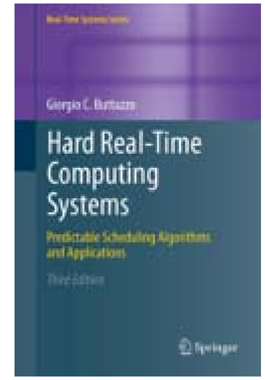
Embedded Systems 14

Scheduling



Scheduling: determine the order in which tasks are to be executed

- **Task** (≈ process ≈ thread): computation to be executed by the CPU in sequential fashion
- **Resources**: processor(s) (also: memory, disks, busses, communcitation channels, ...)
- Scheduler assigns **resources** to **tasks** for **durations of time**
- Other shared resources with exclusive access may complicate scheduling

Point of departure: Scheduling general IT systems

- In general IT systems, not much is known about the computational processes a priori
  - The set of processes to be scheduled is open:
    - New software may be inserted into the running system
    - Software is run with “random” activation patterns
  - The power of schedulers thus is inherently limited by lack of knowledge → only online scheduling is possible

Scheduling processes in ES:
The difference in process charaterization

- Most ES are “closed shops”
  - Task set of the system is known
  - at least part of their activation pattern is known
    - Periodic activation in, e.g., signal processing
    - Maximum activation frequencies of asynchronous events determinable from environment dynamics
      → minimal inter-arrival times
  - Possible to determine bounds on their execution time (WCET)
    (If they are well-built and we invest enough analysis effort)

→ Much better prospects for **guaranteeing** response times!

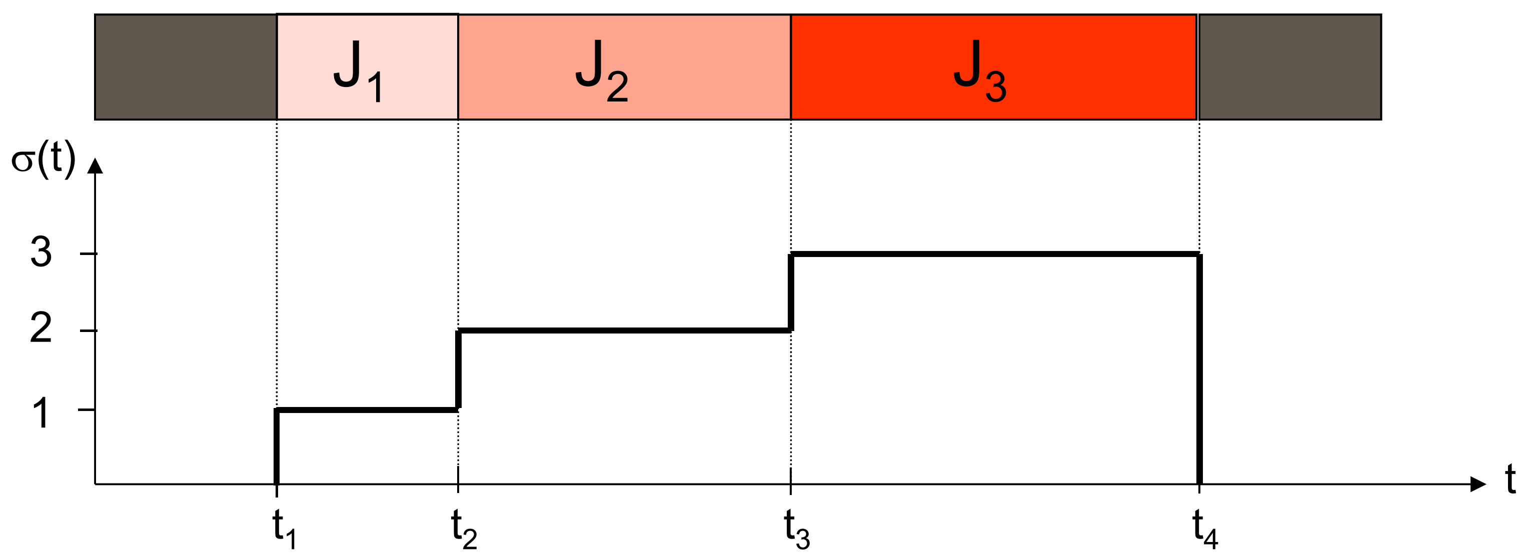
Scheduling processes in ES: The difference in goals

- In **classical OS**, quality of scheduling is normally measured in terms of performance (throughput, reaction times) in the **average case**
- In **embedded real-time systems** the schedules often have to meet stringent quality criteria under all possible execution scenarios:
  - Tasks are often connected with **hard deadlines**, which must be met under all circumstances
  - Real-time systems have to be designed for **peak load**. Scheduling should work for all anticipated situations.

