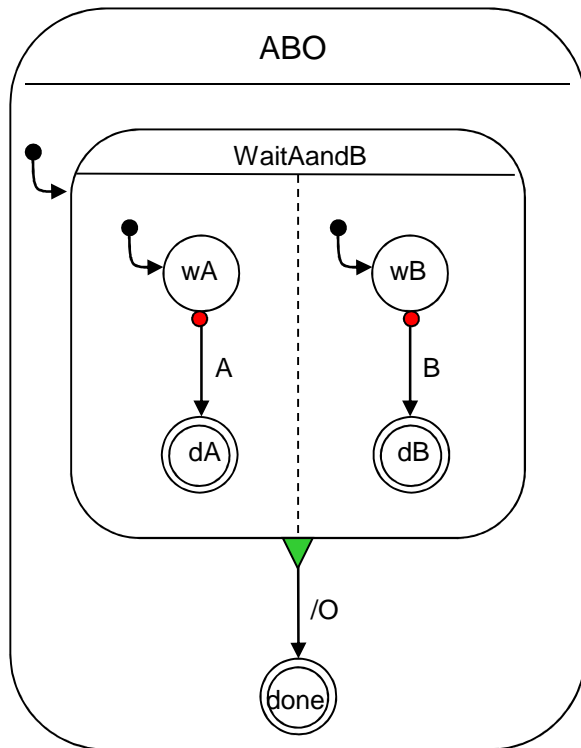
Detailed Example



- Macrostate $ABO=TOP$:
 - $M_{ABO}.G = \{ABO.g\}$
 - $M_{ABO}.I = \{A, B\}$
 - $M_{ABO}.O = \{O\}$
 - $M_{ABO}.L = \emptyset$
- State-Transition Graph $ABO.g$:
 - $ABO.g.S = \{RC_{WaitAandB}, RC_{done}\}$
 - $ABO.g.ini = RC_{WaitAandB}$
- Reactive cell $RC_{WaitAandB}$
 - Body: Macrostate $M_{WaitAandB}$
 - $RC_{WaitAandB}.out = \{<nT,, O, RC_{done}>\}$
- Macrostate $M_{WaitAandB}$:
 - $M_{WaitAandB}.G = \{WaitAandB.g_1, WaitAandB.g_2\}$
 - $M_{WaitAandB}.I = \{A, B\}$
 - $M_{WaitAandB}.O = \emptyset$
 - $M_{WaitAandB}.L = \emptyset$
- Reactive cell RC_{done} :
 - Body: Simple state S_{done}
 - $RC_{done}.out = \emptyset$



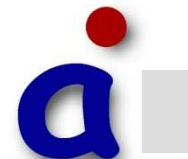
Detailed Example



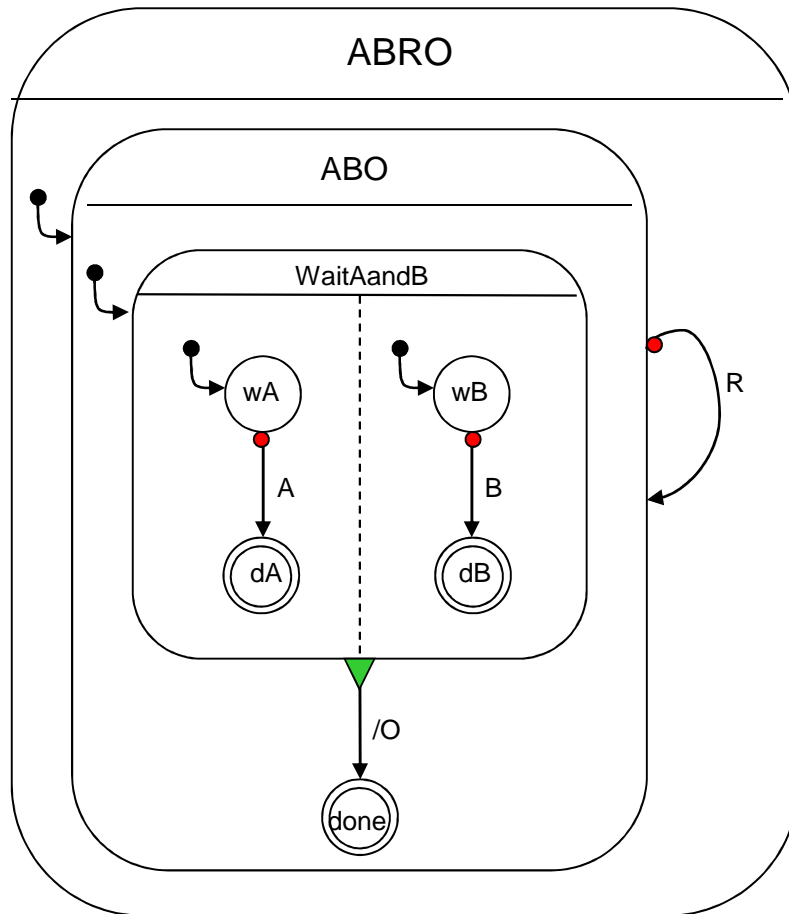
- State-Transition Graph WaitAandB.g₁
 - WaitAandB.g₁.S={RC_{wA},RC_{dA}}
 - WaitAandB.g₁.ini=RC_{wA}
- State-Transition Graph WaitAandB.g₂
 - WaitAandB.g₂.S={RC_{wB},RC_{dB}}
 - WaitAandB.g₂.ini=RC_{wB}
- Reactive cell RC_{wA}
 - Body: simple state S_{wA}
 - RC_{wA}.out = {<sA,A,,dA>}
- Reactive cell RC_{dA}
 - Body: simple state S_{dA}
 - RC_{dA}.out = ∅
- Reactive cell RC_{wB}
 - Body: simple state S_{wB}
 - RC_{wB}.out = {<sA,B,,dB>}
- Reactive cell RC_{dB}
 - Body: simple state S_{dB}
 - RC_{dB}.out = ∅

