Embedded Systems

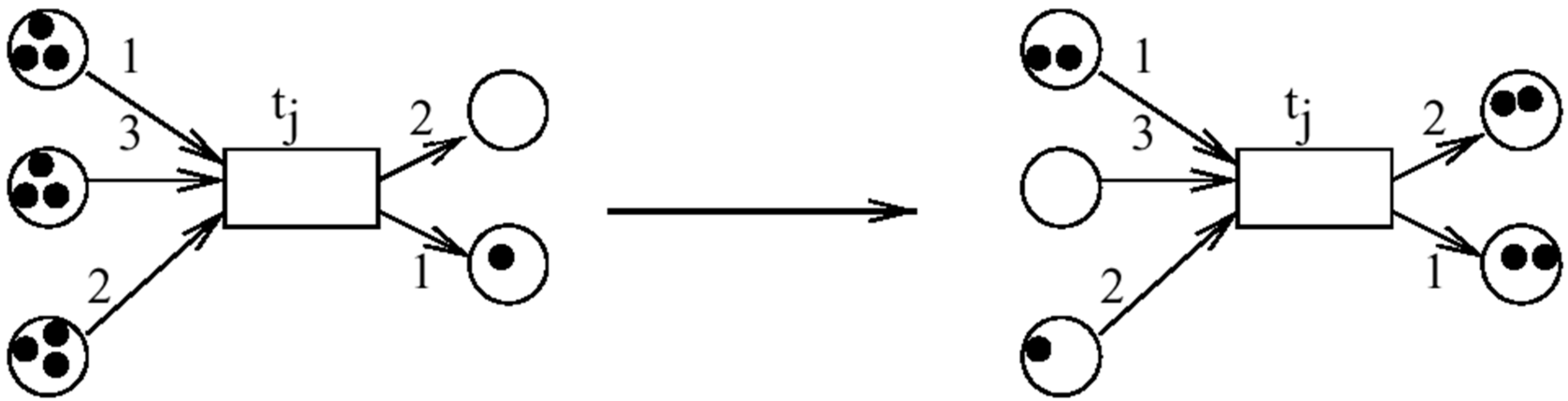
Petri Nets

Computing changes of markings

REVIEW

- „Firing" transitions *t* generate new markings on each of the places *p* according to the following rules:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ M(p) + W(t,p), & \text{if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^{\bullet}t \cap t^{\bullet} \\ M(p) & \text{otherwise} \end{cases}$$



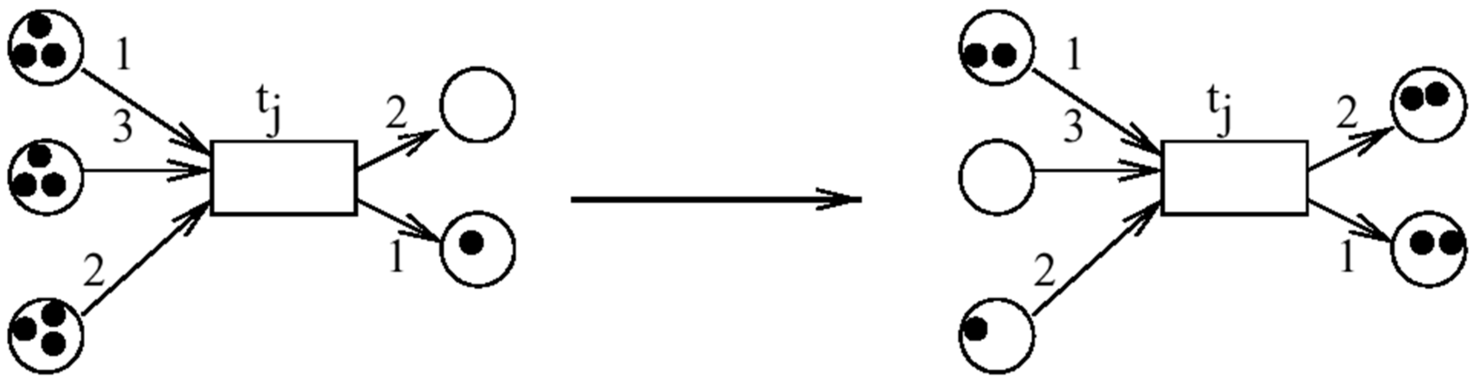
When a transition t *fires* from a marking M, w(p, t) tokens are deleted from the incoming places of t (i.e. from places p $\in {}^{\bullet}t$), and w(t, p) tokens are added to the outgoing places of t (i.e. to places p $\in t^{\bullet}$), and a new marking M' is produced

Activated transitions

REVIEW

- Transition *t* is „activated“
  iff

$$(\forall p \in {}^{\bullet}t : M(p) \geq W(p,t)) \wedge (\forall p \in t^{\bullet} : M(p) + W(t,p) \leq K(p))$$



Activated transitions can „take place“ or „fire“,
but don‘t have to.
The order in which activated transitions fire is not fixed
(it is non-deterministic).

Boundedness

REVIEW

- A place is called ***k-safe*** or ***k-bounded*** if it contains in the initial marking m_0 and in all other reachable from there markings at most **k** tokens.

- A net is **bounded** if each place is bounded.

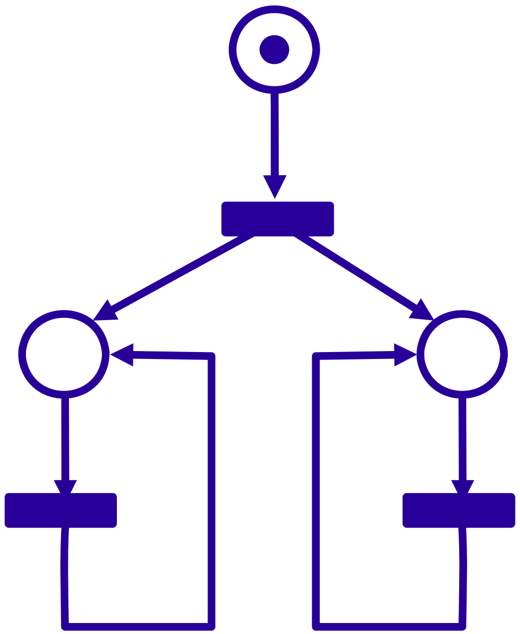
- **Boundedness: the number of tokens in any place cannot grow indefinitely**
- **Application: places represent buffers and registers (check there is no overflow)**

- A Petri net is inherently bounded if and only if all its reachability graphs (i.e. reachability graphs with all possible starting states) all have a finite number of states.

Liveness

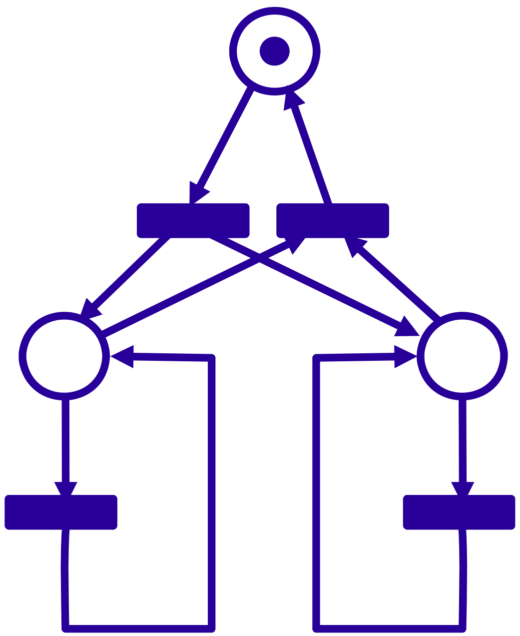
REVIEW

- A transition T is live if in any marking there exists a firing sequence such that T becomes enabled
- An entire net is live if all its transitions are live
- Important for checking deadlock



NO

Live ?



YES

Liveness (more precisely)

- A **transition** t is **dead** at M if no marking M‘ is reachable from M such that t can fire in M‘.
- A **transition** t is **live** at M if there is no marking M‘ reachable from M where t is dead.
- A **marking** is **live** if all transitions are live.
- A **P/T net** is **live** if the initial marking is live.

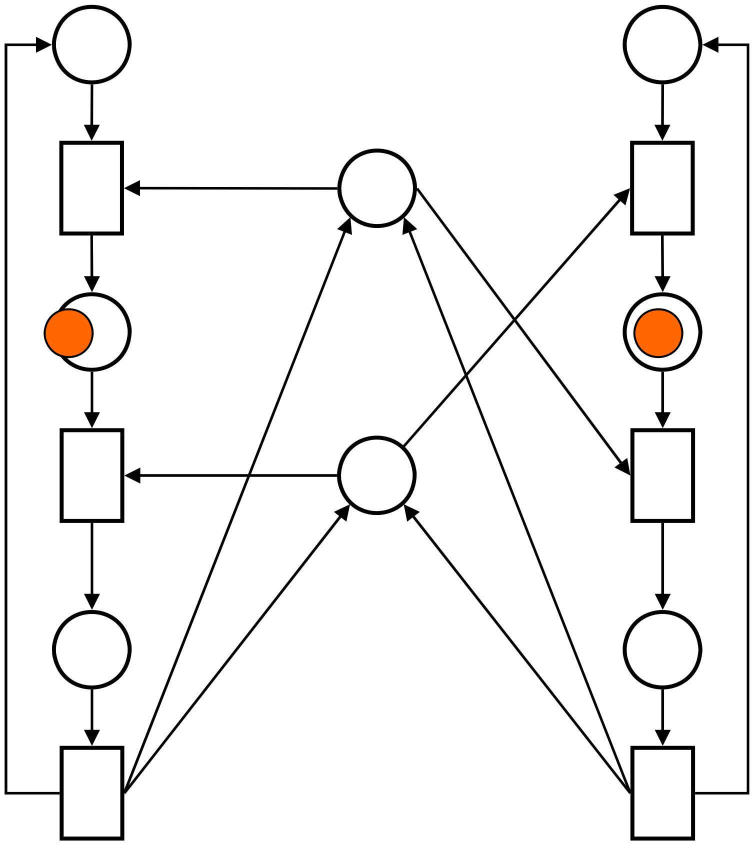
Observations:

- A live net is deadlock-free.
- No transition is live if the net is not deadlock-free.

Deadlock

REVIEW

- A **dead marking (deadlock)** is a marking where no transition can fire.
- A Petri net is **deadlock-free** if no dead marking is reachable.



Shorthand for changes of markings **REVIEW**

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

$\Rightarrow$ $\forall p \in P$: $M'(p) = M(p) + \underline{t}(p)$

$\Rightarrow$ $M' = M + \underline{t}$ +: vector add

Matrix *N* describing all changes of markings

$$\underline{t}(p)=\begin{cases}-W(p,t) \text{ if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ +W(t,p) \text{ if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ -W(p,t)+W(t,p) \text{ if } p \in t^{\bullet} \cap {}^{\bullet}t \\ 0 \end{cases}$$

Def.: Matrix $\underline{N}$ (incidence matrix) of net *N* is a mapping

$\underline{N}: P \times T \rightarrow Z$ (integers)

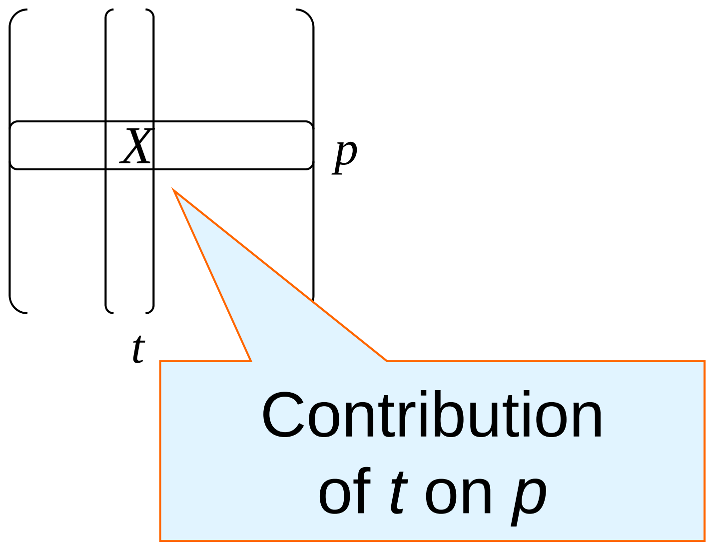
such that $\forall\, t \in T$: $\underline{N}(p,t)=\underline{t}(p)$

Component in column *t* and row *p* indicates the change of the marking of place *p* if transition *t* takes place.