Schedules

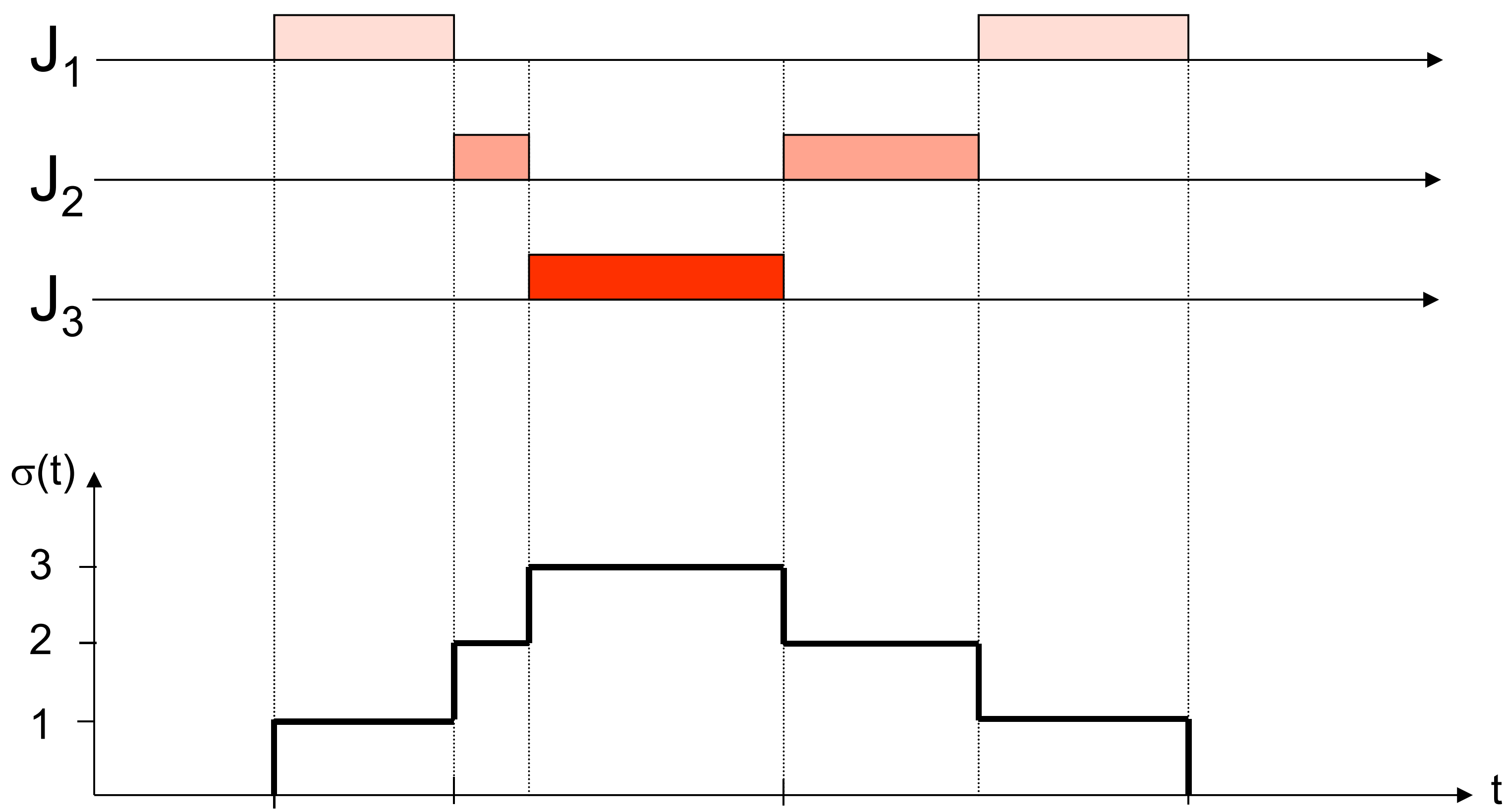
Def.: Given a set of tasks $J=\{J_1, \ldots, J_n\}$,
a **schedule** is a function $\sigma : R^+ \rightarrow \{0..n\}$ such that
$\forall\, t \in R^+, \exists\, t_1, t_2 \in R^+.\;\; t \in [t_1, t_2)\; \forall\, t' \in [t_1, t_2)\; \sigma(t) = \sigma(t')$.

In other words: σ is an integer step function
$\sigma(t) = k$, with $k > 0$, means that task J_k is executed at time t,
$\sigma(t) = 0$ means that the CPU is idle.