Configurations

- A **configuration** is a maximal set of states (macrostates or simple states) the system could be in simultaneously. (Note that formally status is associated with reactive cells.)
- Let T be the top macrostate associated with an SSM. A **legal configuration** C for T must satisfy the following rules:
 1. T in C
 2. If a macrostate M is in C , then C must also contain for each STG G directly contained in M exactly one state directly contained in G .
 3. C is maximal and contains only states satisfying rules 1 and 2.
- A **stable configuration** is a legal configuration that the SSM can reach after a sequence of reactions. Only the stable configurations are of interest for the user.



Example

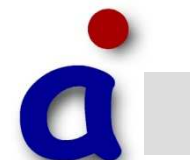


- Legal configurations:
 - $\{ABRO, ABO, done\}$
 - $\{ABRO, ABO, WaitAandB, wA, wB\}$
 - $\{ABRO, ABO, WaitAandB, wA, dB\}$
 - $\{ABRO, ABO, WaitAandB, dA, wB\}$
 - $\{ABRO, ABO, WaitAandB, dA, dB\}$

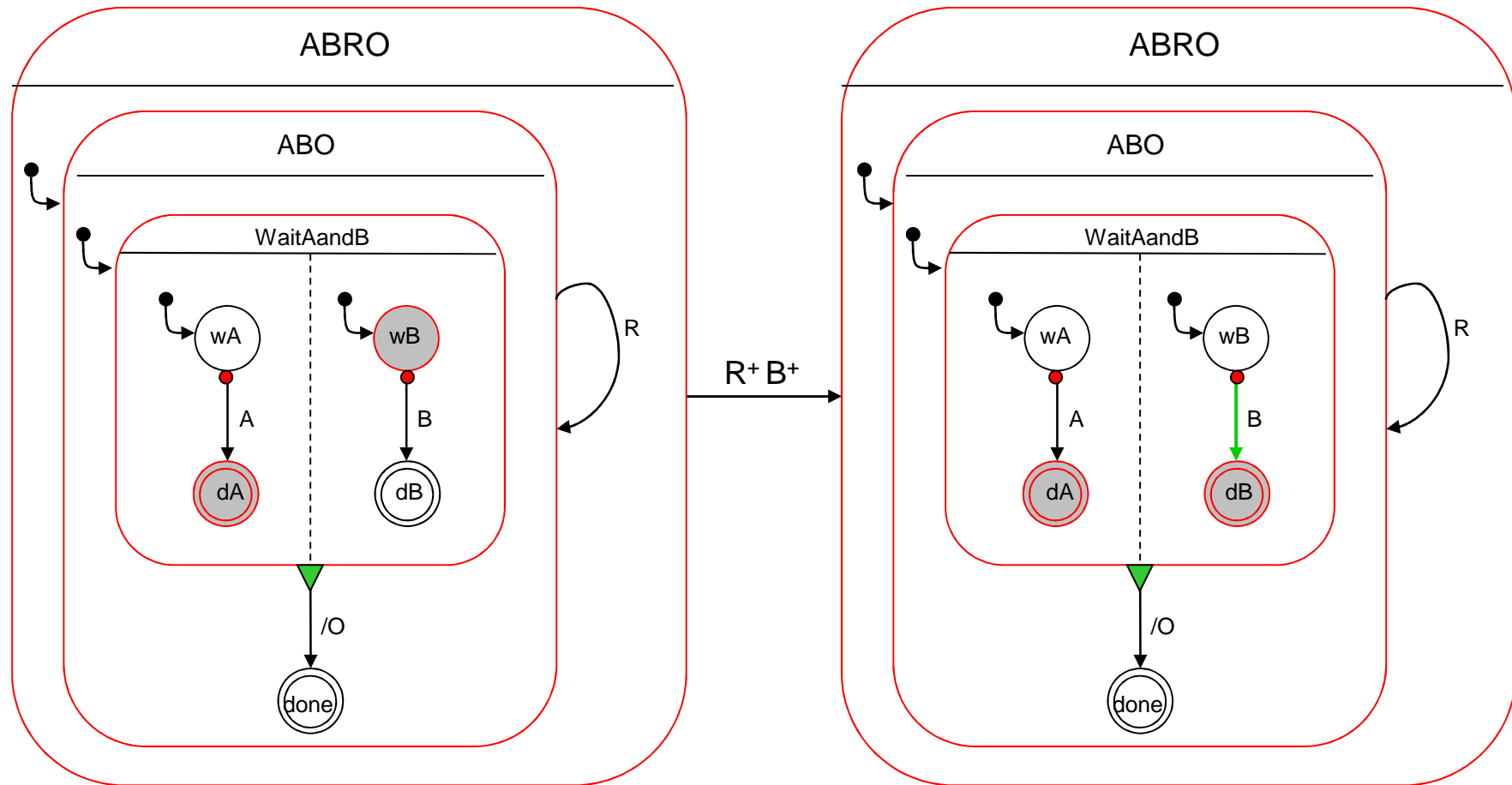
- Stable configurations:
 - all legal configurations except $\{ABRO, ABO, WaitAandB, dA, dB\}$