Incidence matrix

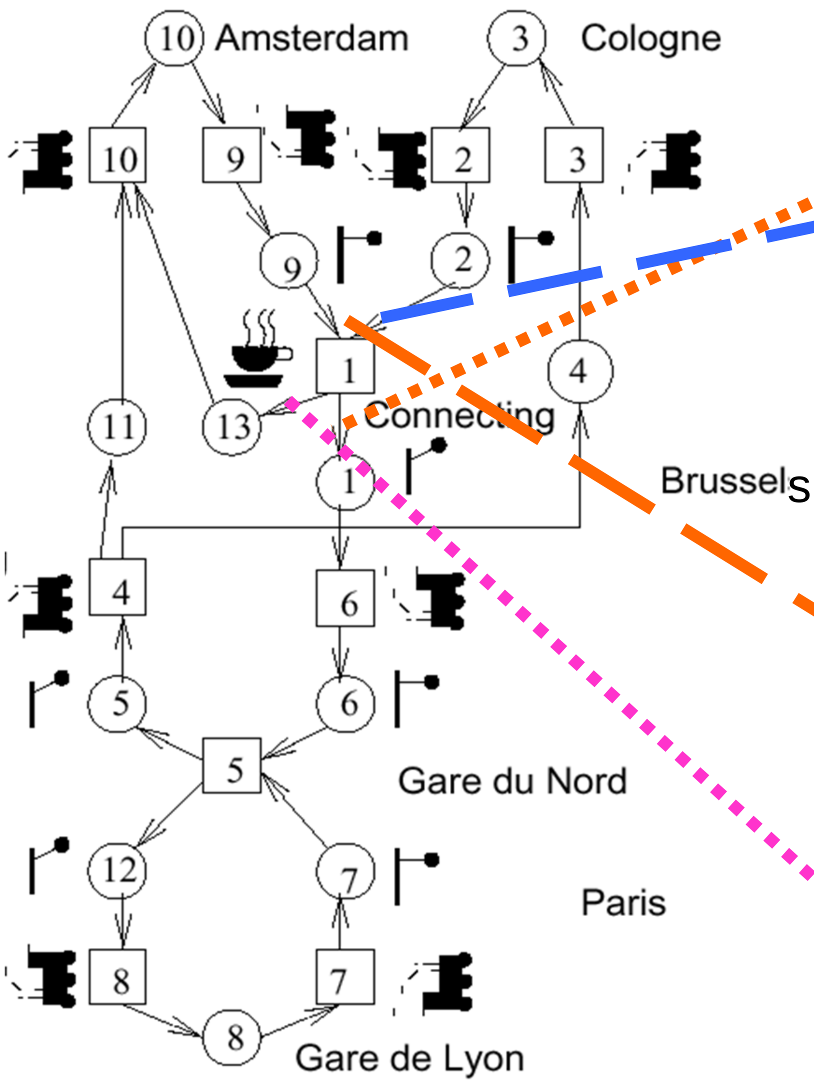
incidence matrix N of a pure (no elementary loops) place/transition-net:

$$N_{p,t} := \begin{cases} -W(t,p), & arc \text{ from } p \text{ to } t \\ +W(t,p), & arc \text{ from } t \text{ to } p \\ 0, & \text{otherwise} \end{cases}$$



Example: $\underline{N}$ =

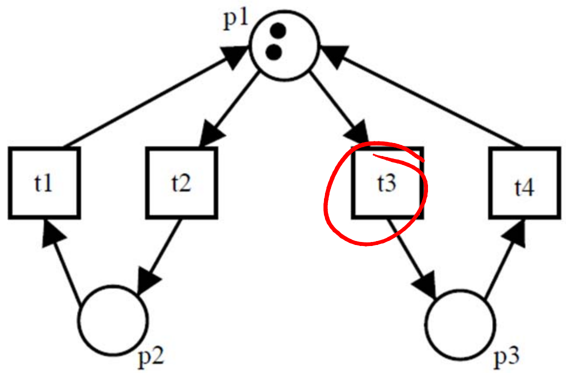
REVIEW



	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
p_1	1					–1				
p_2	–1	1								
p_3		–1	1							
p_4			–1	1						
p_5				–1	1					
p_6					–1	1				
p_7					–1		1			
p_8							–1			
p_9	–1							1	1	
p_{10}									–1	1
p_{11}				1						–1
p_{12}					1			–1		
p_{13}	1									–1

State equation

REVIEW



$N_{p,t}$ =

	t_1	t_2	t_3	t_4
p_1	1	-1	-1	1
p_2	-1	1	0	0
p_3	0	0	1	-1

$$m_0 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$m' = m_0 + N \cdot u_i$$

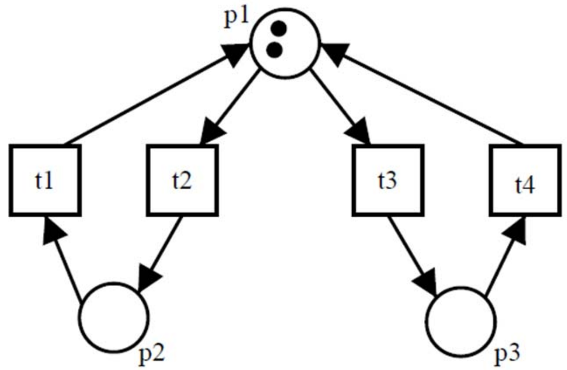
$$u_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \text{ for } t_3$$

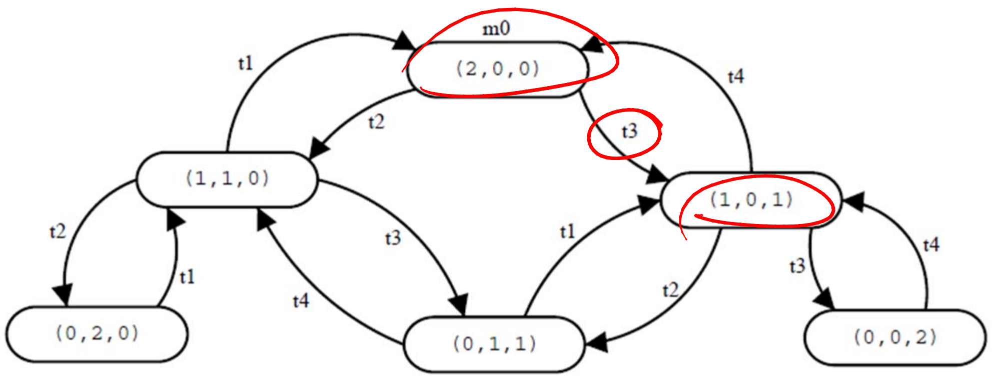
$$m' = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

State equation

REVIEW





reachability graph

Computation of Invariants

REVIEW

We are interested in subsets *R* of places whose number of labels remain invariant under fireing of transitions:

- e.g. the number of trains commuting between Amsterdam and Paris (Cologne and Paris) remains constant

Important for correctness proofs

Place - invariants

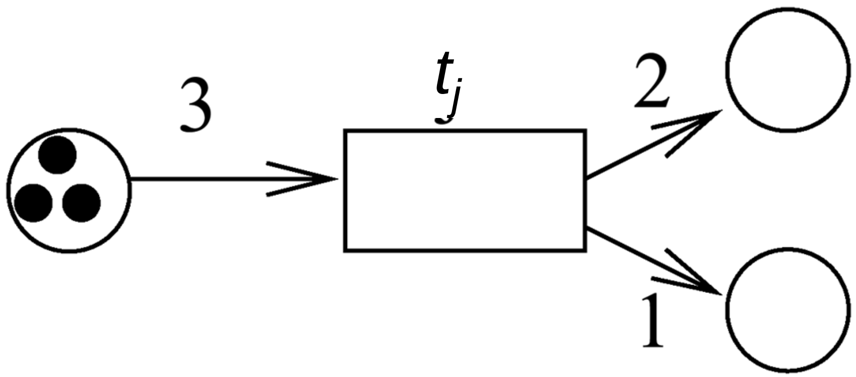
REVIEW

Standardized technique for proving properties of system models

For any transition $t_j \in T$ we are looking for sets $R \subseteq P$ of places for which the accumulated marking is constant:

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Example:



Characteristic Vector

REVIEW

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Let:

$$\underline{c}_R(p) = \begin{cases} 1 \text{ if } & p \in R \\ 0 \text{ if } & p \notin R \end{cases}$$

$$\Rightarrow \quad \sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p) \, \underline{c}_R(p) = 0$$

↑ Scalar product

Condition for place invariants

REVIEW

$$\sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Accumulated marking constant for **all** transitions if

$$\begin{array}{ccc} \underline{t}_1 \cdot \underline{c}_R & = & 0 \\ \dots & \dots & \dots \\ \underline{t}_n \cdot \underline{c}_R & = & 0 \end{array}$$

Equivalent to $\underline{N}^T \underline{c}_R = 0$ where $\underline{N}^T$ is the transposed of $\underline{N}$

More detailed view of computations **REVIEW**

$$\begin{pmatrix} \underline{t}_1(p_1) \ldots \underline{t}_1(p_n) \\ \underline{t}_2(p_1) \ldots \underline{t}_2(p_n) \\ \ldots \\ \underline{t}_m(p_1) \ldots \underline{t}_m(p_n) \end{pmatrix} \begin{pmatrix} \underline{c}_R(p_1) \\ \underline{c}_R(p_2) \\ \ldots \\ \underline{c}_R(p_n) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

System of linear equations.

Solution vectors must consist of zeros and ones.

Different techniques for solving equation system (Gauss elimination, tools e.g. Matlab, …)

Application to Thalys example

REVIEW

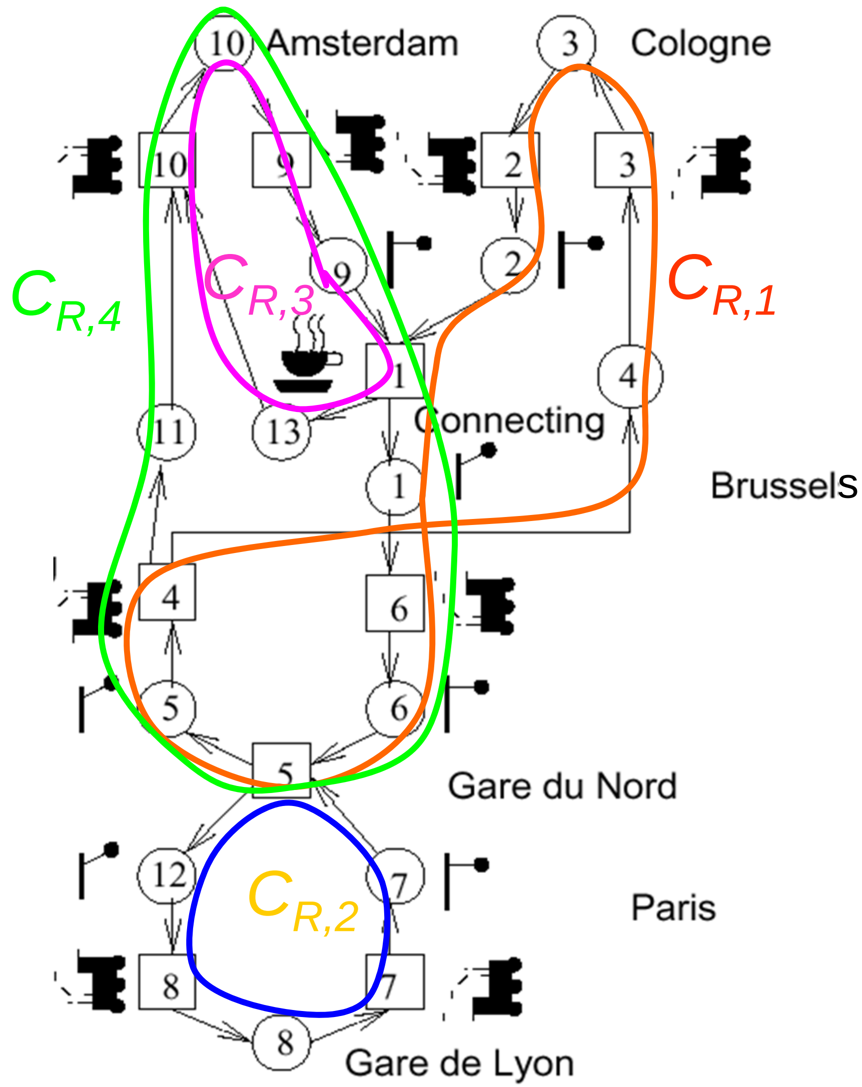
$\underline{\mathbf{N}}^T \underline{\mathbf{c}}_R = \mathbf{0}$, with $\underline{\mathbf{N}}^T =$