Preemptive schedules

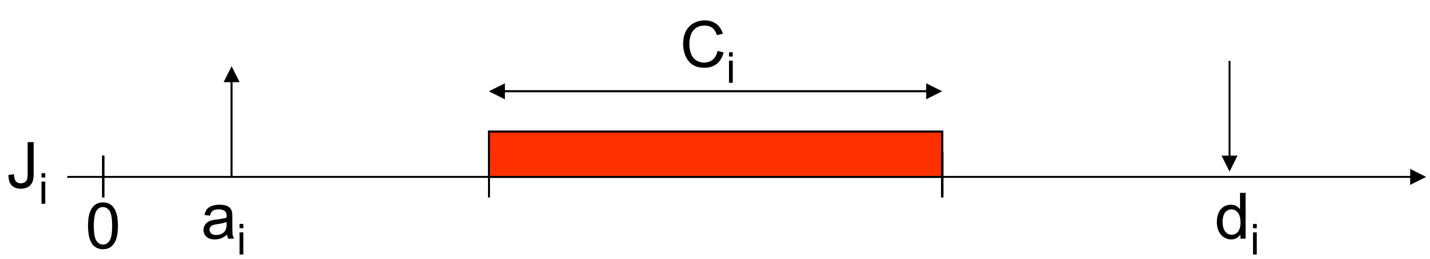
Preemption: the running task is interrupted



Timing constraints of an aperiodic task J_i

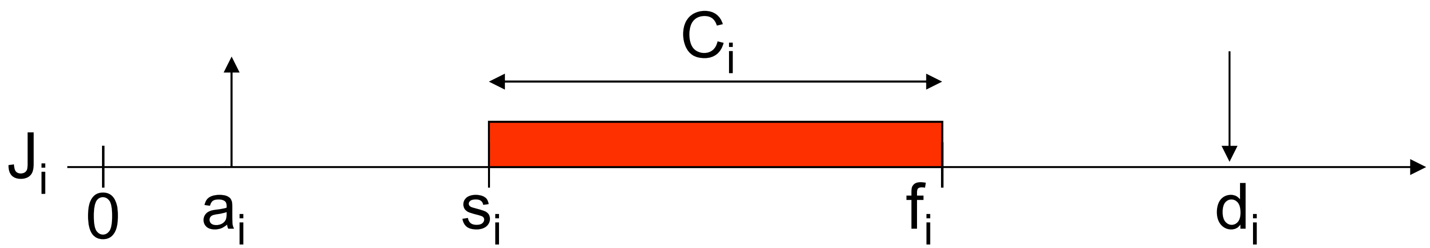
- **Arrival time a_i:** time at which task becomes ready for execution
- **Computation time C_i:** time necessary to the processor for executing the task without interruption
- **Deadline d_i:** time before which a task should be complete to avoid damage to the system

- **Slack time X_i:** $X_i = d_i - a_i - C_i$, maximum time a task can be delayed on its activation to complete within its deadline



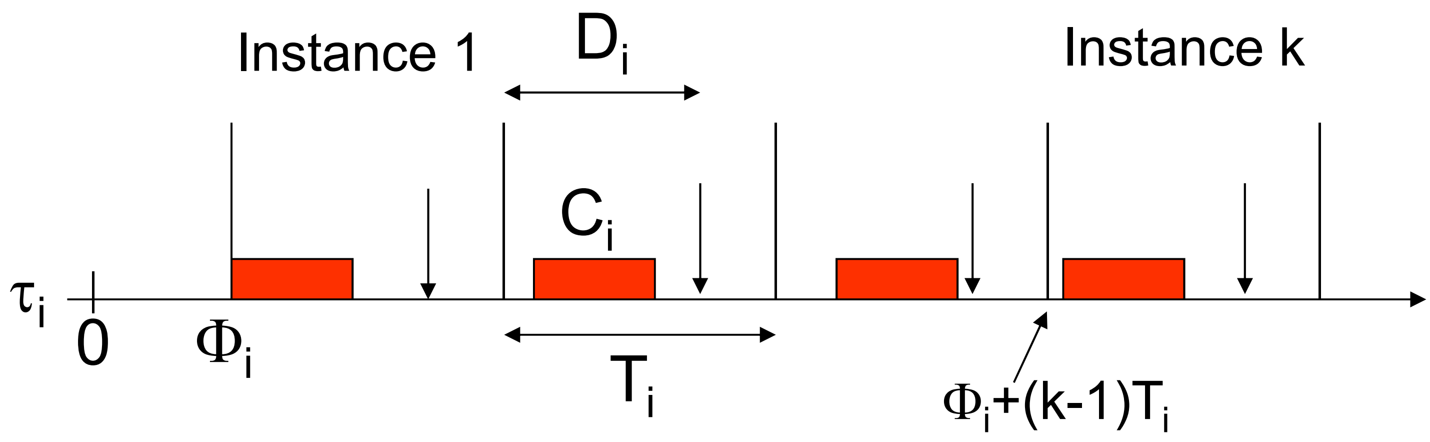
Timing parameters of J_i in schedule

- **Start time s_i:** time at which a tasks starts its execution
- **Finishing time f_i:** time at which task finishes its execution
- **Lateness L_i:** $L_i = f_i – d_i$, delay of task completion with respect to deadline
- **Exceeding time E_i:** $E_i = \max(0, L_i)$