Computing a reaction

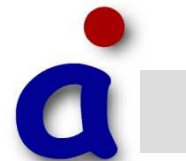
- Computing a reaction is done by **concurrent threads** which suspend their execution when a trigger cannot be evaluated and can resume when new signal statuses are broadcast.
- Reactions are computed as a **sequence of microsteps**, all executed during the same instant but in the order that respects **causality**.
- A transition is taken (ie a **microstep** is executed) only when its trigger is **surely satisfied** (no possibility of backtracking).
- Termination codes of components (reactive cell, STG, macrostate, simple state):
 - **DONE**: execution has been terminated, but can go on in same instant
 - **PAUSE**: nothing left to do until next instant
 - **DEAD**: nothing left to do at the current instant and in the future (final state); component is candidate to join a normal termination.
 - Partial order: DEAD < PAUSE



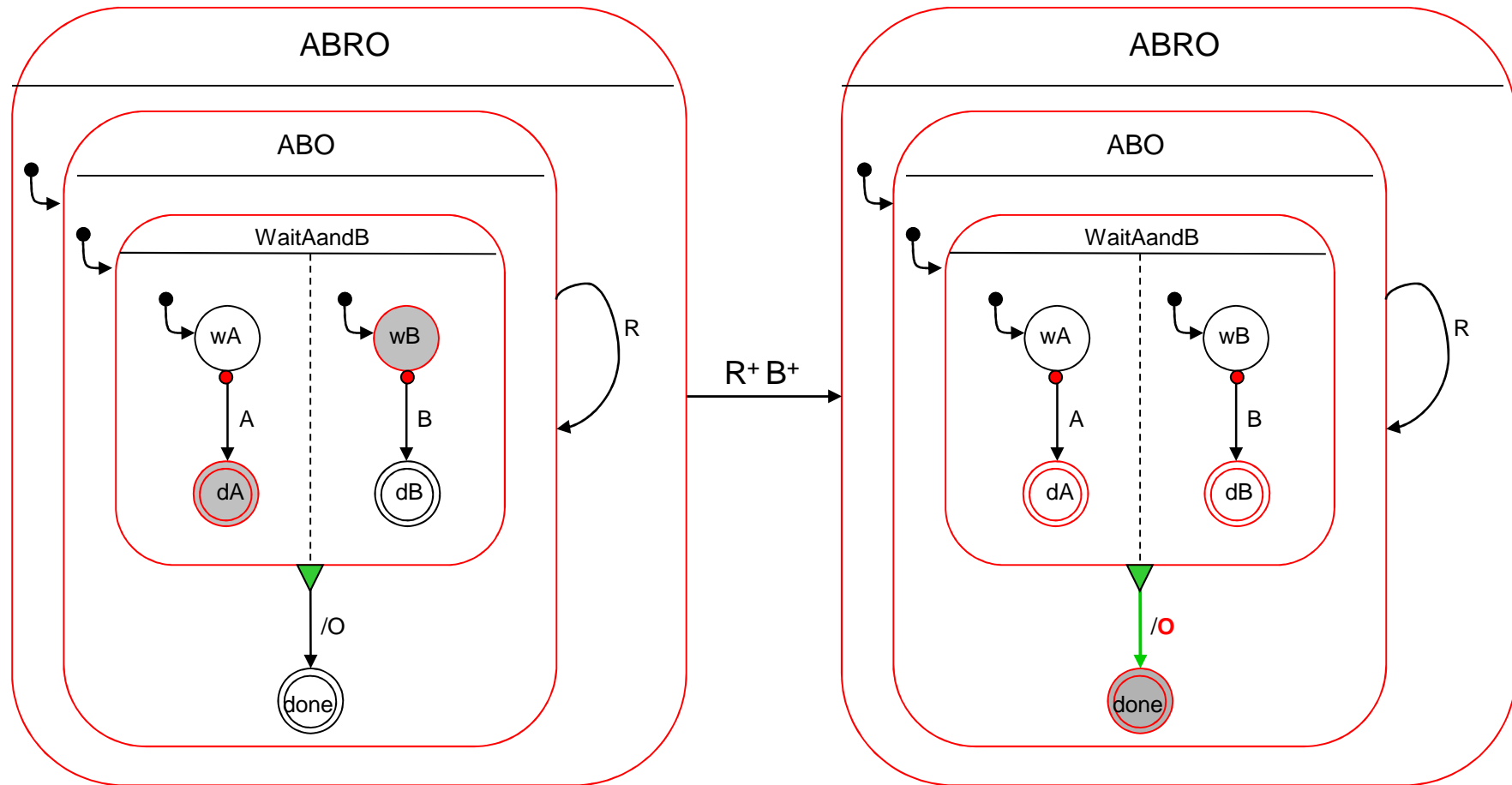
Concurrency and Weak Abort



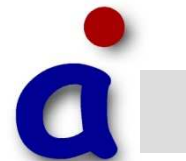
Microstep 1



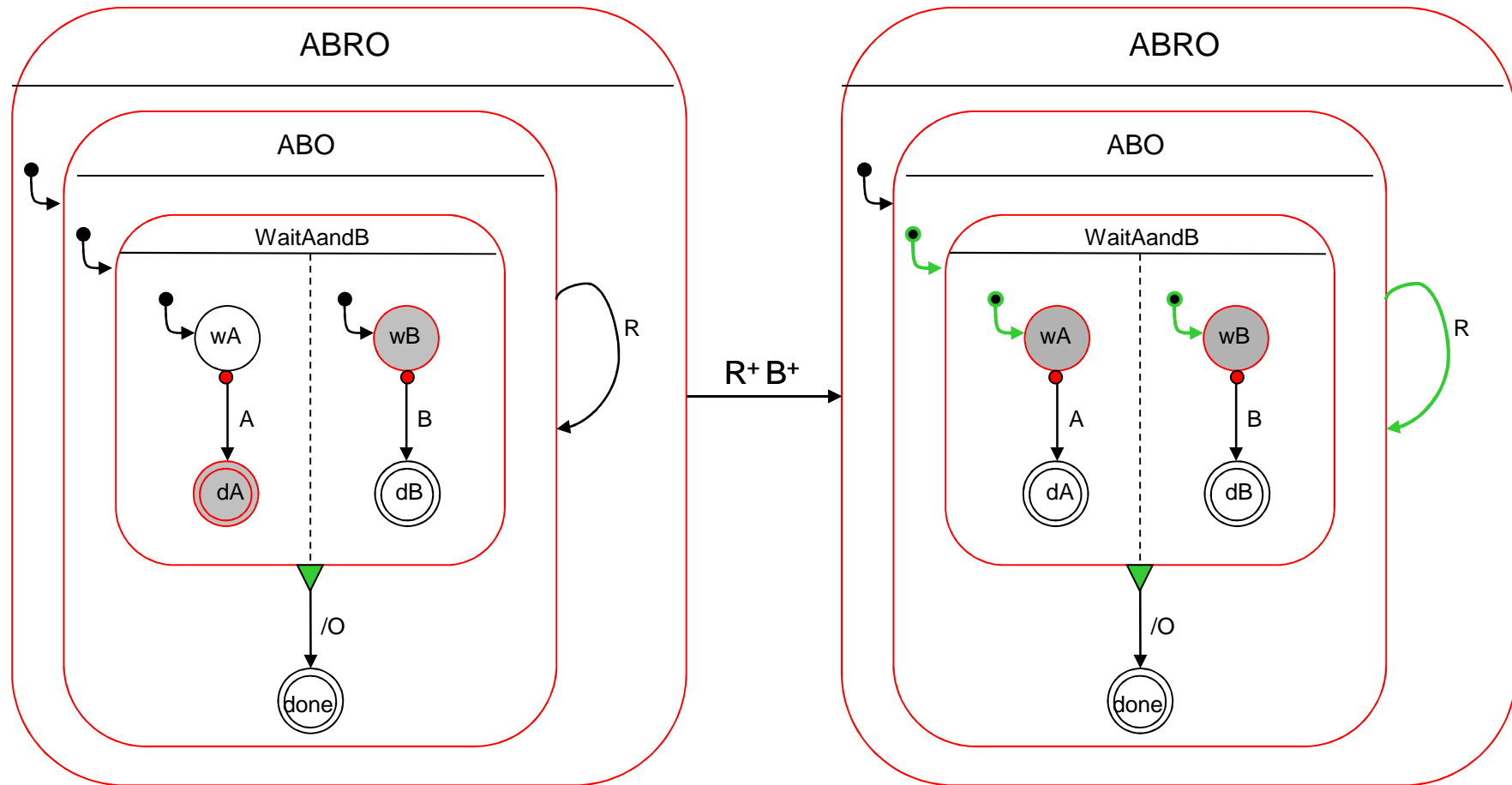
Concurrency and Weak Abort



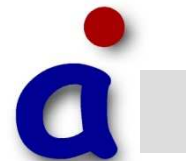
Microstep 2



Concurrency and Weak Abort

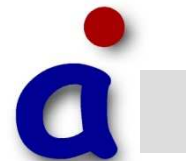


Microstep 3



Computing the reaction of an SSM

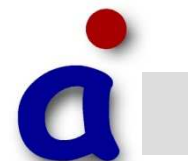
- Reaction of an SSM. Given a stable configuration, a reaction is computed by:
 1. Read input signals (presence status of all input signals is known)