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}
t_1	1	-1							-1				1
t_2		1	-1										
t_3			1	-1									
t_4				1	-1						1		
t_5					1	-1	-1					1	
t_6	-1					1							
t_7							1	-1					
t_8								1				-1	
t_9									1	-1			
t_{10}										1	-1		-1

$$c_{R,1} = \begin{pmatrix} 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \end{pmatrix}$$

Solution vectors for Thalys example **REVIEW**

$$c_{R,1} = (1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$$
$$c_{R,2} = (0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0)$$
$$c_{R,3} = (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1)$$
$$c_{R,4} = (1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0)$$



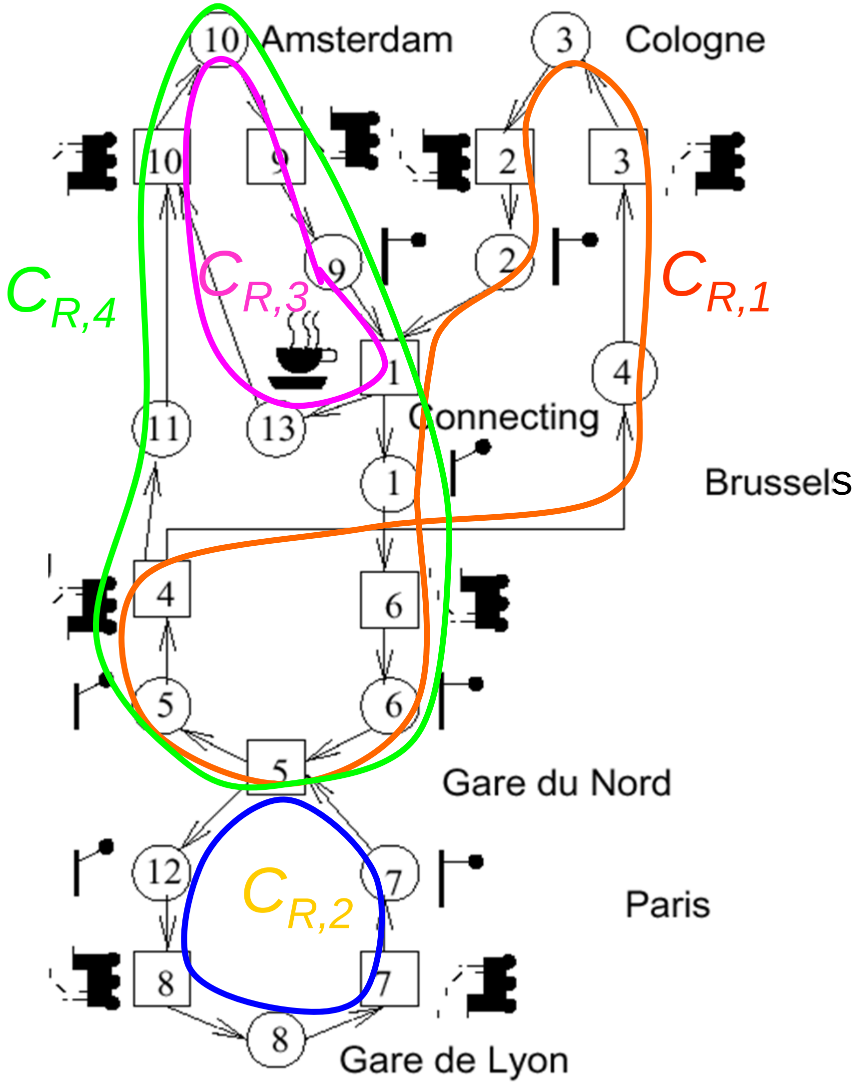
We proved that:

- the number of trains serving Amsterdam, Cologne and Paris remains constant.
- the number of train drivers remains constant.

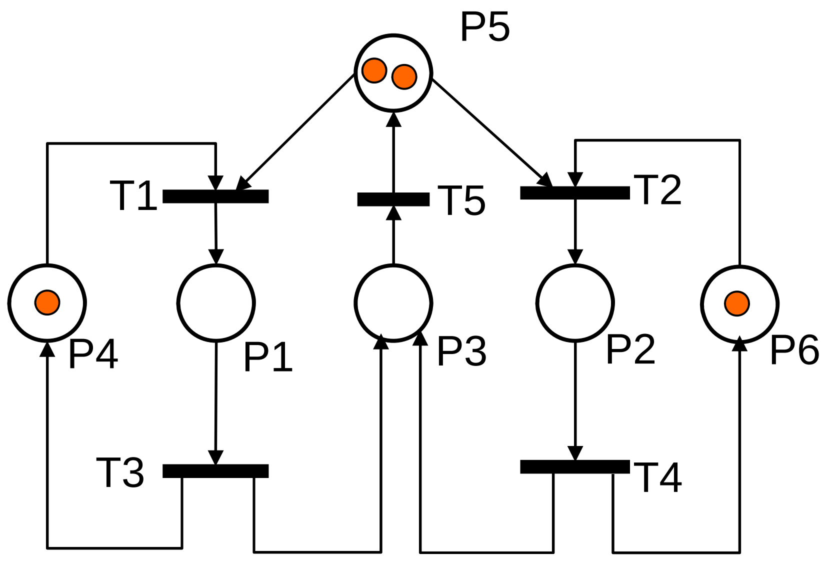
Solution vectors for Thalys example REVIEW

It follows:

- each place invariant must have at least one label at the beginning, otherwise “dead”
- at least three labels are necessary in the example



REVIEW

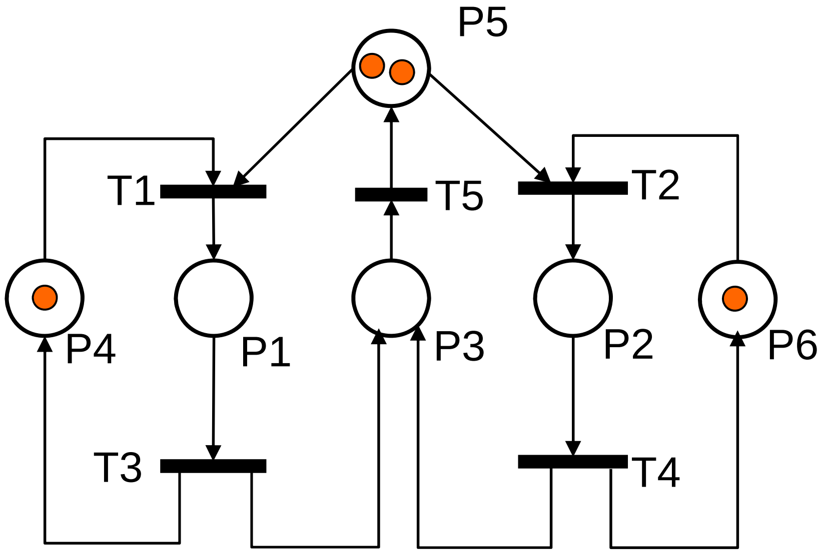


$\underline{\boldsymbol{N}}^{T}\ \underline{\boldsymbol{c}}_{R} = \mathbf{0}$, with $\underline{\boldsymbol{N}}^{T}=$

	P1	P2	P3	P4	P5	P6
T1	1	0	0	-1	-1	0
T2	0	1	0	0	-1	-1
T3	-1	0	1	1	0	0
T4	0	-1	1	0	0	1
T5	0	0	-1	0	1	0

	P1	P2	P3	P4	P5	P6
T1	1	0	0	-1	-1	0
T2	0	1	0	0	-1	-1
T3	-1	0	1	1	0	0
T4	0	-1	1	0	0	1
T5	0	0	-1	0	1	0

Place - invariants

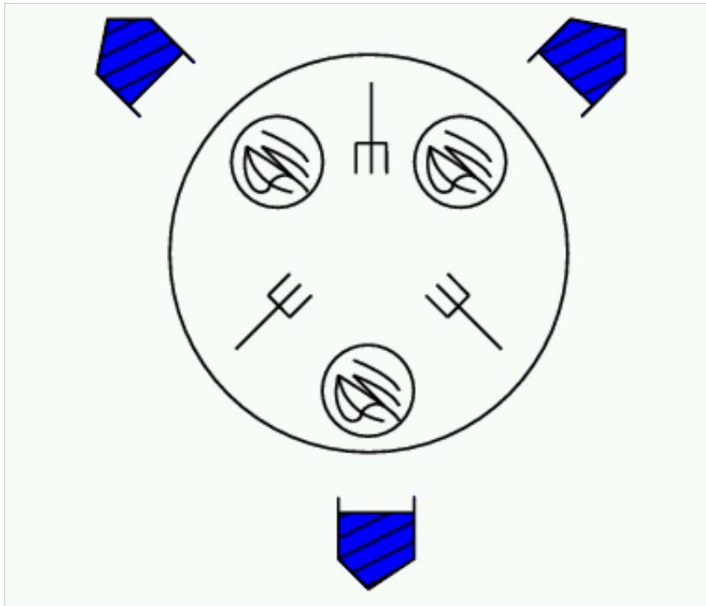


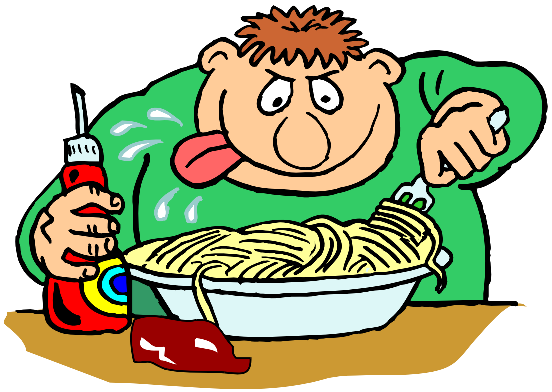
Predicate/transition nets

- Goal: compact representation of complex systems.
- Key changes:
  - Tokens are becoming individuals;
  - Transitions enabled if functions at incoming edges true;
  - Individuals generated by firing transitions defined through functions
- Changes can be explained by folding and unfolding C/E nets

Example: Dining philosophers problem

- $n>1$ philosophers sitting at a round table;
- n forks,
- n plates with spaghetti;
- philosophers either thinking or eating spaghetti (using left and right fork).





2 forks needed!

How to model conflict for forks?

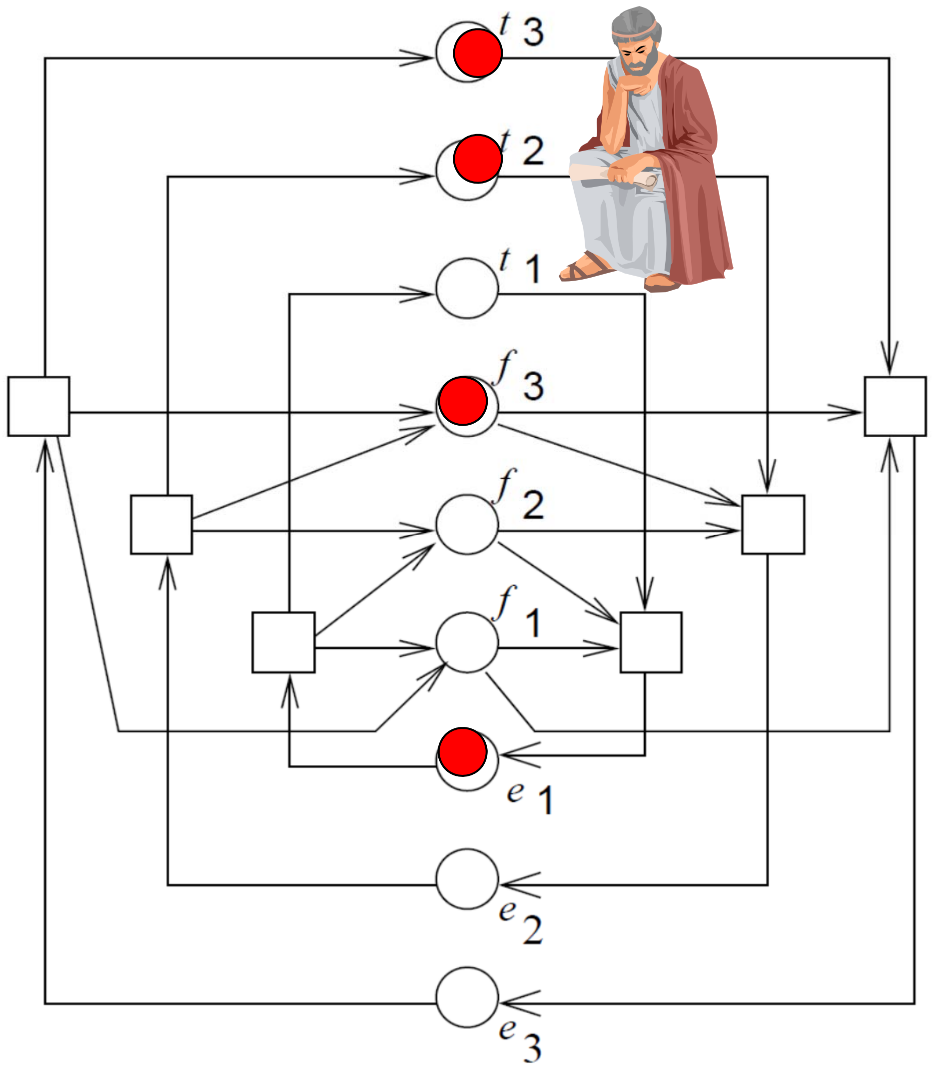
How to guarantee avoiding starvation?