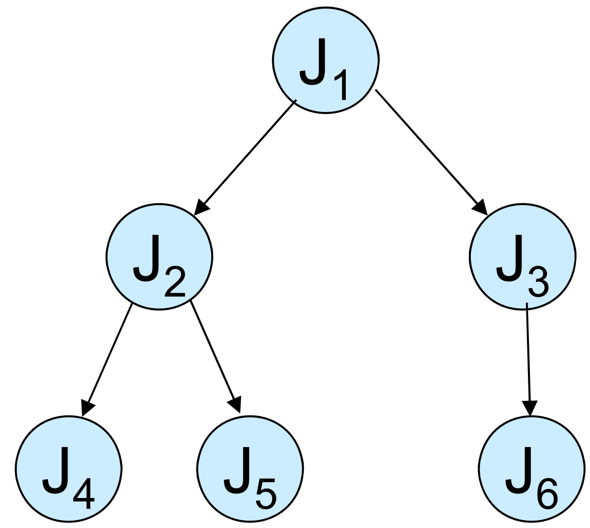
Timing constraints of periodic task τ_i

- **Phase Φ_i:** activation time of first periodic instance
- **Period T_i:** time difference between two consecutive activations
- **Relative deadline D_i:** time after activation time of an instance at which it should be complete



Precedence constraints

Precedence constraints describe a partial order $\langle$ in which the tasks can be executed:



$J_1 \langle J_2$ $J_1 \langle J_3$
$J_2 \langle J_4$ $J_2 \langle J_5$
$J_3 \langle J_6$

$J_1 \langle J_2$: J_1 is a **predecessor** of J_2
$\rightarrow$ J_1 must be executed before J_2

Schedulability

A schedule is **feasible**, if all tasks can be completed according to a set of specified constraints.

A set of tasks is **schedulable** if there exists at least one feasible schedule.

Optimal schedule: Scheduling algorithms often aim at an optimal schedule with respect to a **cost function**
Example: Maximum lateness (max_i L_i)

Spectrum of scheduling algorithms

- **preemptive vs. non-preemptive:**
  Preemptive: tasks may be interrupted
  Non-preemptive: tasks always run to completion

- **static vs. dynamic:**
  Static scheduling: takes decisions at compile time
  Dynamic scheduling: takes decisions at runtime

- **uniprocessor vs. multiprocessor**

- **optimal vs. heuristic**

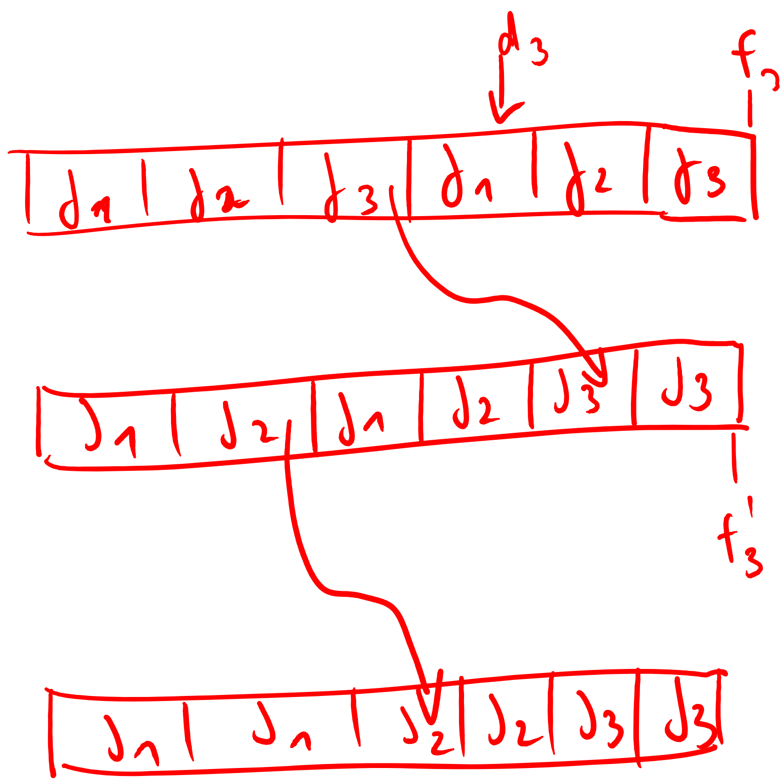
Uniprocessor aperiodic task scheduling

Aperiodic tasks with synchronous release

- A set of (aperiodic) tasks $\{J_1, \ldots, J_n\}$ with
  - arrival times $a_i = 0 \ \forall\ 1 \leq i \leq n$, i.e. "synchronous" arrival times
  - deadlines d_i,
  - computation times C_i
  - no precedence constraints, i.e. "independent tasks"
- non-preemptive
- single processor
- optimal
- find schedule which minimizes maximum lateness (variant: find feasible solution)

Preemption

- **Lemma**:
  If arrival times are synchronous, then preemption does not help, i.e. if there is a preemptive schedule with maximum lateness L_{max}, then there is also a non-preemptive schedule with maximum lateness L_{max}.