 2. Set all output signals to the unknown presence status (\perp)
 3. Compute reaction of the top macrostate:
react(T)
/* yields emitted signals and the next stable configuration */



State Reaction

- Reaction of **macrostate** M:
 1. Set all local signals to \perp
 2. For each STG g directly contained in M.G do in parallel
 - Compute reaction of STG g and store the termination code in $c(g)$:
 $c(g) = \text{react}(g)$
 3. When all parallel executions are done
 - Compute $C = \text{maximum of } c(g) \text{ for all STGs } g \text{ in } M$
 4. Return C

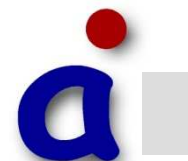
- Reaction of a **simple state** S
 1. If S is a final state return DEAD.
 2. If an effect is associated with S, then emit all signals of the effect.
 3. return PAUSE



State Transition Graph Reaction

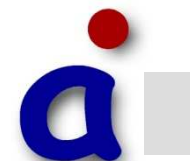
- Reaction of a **STG** g
 1. If there is no current state in g then set current to the initial reactive cell: $g.current = g.ini$;
 2. Compute the reaction of the **reactive cell** $C=(M,t)$ whose body M is the current state ($M=g.current$):
 $r = react(C)$
 3. If $r = DONE$ then $G.current = C.nextState$; goto 2.
 4. return r /* here r cannot be $DONE$ */

- Comments:
 - When entering a macrostate the current state of each STG is undefined. If the STG is already active, the current state is the (unique) currently active state.
 - Reactions of all STGs from a macrostate are computed in parallel (fork).