Condition/event net model of the dining philosophers problem

- Let $x \in \{1..3\}$
- t_x: x is thinking
- e_x: x is eating
- f_x: fork x is available

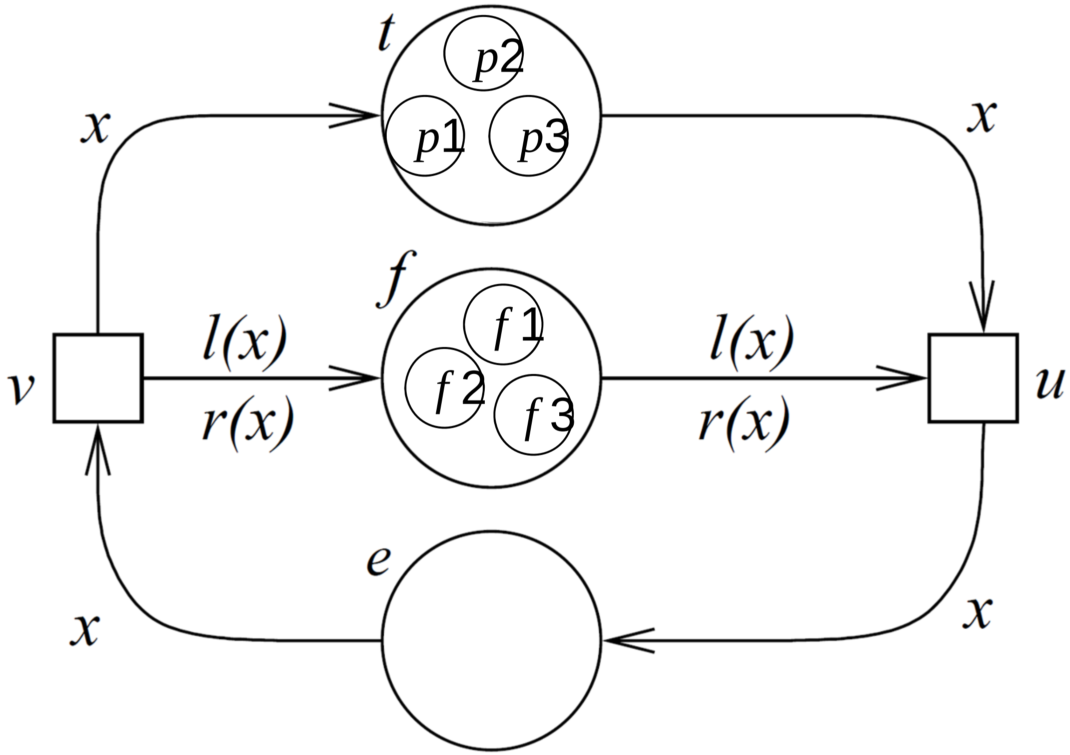
Model quite clumsy.

Difficult to extend to more philosophers.



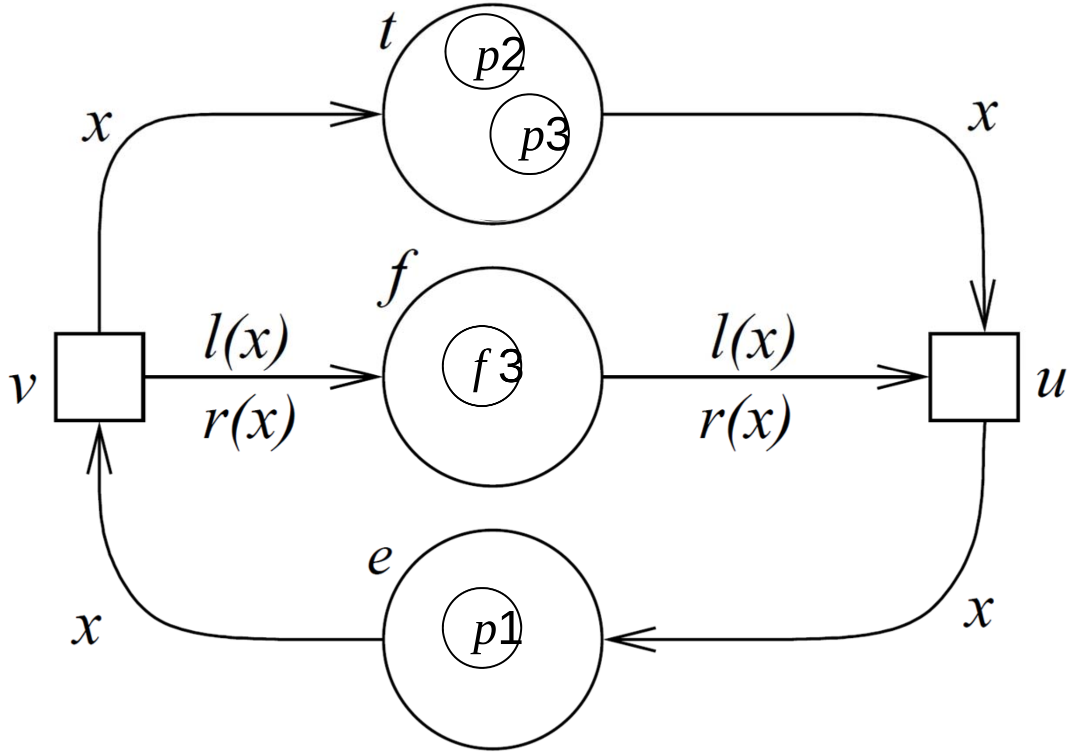
Predicate/transition model of the dining philosophers problem (1)

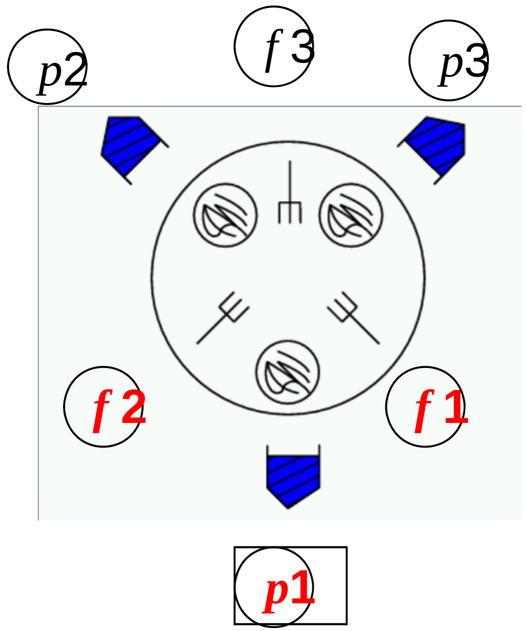
- Let x be one of the philosophers,
- let $l(x)$ be the left spoon of x,
- let $r(x)$ be the right spoon of x.



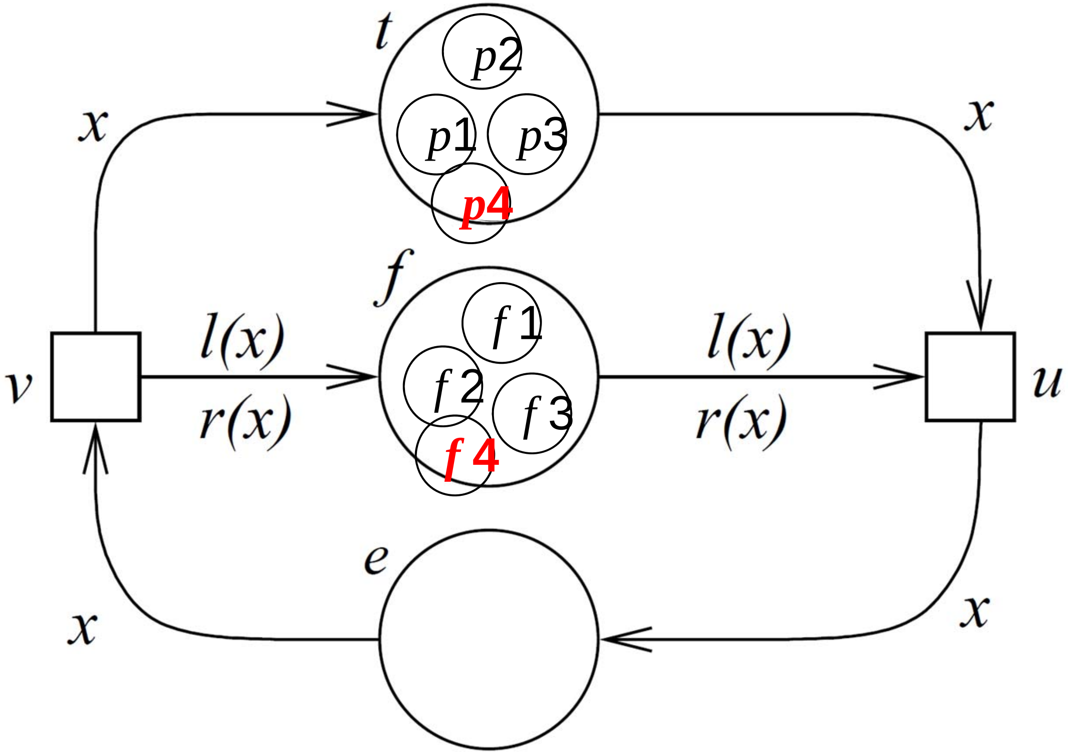
- Tokens individuals
- Edges can be labeled with variables and functions

Predicate/transition model of the dining philosophers problem (1)

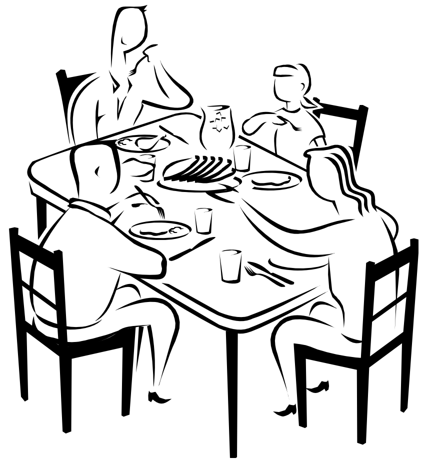




Predicate/transition model of the dining philosophers problem (2)



- Model can be extended to arbitrary numbers of people.
- No change of the structure.