$f_3' - d_3 = f_3 - d_3$

- J_3 has same lateness
- others may have smaller lateness

Proof

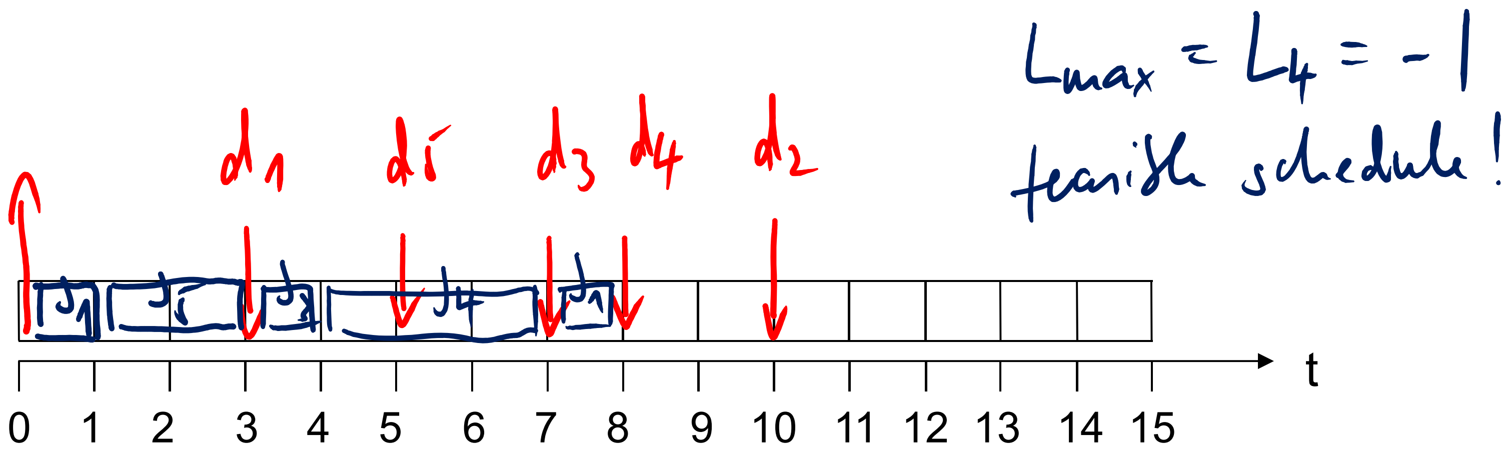
- Consider a preemptive schedule with maximum lateness L_{max}.
- If there is a task which is preempted, then choose the last point t of preemption. Let J_l be the task preempted in the schedule. Reshuffle the schedule by postponing all of J_l's runtime allocated immediately before t s.t. that it happens immediately before the time t' of resumption of J_l, thus removing the preemption at t. This will not change the lateness of J_l and will at most reduce lateness of all other tasks, as those are unaffected or shuffled forward.
- Repeat this reshuffling until there is no further preemption.

EDD – Earliest Due Date

EDD: execute the tasks in order of non-decreasing deadlines

Example 1:

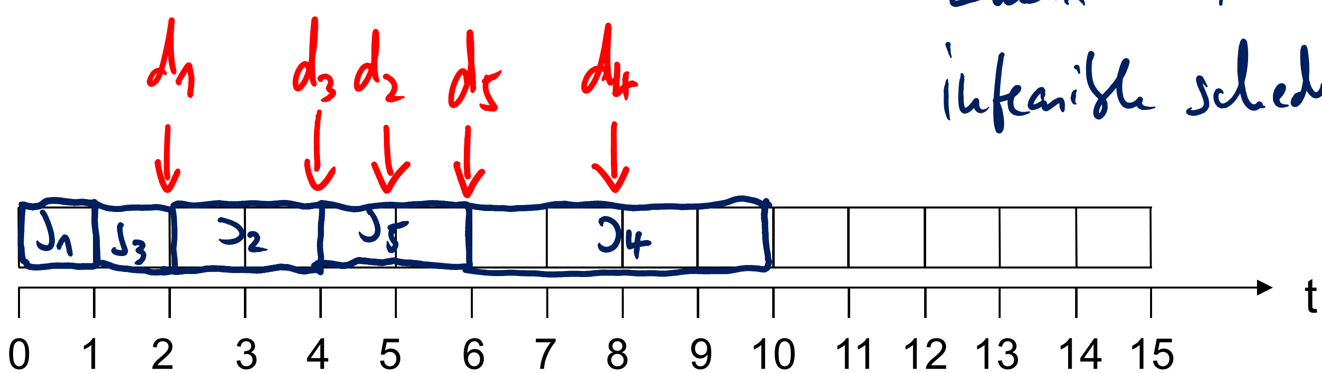
	J_1	J_2	J_3	J_4	J_5
C_i	1	1	1	3	2
d_i	3	10	7	8	5



EDD

Example 2:

	J_1	J_2	J_3	J_4	J_5
C_i	1	2	1	4	2
d_i	2	5	4	8	6



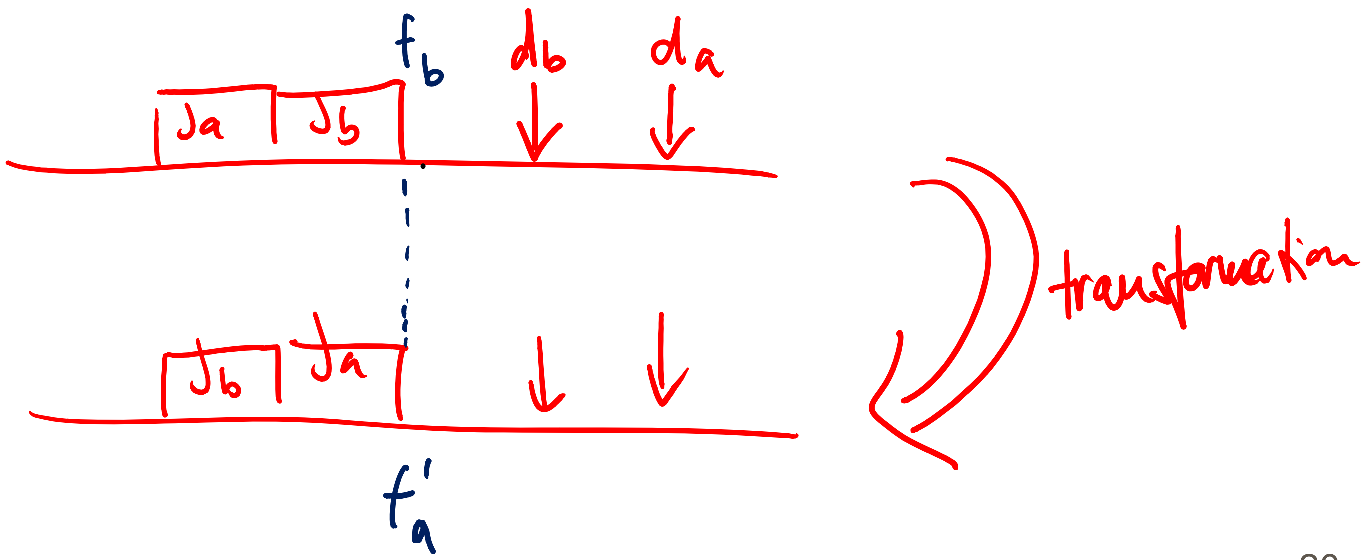
$L_{max} = L_4 = 2$

infeasible schedule!

EDD

- **Theorem (Jackson ’55):**
  Given a set of independent tasks with synchronous arrival times, any algorithm that executes the tasks in order of non-decreasing deadlines is optimal with respect to minimizing the maximum lateness.

- **Remark:** Minimizing maximum lateness includes finding a feasible schedule, if it exists. The reverse is not necessarily true.



$$L'_{max_{a,b}} = \max \{ f'_a - d_a , f'_b - d_b \}$$

$$f'_a - d_a = f_b - d_a \leq f_b - d_b$$

$$f'_b - d_b \leq f_b - d_b$$

$$\Rightarrow L'_{max_{a,b}} \leq L_{max_{a,b}}$$

EDD

- **Complexity of EDD scheduling:**
  - Sorting n tasks by increasing deadlines

  $\rightarrow$ O(n log n)

- **Test of Schedulability:**
  If the conditions of the EDD algorithm are fulfilled, schedulability can be checked in the following way:
  - Sort task wrt. non-decreasing deadline.
    Let w.l.o.g. $J_1, \ldots, J_n$ be the sorted list.
  - Check whether in an EDD schedule $f_i \leq d_i \ \forall\ i = 1, \ldots, n$.
    Since $f_i = \sum_{k=1}^{i} C_k$, we have to check

$$\forall\ i = 1, \ldots, n \quad \sum_{k=1}^{i} C_k \leq d_i$$

- Since EDD is optimal, non-schedulability by EDD implies non-schedulability in general.

Aperiodic tasks with asynchronous release

- A set of (a-periodic) tasks $\{J_1, \ldots, J_n\}$ with
  - **arbitrary** arrival times a_i
  - deadlines d_i,
  - computation times C_i
  - no precedence constraints, i.e., “independent tasks”
- **preemptive**
- Single processor
- Optimal
- Find schedule which minimizes maximum lateness (variant: find feasible solution)