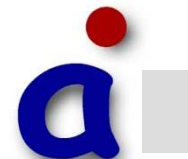
Reactive Cell Reaction

- Reaction of a reactive cell C:
 1. if (!firstInstant) Strong abort test:
 - If a strong abort transition is enabled then take this transition
 2. Execute the body of the reactive cell
 - If it is a macrostate M, then recursive call: $B = \text{react}(M)$
 - If it is a simple state S, then terminal call: $B = \text{react}(S)$
 3. if (!firstInstant) Weak abort test:
 - /* Note: body has completed execution. */
 - If a weak abort transition is enabled then take this transition
 4. Normal termination test:
 - If $(B == \text{DEAD})$ then take normal termination transition
 5. End of reaction:
 - Set C.status=ACTIVE
 - return PAUSE



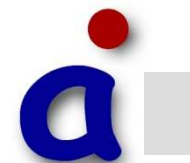
Reactive Cell Reaction

- The triggers are not tested at the **first instant** when the reactive cell is activated (strong/weak abort test).
- If the presence status of a triggering signal is **unknown**, the execution is **suspended** till another concurrent execution thread will fix the status of the tested signal.
- **Taking** a transition t (strong/weak abort, or normal termination) means:
 - Recursively “kill” the body of the reactive cell C :
 - set $k.status=IDLE$ for all transitively contained reactive cells k
 - reset $G.current$ for all transitively contained STGs G
 - Execute the effect associated with t and set $C.nextstate = t.target$;
 - Set $C.status = IDLE$;
 - return **DONE**;



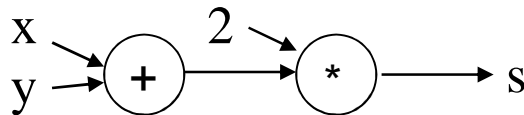
Naming Confusion

- **Scade Suite**: model based development IDE
 - Graphical modelling language
 - ✓ Control part: **SyncCharts (SSM)**
 - Data flow part: **SCADE** graphical syntax
 - Textual representation: **SCADE** textual language. Semantics comprise
 - constructive semantics of **ESTEREL**
 - data flow semantics of **LUSTRE**



LUSTRE

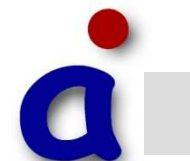
- Programs are structured into **nodes**:
 - Node: subprogram defining its output parameters as functions of its input parameters.
 - Definition given by **unordered set of equations** (\rightarrow declarative language)
- Based on **synchronous data-flow model**:
 - Functional: no side effects.
 - All nodes work simultaneously, ie at the same speed.
 - No broadcasting of signals; sequencing and synchronization only from data dependences.
 - Each variable takes a value **at every cycle** of the program.



At any cycle n:
 $s_n = 2 * (x_n + y_n)$

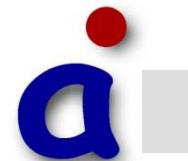
- Basis of **SCADE**.
- Example:


```
node Counter (init, incr: int; reset: bool)
  returns (count:int);
let
  count = init -> if reset then init
                 else pre(count)+incr;
tel
```



Required Properties

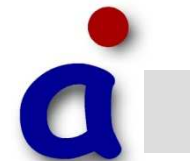
- **Causality**: The output at any instant t may only depend upon input received before or at t .
- **Bounded memory**: There must be a finite bound such that, at each instant the number of past input values necessary to produce a new output value remains smaller than that bound.
- **Efficient** code generation.
- Execution time **predictability**: no unbounded loops, no recursion.



Flows and Clocks

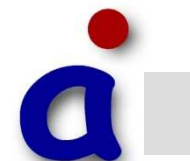
- Any variable and expression denotes a **flow**, ie a pair made (x, b_x) of
 - a possibly infinite sequence x of values of a given type
 - a clock b_x , representing a sequence of times.
- x is defined at instant i iff $b_x(i) = \text{true}$.
- A flow takes its n -th value in the n -th time of its clock.
- Input variables are defined at every instant: their clock is called the **basic clock**.
- Example: Let x run on the basic clock C , y on a slower clock. This gives the following time scales:

Basic time-scale	1	2	3	4	5	6	7	8
b_x	t	t	t	t	t	t	t	t
x time-scale	1	2	3	4	5	6	7	8
b_y	t	f	t	f	t	f	t	f
y time-scale	1		2		3		4	



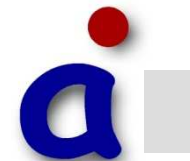
Types, Equations, Assertions

- Variables are declared with their type:
 - Basic types: boolean, integer, real.
 - Type constructor tuple.
 - Semantics is Cartesian product.
 - Abstract types via import.
- Equations:
 - Variables are defined via **equations**, e.g. $X=E$ with variable X and expression E .
 - **Substitution** principle: X can be substituted to E anywhere in the program and vice versa.
 - **Definition** principle: The behavior of X must be completely specified by this equation.
- Assertions:
 - Assertions $\text{assert}(E)$: E must hold during execution.
 - Used to optimize code generation, for simulation and for verification.