Time and Petri Nets

- e.g.: Petri nets tell us that ""a new request can be issued only after the resource is released”

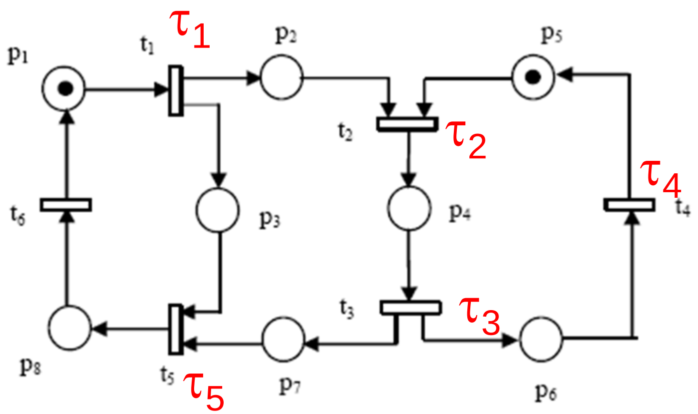
- Nothing about time

- In literature, time has been added to PNs in many different ways (notion of temporal constraints for: transitions, places, arcs) → TPN

Timed Petri Nets

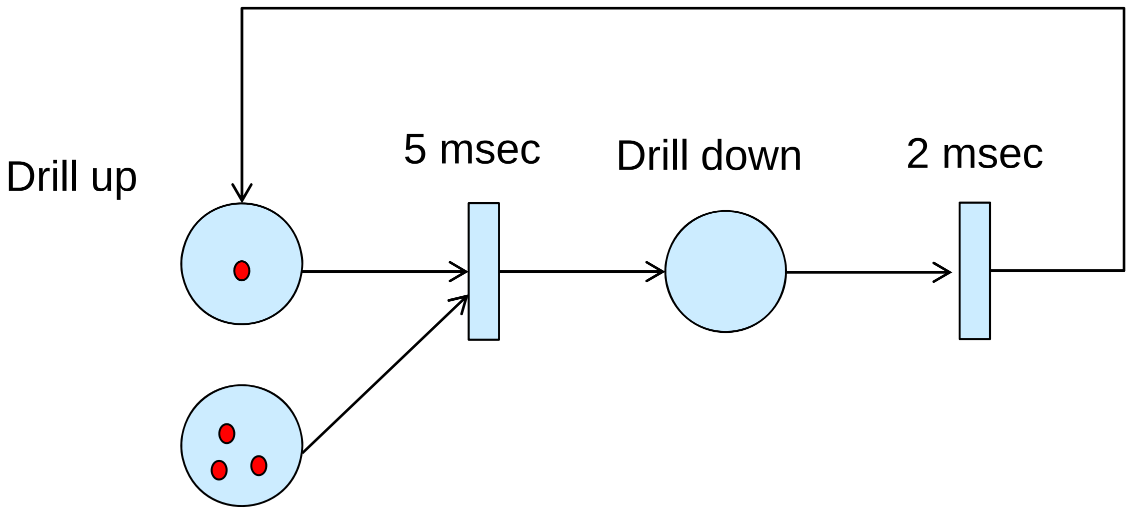
- TPN
  - Each transition is defined precisely based on connectivity and tokens needed for transition
  - Given an initial condition, the exact system state at an arbitrary future time *T* can be determined

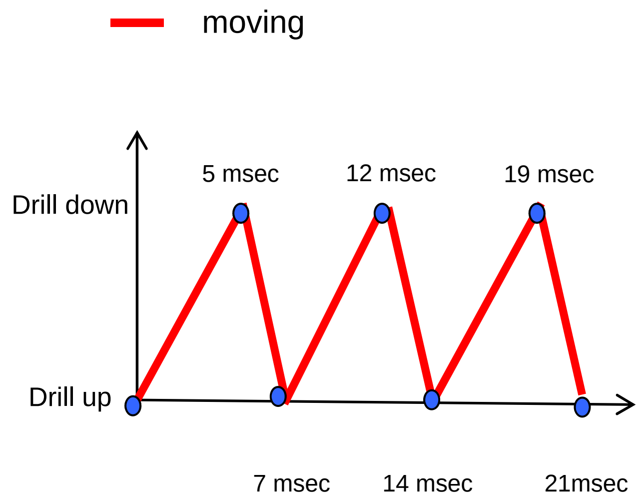
- Timed Petri Nets becomes a 7-tuple system
  - PN = (**P**,**T**,**F**,**W**,**K**, $\mathbf{M_0}$, τ)
  - $\tau = \{\tau_1, \tau_2, \ldots \tau_n\}$ is a finite set of deterministic time delays to corresponding t_i



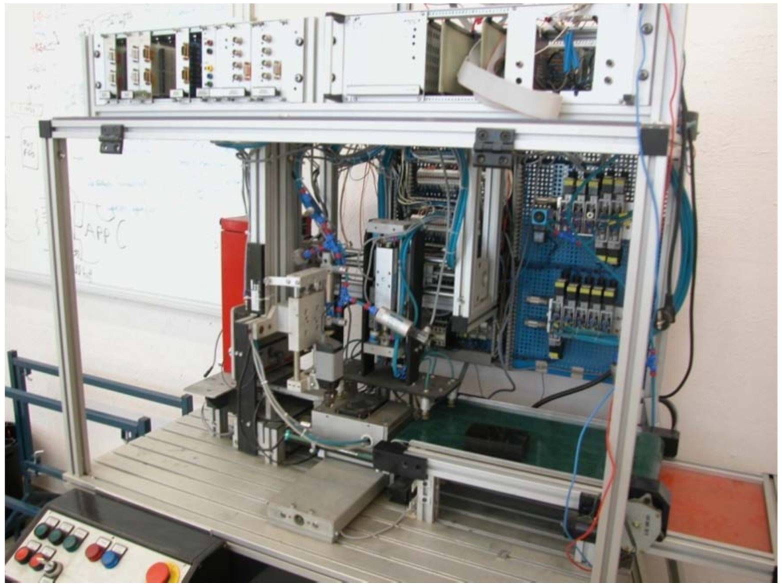
Time and Petri Nets (TPN)

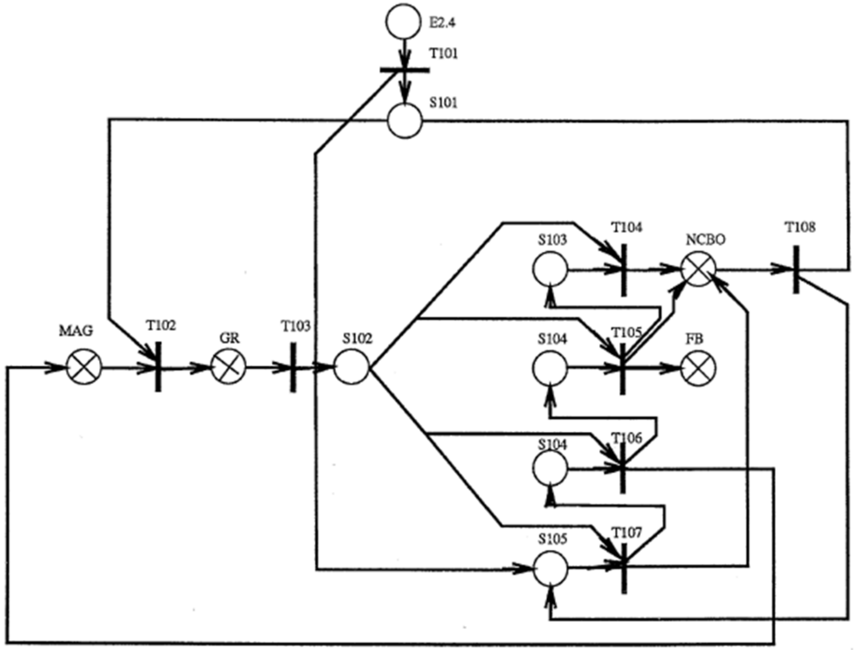
- adding (quantitative) time to PNs is to introduce temporal constraints on its elements:
  - e.g., a transition must fire after 5 msec