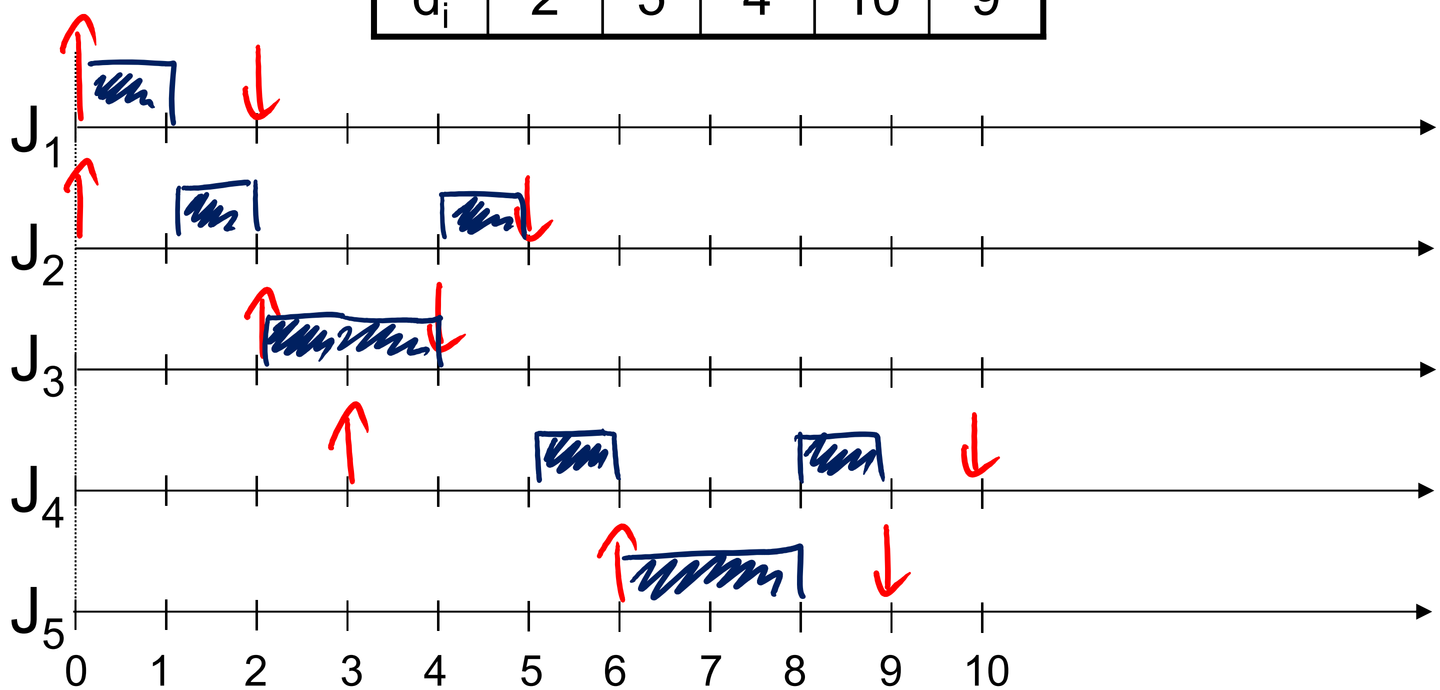
EDF – Earliest Deadline First

- At every instant, execute the task with the earliest deadline among all the ready tasks.

- Remark:
  1. If a new task arrives with an earlier deadline than the running task, the running task is immediately preempted.
  2. Here we assume that the time needed for context switches is negligible (we'll later see that this is unrealistic).

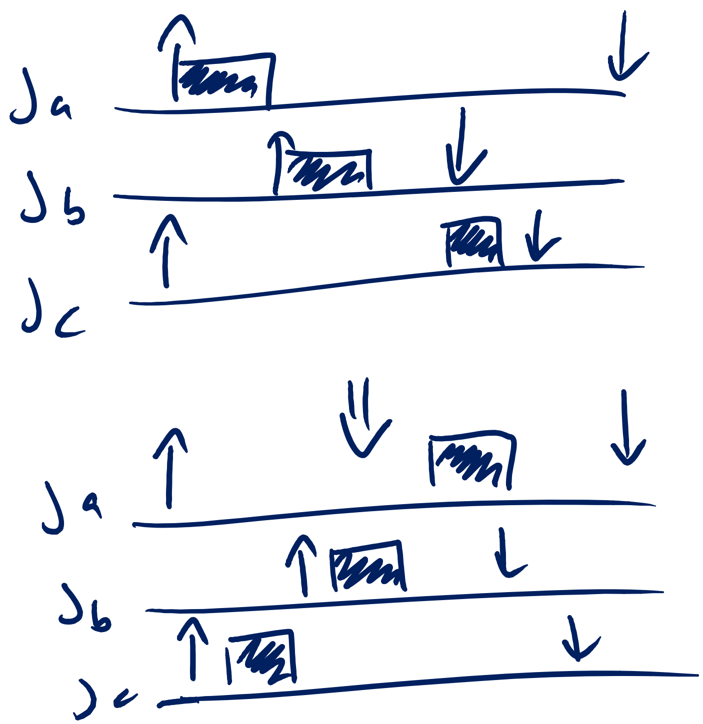
EDF - Example

	J_1	J_2	J_3	J_4	J_5
a_i	0	0	2	3	6
C_i	1	2	2	2	2
d_i	2	5	4	10	9



EDF

- **Theorem (Horn '74):**
  Given a set of independent task with arbitrary arrival times, any algorithm that at every instant executes the task with the earliest deadline among all the ready tasks is optimal with respect to minimizing the maximum lateness.



$$f_a' - d_a \le f_c - d_c$$

$$f_c' - d_c \le f_c - d_c$$

(same idea as for Jackson's thm.)