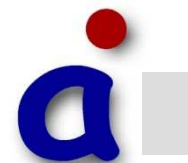
Variables and Expressions

- Operators only operate on operands sharing the **same clock**.
- As variables and expressions are streams, operators also produce streams. Example: With $x = (0,1,2,3,4,\dots)$ and $y = (2,4,6,8,10,\dots)$:
 $x+y=(2,5,8,11,14,\dots)$
- **Expressions** are build from variables, constants and operators.
- Three types of **operators**:
 - Data operators:
 - arithmetic, boolean and relational expressions
 - conditional expressions: *if E then X else Y*
 - Imported operators:
 - functions imported from host language
 - Temporal (sequence) operators.



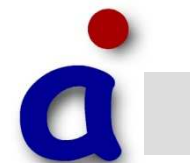
Temporal Operators

- 'previous' operator **pre**:
 - $(pre(E))_0 = \perp$ (undefined, also denoted *nil*)
 - $(pre(E))_n = E_{n-1}$
- 'followed by' operator **->**
 - $(E->F)_0 = E_0$
 - $(E->F)_n = F_n$
- (Down-)Sampling: **when**
 - Let E be an expression and B a boolean expression with the same clock: **(E when B)** is the sequence of values of E when B is true.
- Upsampling/Interpolation/Projection: **current**
 - Let E be an expression and B a boolean expression defining the clock of E : Then **current E** has the same clock as B ; and (**current E**) is the sequence of values of E at the last time when B was true.



Example

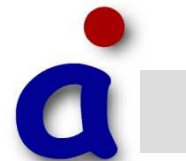
B	false	true	false	true	false	false	true	true
X	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Y = X when B								
Z = current Y								



Example

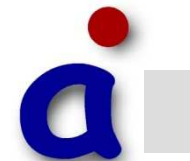
B	false	true	false	true	false	false	true	true
X	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Y = X when B		x_2		x_4			x_7	x_8
Z = current Y								

Note: `gaps` are not filled.



Example

B	false	true	false	true	false	false	true	true
X	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Y = X when B		x_2		x_4			x_7	x_8
Z = current Y	\perp	x_2	x_2	x_4	x_4	x_4	x_7	x_8



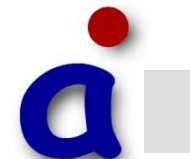
Clock Rules

- Let a **clock environment** ω be a function from identifiers to clocks.
- Let $clk(E, \omega)$ be the clock of the expression E in the environment ω .
- For an equation $X=E$ holds $\omega(X)=clk(E, \omega)$.
- Let \perp be the undefined clock and \top the erroneous clock. Then

$$ck \leq ck' \Leftrightarrow (ck = \perp \vee ck' = \top \vee ck = ck')$$

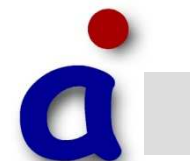
- Let \cup denote the least upper bound operator.
- Constants: For any constant k , $clk(k, \omega)=true$ (the basic clock).
- Variables: For any identifier X , $clk(X, \omega)=\omega(X)$.
- Synchronous operators:

$$clk(op(E_1, E_2, \dots, E_n), \omega) = \bigcup_{i=1}^n clk(E_i, \omega)$$



Clock Rules

- **Downsampling:** The operands of the *when* operator must be on the same clock: $clk(E)=clk(ck) \Leftrightarrow clk(E \text{ when } ck, \omega)=ck$.
- **Upsampling:** Let *ck* be the clock of *E*, $ck \neq true$:
 $clk(current(E), \omega)=clk(ck)=clk(clk(E))$.
- Clock of a node instance: clock of its effective inputs.
- Initialization problem: $current(X \text{ when } C)$ exists but is undefined (\perp) until *C* becomes *true* for the first time.
- Solution: **activation conditions**
 - Not an operator, rather a macro.
 - $y = CONDUCT(OP, C, X, dflt)$ equivalent to
 $Y = \text{if } C \text{ then } current(OP(X \text{ when } C))$
 $\text{else } (dflt \rightarrow pre(Y))$
 - Provided by SCADE (not part of LUSTRE).



Example

C	true	true	false	false	true	false	true
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
$n = (0 \rightarrow \text{pre}(n) + 1)$							
$e = (1 \rightarrow \text{not pre}(e))$							
n when e							
current(n when e)							
Counter((1,1,false) when C)							
Counter(1,1,false) when C							

```

node Counter (init, incr: int; reset: bool)
  returns (count:int);
let
  count = init -> if reset then init
                  else pre(count)+incr;
tel

```



Example

C	true	true	false	false	true	false	true
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
$n = (0 \rightarrow \text{pre}(n) + 1)$	0	1	2	3	4	5	6
$e = (1 \rightarrow \text{not pre}(e))$							
n when e							
current(n when e)							
Counter((1,1,false) when C)							
Counter(1,1,false) when C							

```

node Counter (init, incr: int; reset: bool)
  returns (count:int);
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  count = init -> if reset then init
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tel

```



Example

C	true	true	false	false	true	false	true
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
$n = (0 \rightarrow \text{pre}(n) + 1)$	0	1	2	3	4	5	6
$e = (1 \rightarrow \text{not pre}(e))$	1	0	1	0	1	0	1
n when e							
current(n when e)							
Counter((1,1,false) when C)							
Counter(1,1,false) when C							

```

node Counter (init, incr: int; reset: bool)
  returns (count:int);
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$e = (1 \rightarrow \text{not pre}(e))$	1	0	1	0	1	0	1
n when e	0		2		4		6
current(n when e)	0	0	2	2	4	4	6
Counter((1,1,false) when C)	1	2			3		4
Counter(1,1,false) when C	1	2			5		7