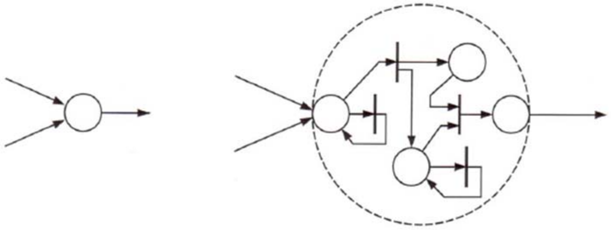


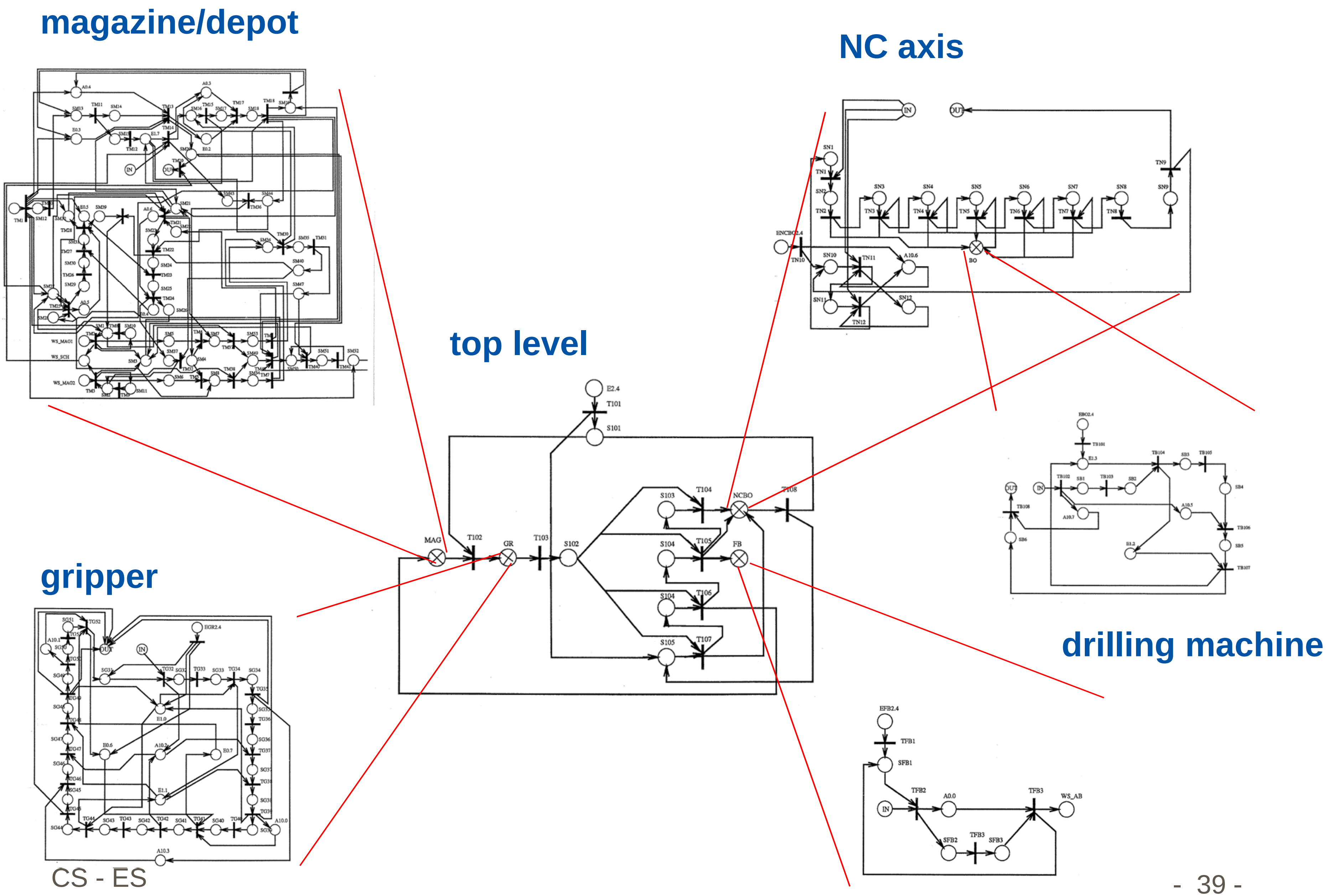


Production system - Top level petri net

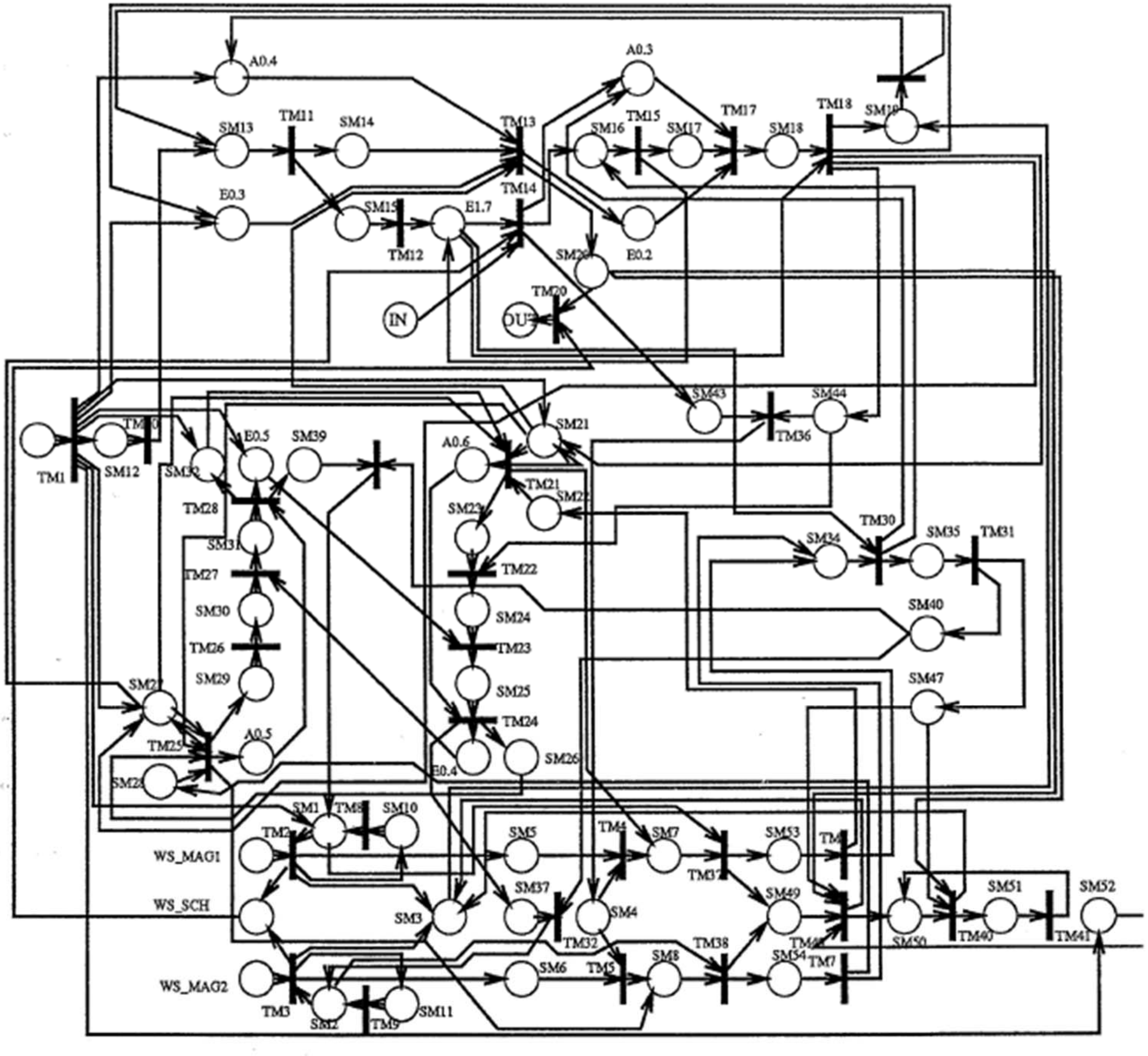




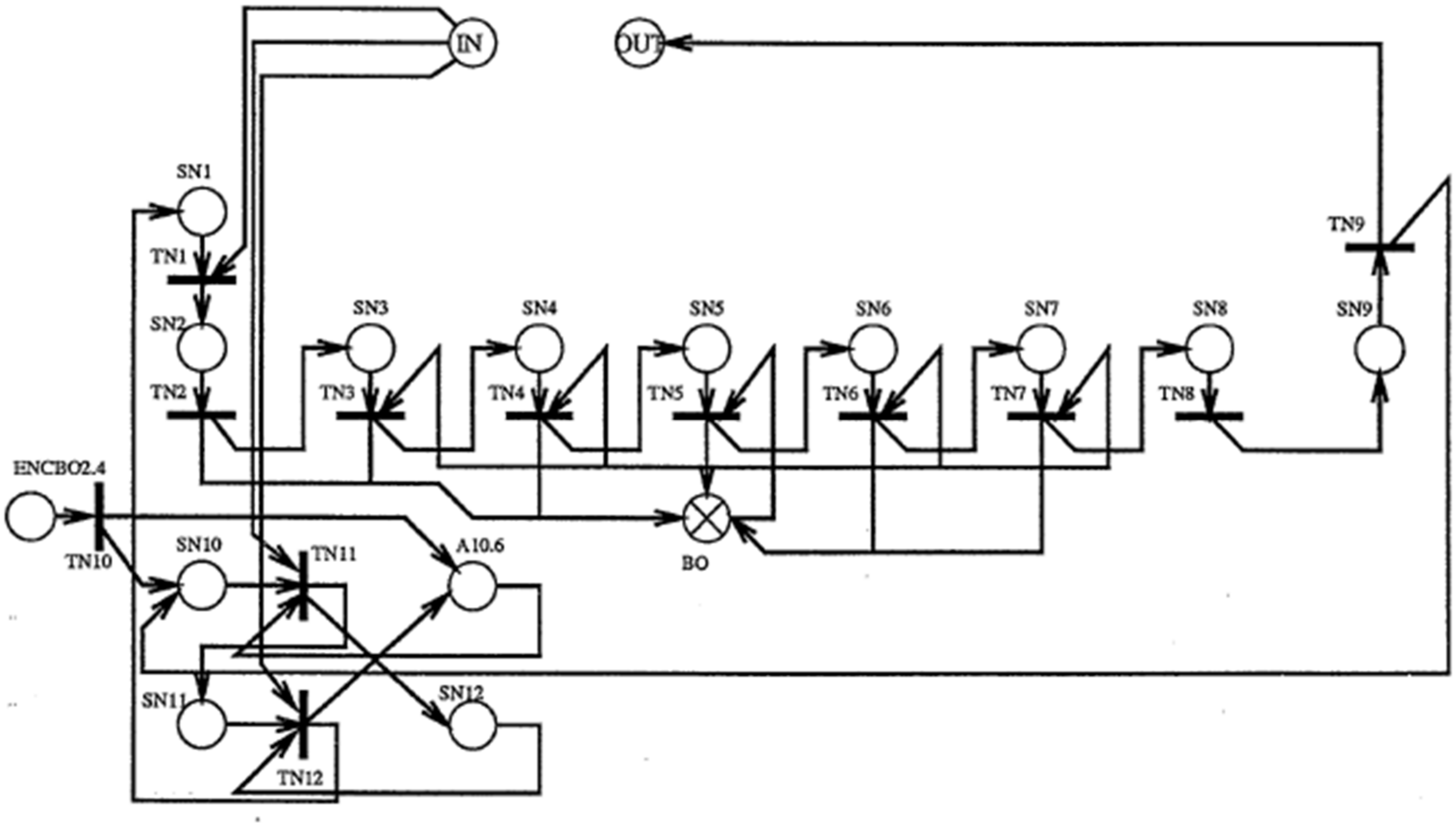



magazine/depot
NC axis
top level
gripper
drilling machine
E2.4
T101
S101
MAG
T102
GR
T103
S102
S103
T104
NCBO
T108
S104
T105
FB
T106
T107
S105
EFB2.4
TFB1
SFB1
TFB2
A0.0
TFB3
WS_AB
IN
SFB2
SFB3

magazine/depot



NC axis



IN
OUT
SN1
TN1
SN2
TN2
SN3
TN3
SN4
TN4
SN5
TN5
SN6
TN6
SN7
TN7
SN8
TN8
SN9
TN9
ENCBO2.4
TN10
SN10
TN11
A10.6
BO
SN11
SN12
TN12

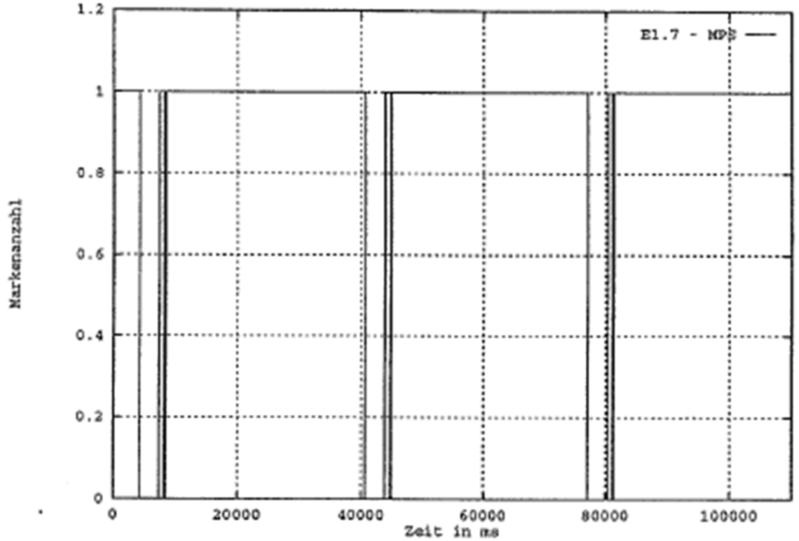


Abbildung 5.7: E1.7 – MPS (Teil 1)

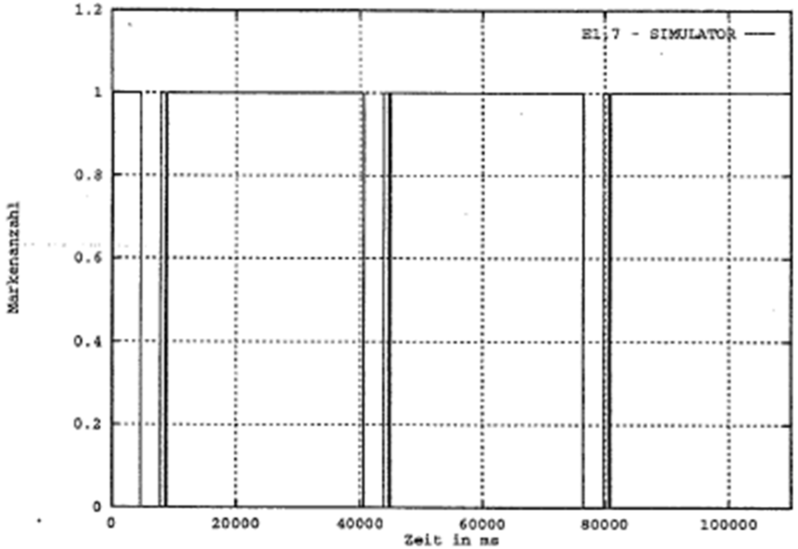


Abbildung 5.8: E1.7 – SIMULATOR (Teil 1)

Evaluation

- **Pros:**
  - Appropriate for distributed applications,
  - Well-known theory for formally proving properties,
- **Cons :**
  - PN problems with modeling timing (extensions in TPN)
  - no programming elements, no hierarchy (extensions available)
- **Extensions:**
  - Enormous amounts of efforts on removing limitations.
- **Remark:**
  - A FSM can be represented by a subclass of Petri nets, where each transition has exactly one incoming edge and one outgoing edge.

Summary

- Petri nets: focus on causal dependencies
  - Condition/event nets
    - Single token per place
  - Place/transition nets
    - Multiple tokens per place
  - Predicate/transition nets
    - Tokens become individuals
    - Dining philosophers used as an example
  - Extensions required to get around limitations

SDL - Specification and Description *Language*

SDL - Specification and Description *Language*

- Used here as a (prominent) example of a model of computation based on **asynchronous message passing communication**.
- ☞ appropriate also for distributed systems
- Language designed for specification of distributed systems.
    - Dates back to early 70s,
    - Formal semantics defined in the late 80s,
    - Defined by ITU (International Telecommunication Union): Z.100 recommendation in 1980
      Updates in 1984, 1988, 1992, 1996 and 1999
- Another acronym SDL ("System Design Languages")

SDL - Specification and Description *Language*

- Provides textual (tool processing) and graphical formats (user interaction)
- Ability to be used as a wide spectrum language from requirements to implementation
- Just like StateCharts, it is based on the CFSM (Communicating FSM) model of computation; each FSM is called a **process.**
- With SDL the protocol behaviour is completely specified by communicating FSM.
- The formal basis of SDL enables the use of code generation tool chains, which allows an automated implementation of the specification.

SDL - Specification and Description *Language*

- However, it uses message passing instead of shared memory for communications
- SDL supports operations on data
- object oriented description of components.

Structuring SDL designs

SDL systems can be structured in various means:

- A system consists of a number of blocks connected by channels, each block may contain a substructure of blocks or it may contain process sets connected by signals.
- Processes execute concurrently with other processes and communicate by exchanging signals; or by remote procedure calls.

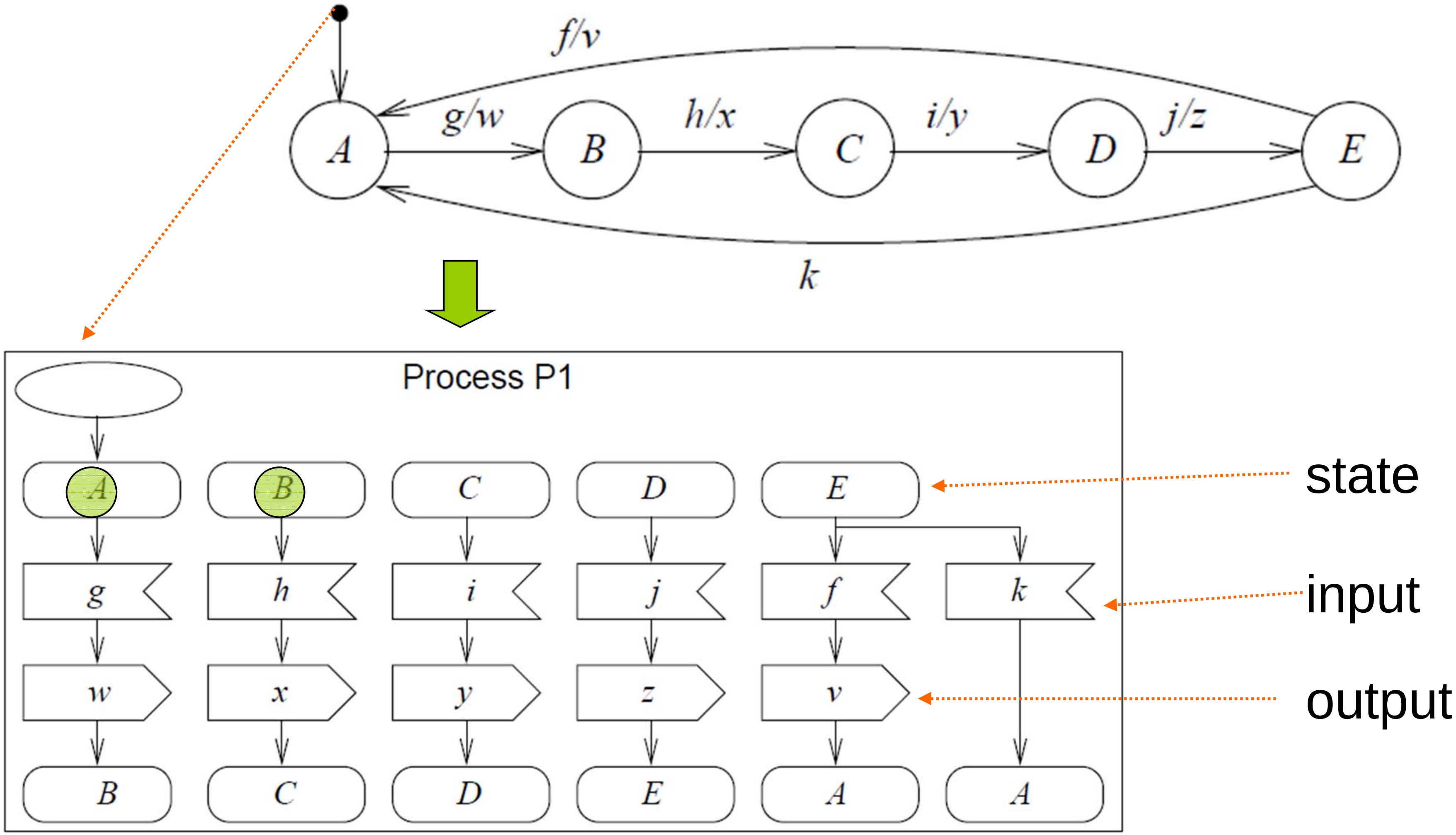
Specifying behaviour

1. The behaviour of a process is described as an extended FSM: When started, a process executes its start transition and enters the first state. (triggered by signals)

2. In transitions, a process may execute actions.

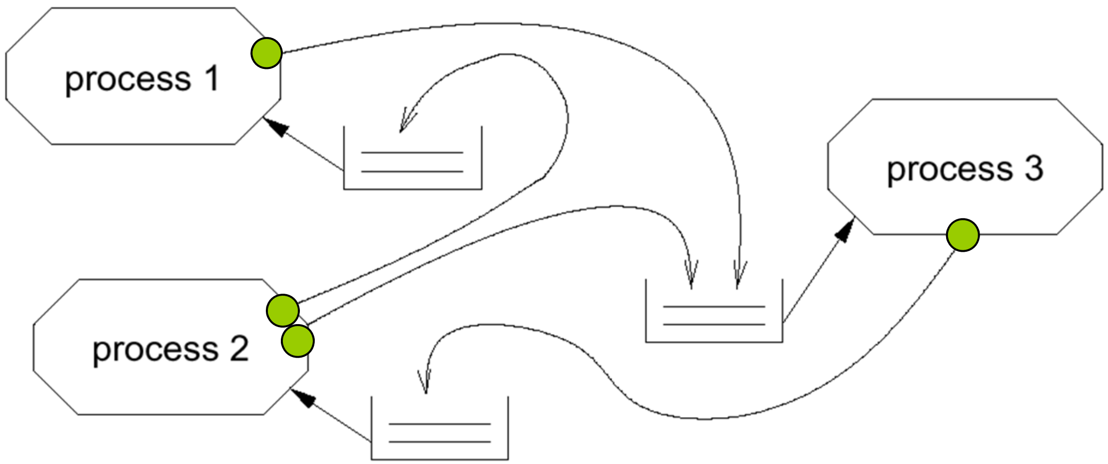
3. E.g.: Actions can assign values to variable attributes of a process, branch on values of expression, call procedures, create new processes, send signal to other processes.

SDL-representation of FSMs/processes



Communication among SDL-FSMs

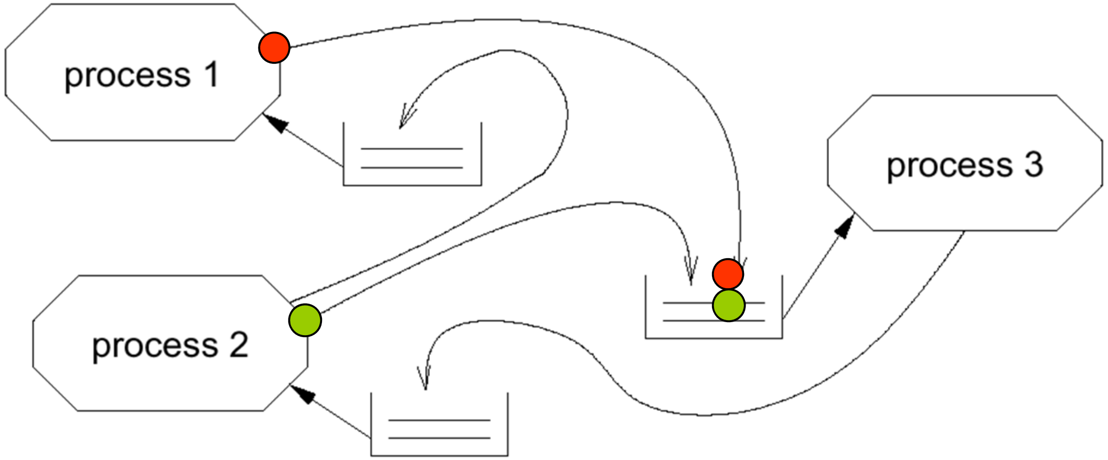
- Communication between FSMs (or “processes“) is based on **message-passing**, assuming a **potentially indefinitely large FIFO-queue**.



- Each process fetches next entry from FIFO,
- checks if input enables transition,
- if yes: transition takes place,
- if no: input is ignored (exception: SAVE-mechanism).

Determinate?

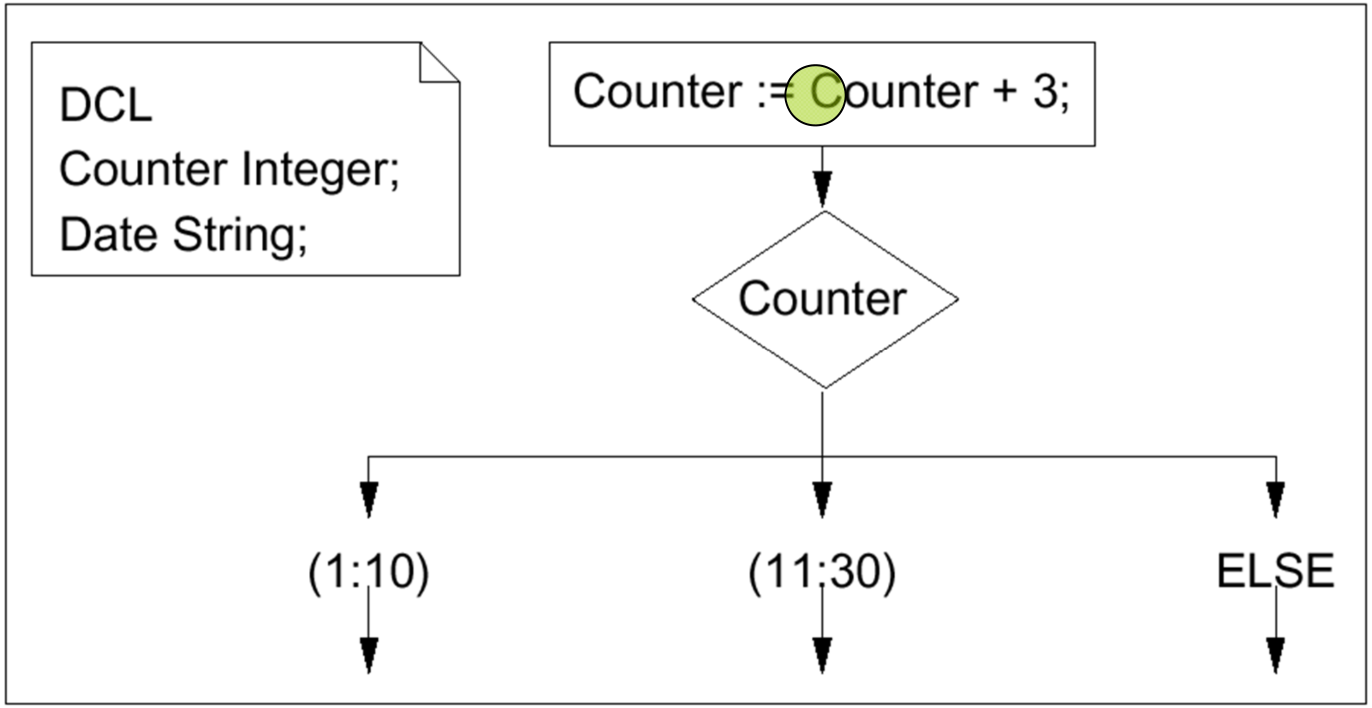
- Let tokens be arriving at FIFO at the same time:
  ☞Order in which they are stored, is unknown:



All orders are legal: ☞simulators can show different behaviors for the same input, all of which are correct.