```

node Counter (init, incr: int; reset: bool)
  returns (count:int);
let
  count = init -> if reset then init
                  else pre(count)+incr;
tel

```



Program Structure

- **Nodes** are LUSTRE subprograms. General structure:

```

node N (  $x_1:\tau_1; x_2:\tau_2; \dots; x_p:\tau_p$  )
returns (  $y_1:\theta_1; y_2:\theta_2; \dots; y_p:\theta_q$  )
var  $z_1:\gamma_1; z_2:\gamma_2; \dots; z_k:\gamma_k$ 
let
     $z_1=E_1; \dots; z_k=E_i;$ 
     $y_1=E_j; \dots; y_p=E_m;$ 
tel

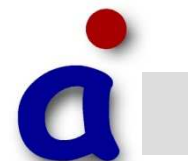
```

- Node **instantiation**: if N is the name of a node with above signature and if E_1, \dots, E_p are expressions of type τ_1, \dots, τ_p , then $N(E_1, \dots, E_p)$ is an expression of type $tuple(\theta_1, \dots, \theta_q)$.
- Conditional and sequence operators are polymorphic and can be applied to tuples.



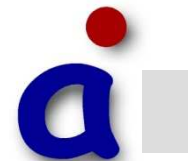
Arrays and Recursion

- Let n be a constant. Given type τ , τ_n defines an **array** with n entries of type τ .
- Example: $x:bool_n$
- The **bounds** of an array must be known at **compile time**; the compiler transforms an array of n values into n **different variables**.
- $X[i]$ denotes i th element.
- $X[i..j]$ denotes the array made of elements i to j of X .
- LUSTRE only allows **static recursion**: the recursion is completely unrolled.
- **Attention**: if the recursion is not bounded the compiler will not stop.



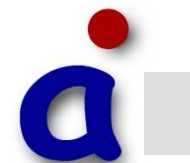
Compilation of LUSTRE Programs

- Static compiler checks:
 - Definition checking: any local and output variable must have exactly one definition.
 - No recursive node calls.
 - Clock consistency.
 - Absence of uninitialized expressions (yielding \perp).
 - Absence of cyclic definitions.
- Compilation to
 - single-loop code
 - automata code.



Causality Problems

- LUSTRE only allows **acyclic** equation systems.
Note: acyclic equations have a unique solution.
- $X=E$ is acyclic if X does not occur in E unless as subterm of the *pre* operator.
- Examples:
 - $X = X$ and $pre(X)$ is cyclic
 - $X = Y$ and $pre(X)$ is acyclic
- Also structural deadlocks which are not true ones are rejected:
 - $X = \text{if } C \text{ then } Y \text{ else } Z;$
 $Y = \text{if } C \text{ then } Z \text{ else } X;$
- Improved causality analysis in SCADE.



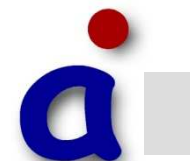
Clock Consistency

- Consider the following (illegal) example:

$b = \text{true} \rightarrow \text{not pre } b;$
 $y = x + (x \text{ when } b);$

x	x_0	x_1	x_2	x_3
b	1	0	1	0
x when b	x_0		x_2	
$x + (x \text{ when } b)$	$x_0 + x_0$	$x_1 + x_2$	$x_2 + x_4$	$x_3 + x_6$

- The computation of the $2n$ th value of y needs the $2n$ th and the n th values of x .
- Problem: not possible with bounded memory.
- Consequence: only streams of the same clock can be combined.
- Problem: **undecidable** whether two boolean expressions denote the same flow.



Clock Consistency (c'ed)

- Thus: two boolean expressions define the same clock iff they can be **syntactically** unified.

- Examples:

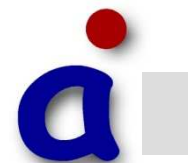
$$x = a \text{ when } (y > z)$$

$$y = b + c$$

$$u = d \text{ when } (b + c > z)$$

$$v = e \text{ when } (z < y)$$

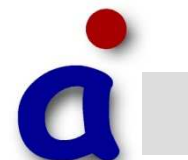
- x and u share the same clock.
- x and v have different clocks.



LUSTRE and SCADE

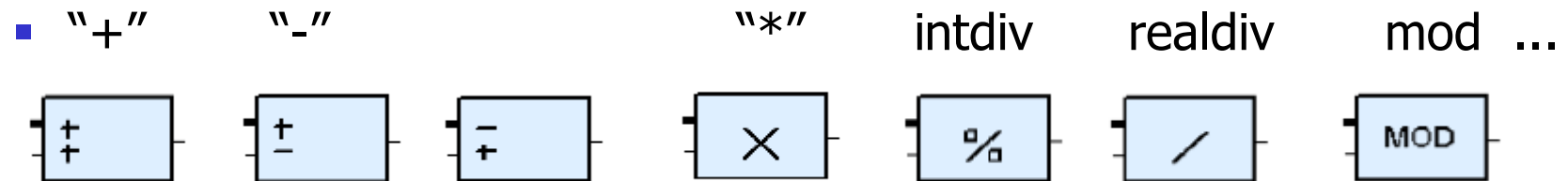
- SCADE: **constant blocks**:
 - delimited by keywords *let const* and *tel*;
 - Example:


```
let const PCSt1
  C2: [real,int] = [C1,3]
  C1: real = 7.2;
  imported Cimp: real;
tel;
```
- **Equation blocks** delimited by keywords *let equa* and *tel*; variable blocks by *var*; global variable blocks by *let global* and *tel*; type blocks by *let type* and *tel*;
- Other additions for **syntactic convenience**, like “don’t care” symbol ``_'`: `_,x = [3,4]`;
- For details see *SCADE Language Reference Manual* [Esterel 2006].

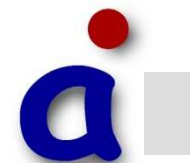
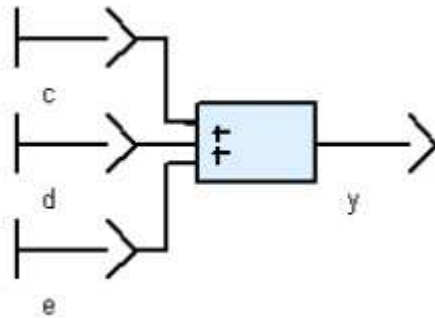


SCADE: The Graphical Language

- See SCADE Technical Manual, Chapter 3: SCADE Language Graphic Syntax.
- Arithmetic Operators:



- Example: $y=c+d+e$



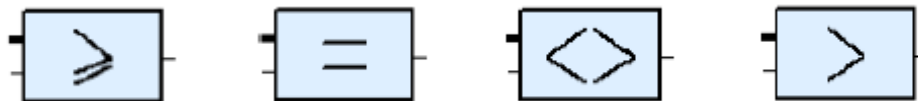
SCADE: The Graphical Language

- Logial Operators:

- "or"
 - "xor"
 - "and"
 - "not"
 - ...

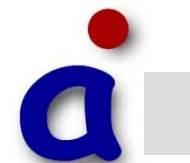
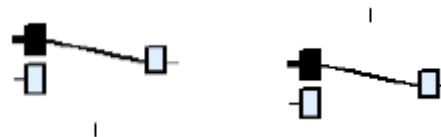


- Some Comparison Operators:



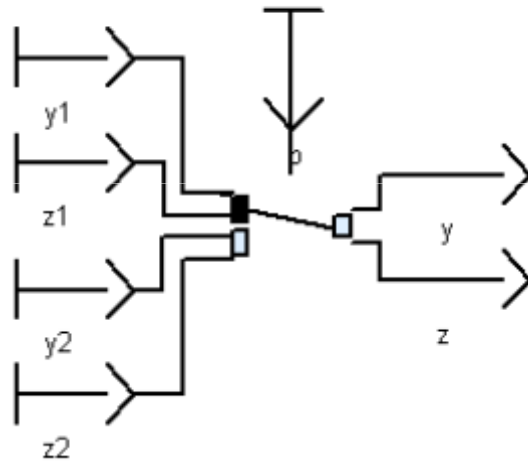
- Control Operators:

- if ... then ... else ...



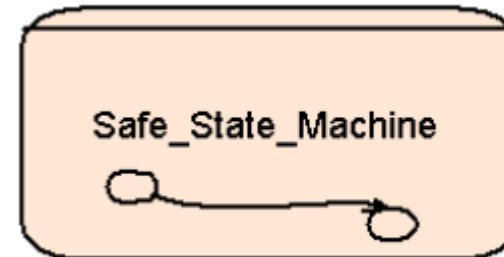
SCADE: The Graphical Language

- Example:
 - $y,z = \text{if } b \text{ then } (y_1, z_1) \text{ else } (y_2, z_2)$



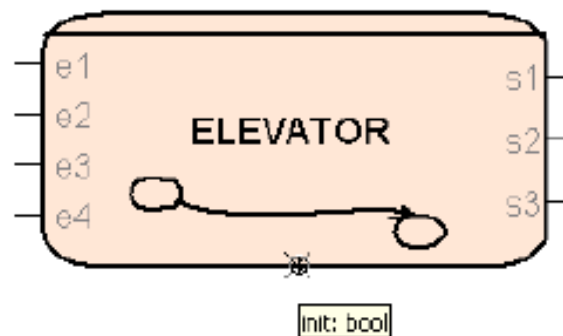
SCADE: The Graphical Language

- SSM (SyncCharts) Nodes



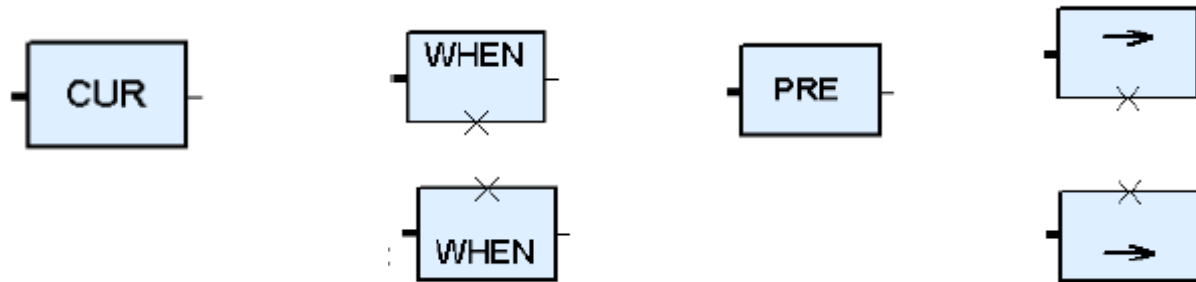
Call of the SSM node ELEVATOR with 4 inputs and 3 outputs:

- ELEVATOR (e1 : int ; e2 : int ; e3 : int ; e4 : int ; hidden input_init : bool)
returns (s1 : bool ; s2 : bool ; s3 : bool)



SCADE: The Graphical Language

- Temporal Operators



- Example: $o1, o2, o3 = (i1, i2, i3)$ when c