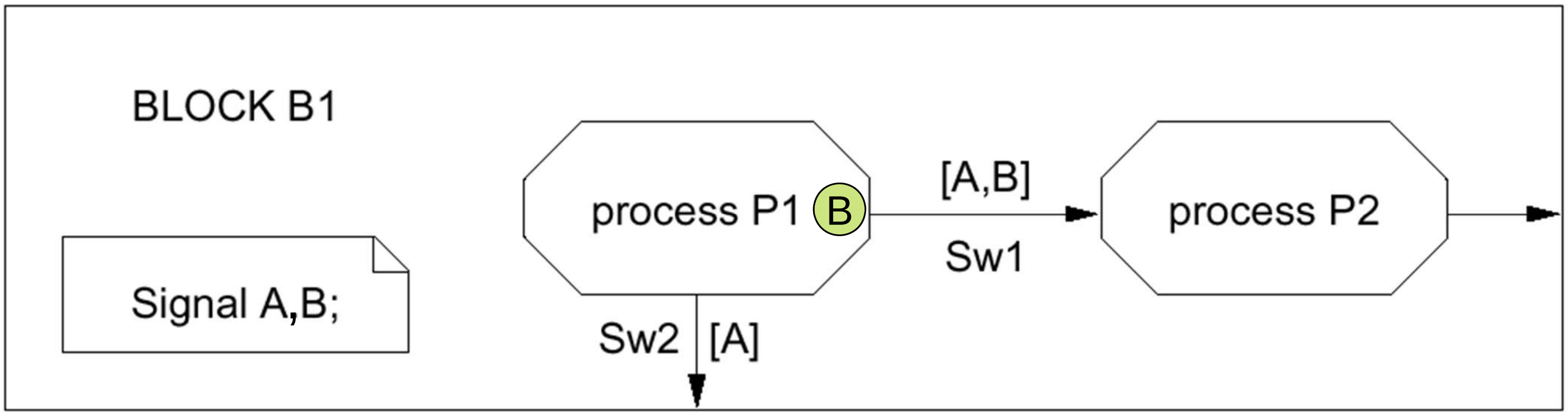
Operations on data

- Variables can be declared locally for processes.
- Their type can be predefined or defined in SDL itself.
- SDL supports abstract data types (ADTs). Examples:



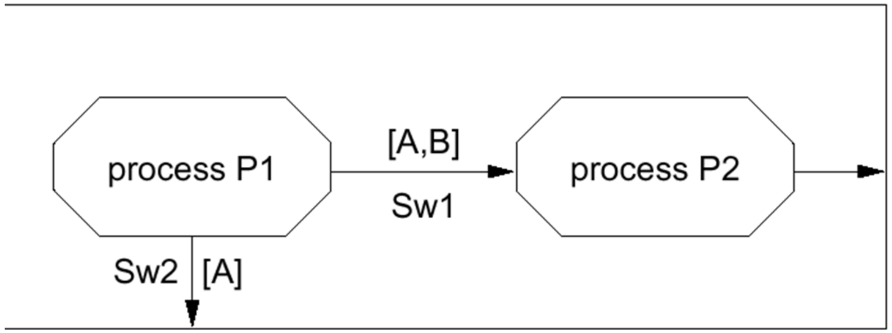
Process interaction diagrams

- Interaction between processes can be described in process interaction diagrams (special case of block diagrams).
- In addition to processes, these diagrams contain channels and declarations of local signals.
- Example:



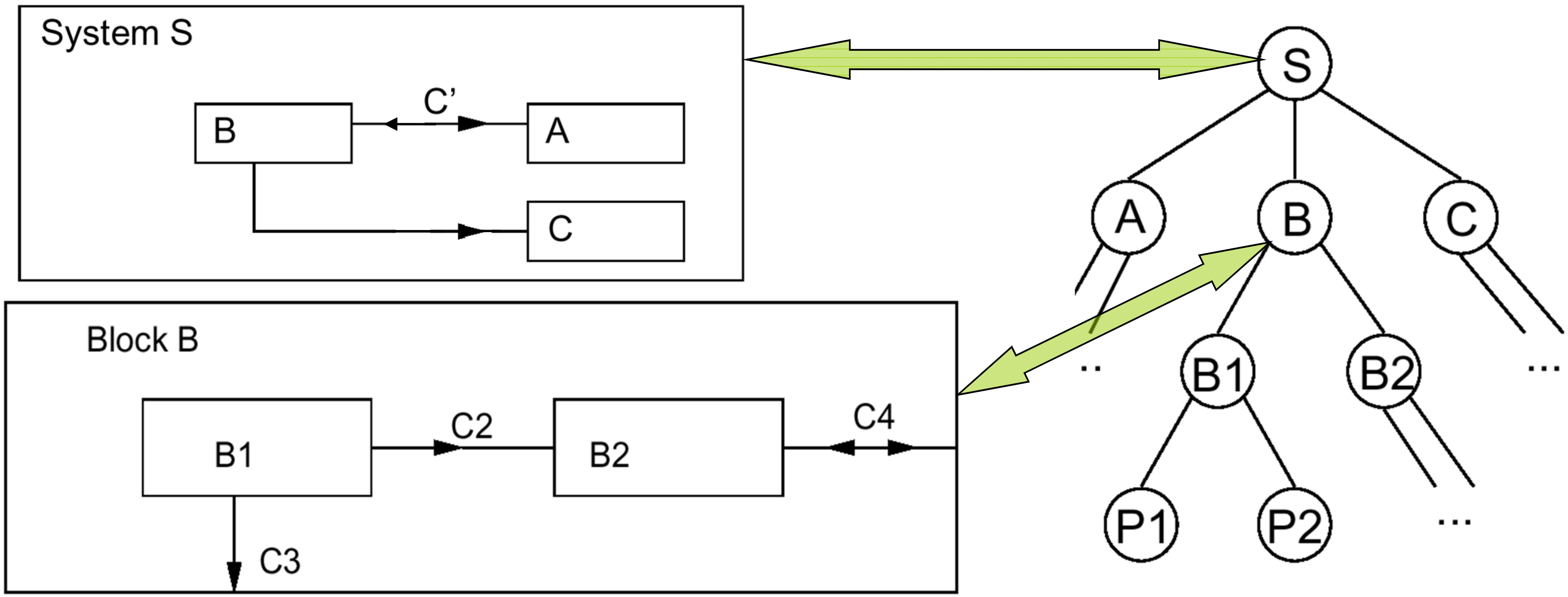
Designation of recipients

1. **Through process identifiers:** Example: OFFSPRING represents identifiers of processes generated dynamically.
2. **Explicitly:** By including the channel name.
3. **Implicitly:** If signal names imply channel names (B → Sw1)



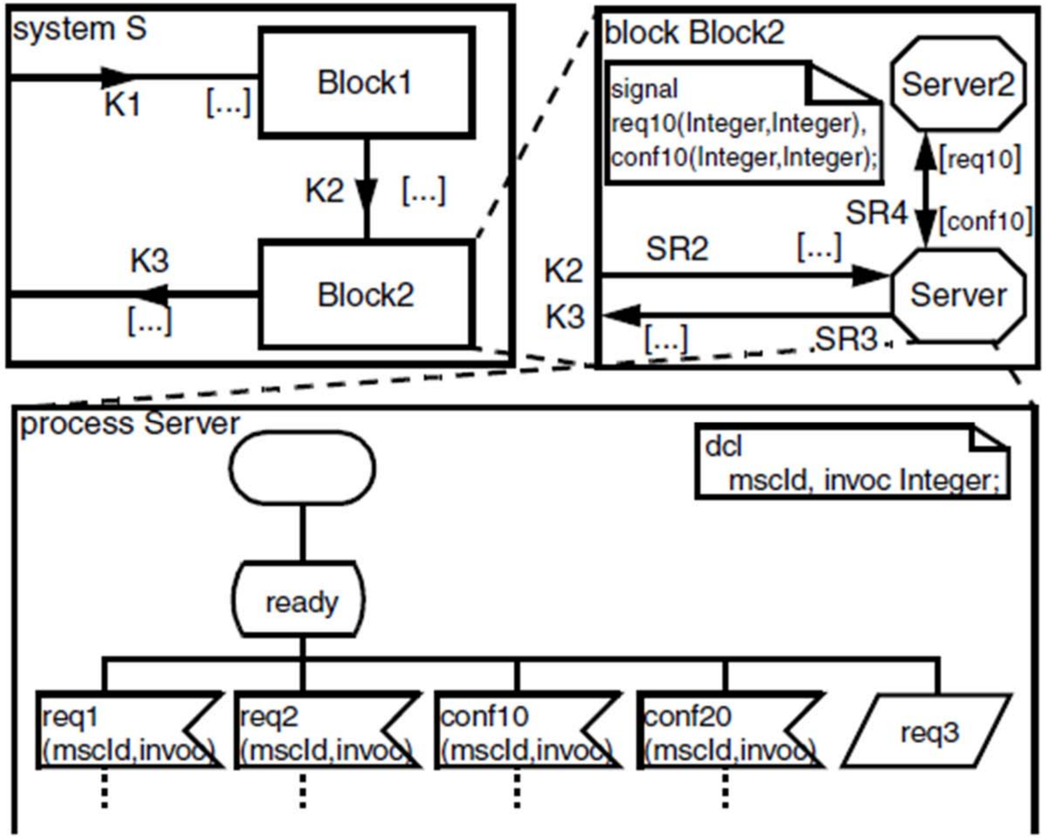
Hierarchy in SDL

- Process interaction diagrams can be included in **blocks.** The root block is called **system.**



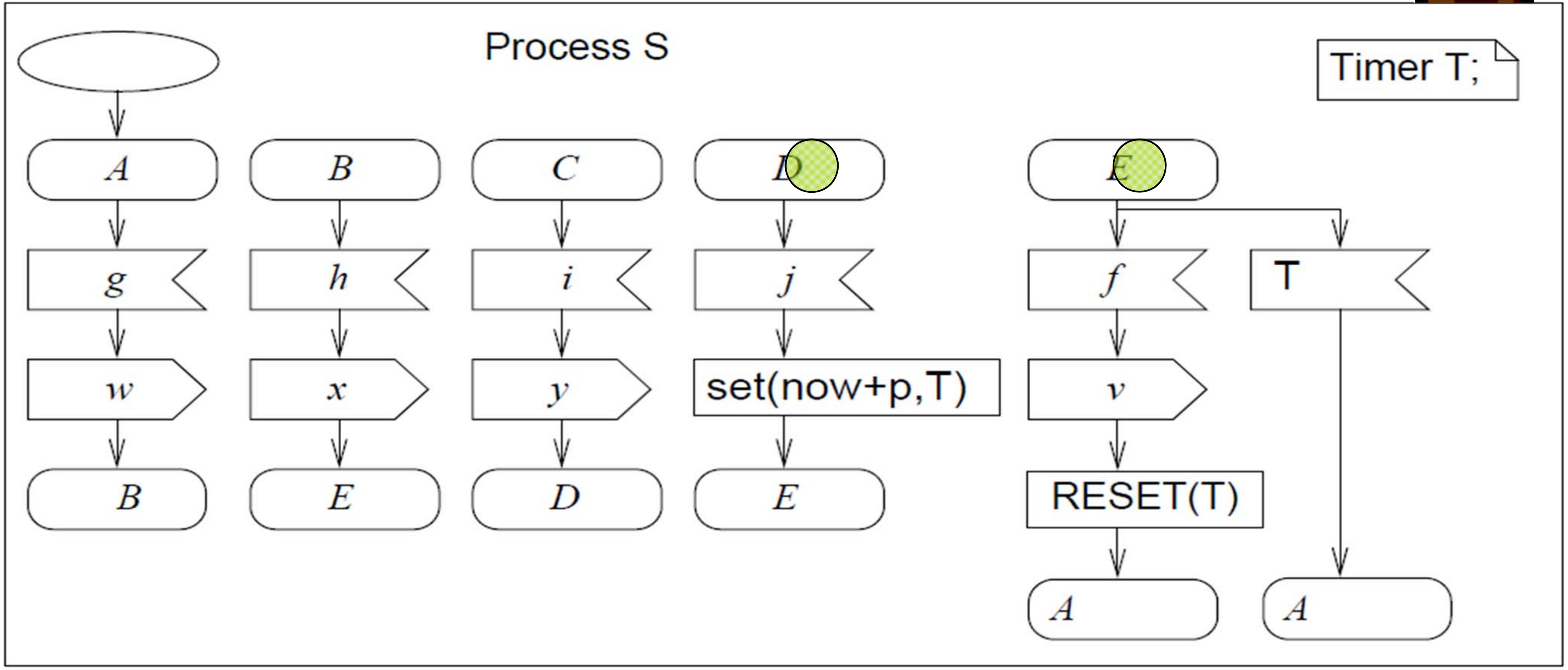
Processes cannot contain other processes, unlike in StateCharts.

Hierarchy of a SDL specification



Timers

- Timers can be declared locally. Elapsed timers put signal into queue (not necessarily processed immediately).
- RESET removes timer (also from FIFO-queue).



SDL application

The semantics of SDL defines the state space of the specification. This state space can be used for various analyses and transformation techniques, e.g.:

- state space exploration, simulation
- checking the SDL-specification for deadlocks/lifelocks
- deriving test cases automatically
- code generation for an executable prototype or end system

Summary

- MoC: finite state machine components
  + non-blocking message passing communication
- Representation of processes
- Communication & block diagrams
- Timers and other language elements
- Excellent for distributed applications (e.g., *Integrated Services Digital Network* (ISDN))
- Commercial tools available from SINTEF, Telelogic, Cinderella (//www.cinderella.dk)