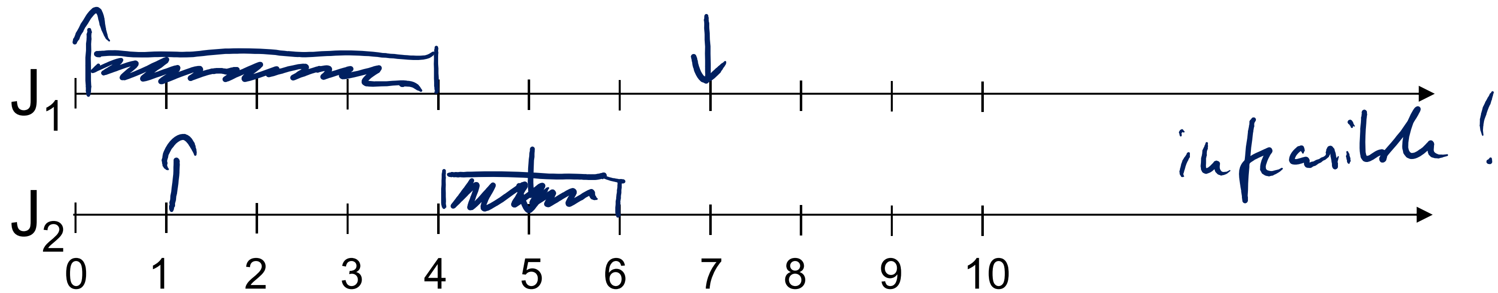
Non-preemptive version

- Changed problem:
  - A set of (a-periodic) tasks $\{J_1, \ldots, J_n\}$ with
    - **arbitrary** arrival times a_i
    - deadlines d_i,
    - computation times C_i
    - no precedence constraints, i.e., “independent tasks”
  - **Non-preemptive** instead of preemptive scheduling
  - Single processor
  - Optimal
  - Find schedule which minimizes maximum lateness (variant: find feasible solution)

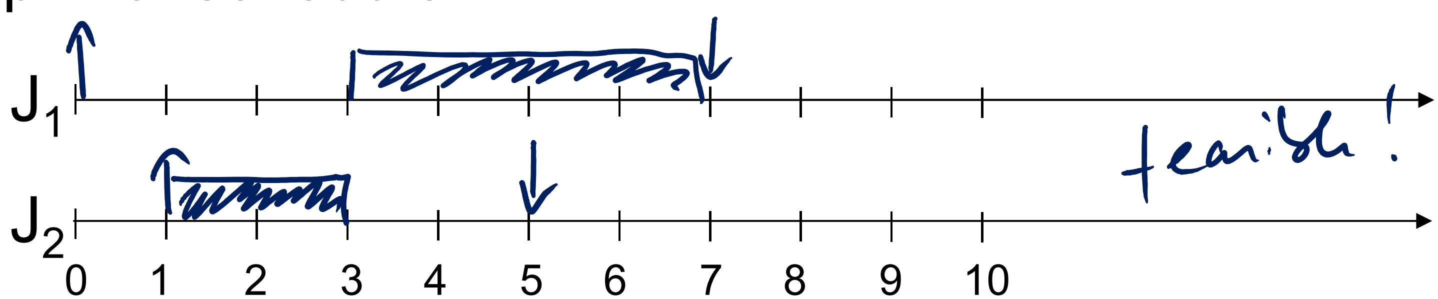
Example

	J_1	J_2
a_i	0	1
C_i	4	2
d_i	7	5

- Non-preemptive EDF schedule:



- Optimal schedule:



Example

- Observation:
  - In the optimal schedule the processor remains idle in intervall [0,1) although task J_1 is ready to execute.
- If arrival times are not known a-priori, then no on-line algorithm is able to decide whether to stay idle at time 0 or to execute J_1.

- **Theorem** (Jeffay et al. ’91): EDF is an optimal *non-idle* scheduling algorithm also in a non-preemptive task model.

Non-preemptive scheduling: better schedules through introduction of idle times

- Assumptions:
  - Arrival times known a priori.
  - Non-preemptive scheduling
  - “Idle schedules” are allowed.
- Goal:
  - Find feasible schedule
- Problem is NP-hard.
- Possible approaches:
  - Heuristics
  - Branch-and-bound

Bratley's algorithm

- Bratley's algorithm
  - Finds feasible schedule by branch-and-bound, if there exists one
  - Schedule derived from appropriate permutation of tasks $J_1, \ldots, J_n$
  - Starts with empty task list
  - Branches: Selection of next task (one not scheduled so far)
  - Bound:
    - Feasible schedule found at current path -> search path successful
    - There is some task not yet scheduled whose addition causes a missed deadline -> search path is blind alley

Bratley's algorithm

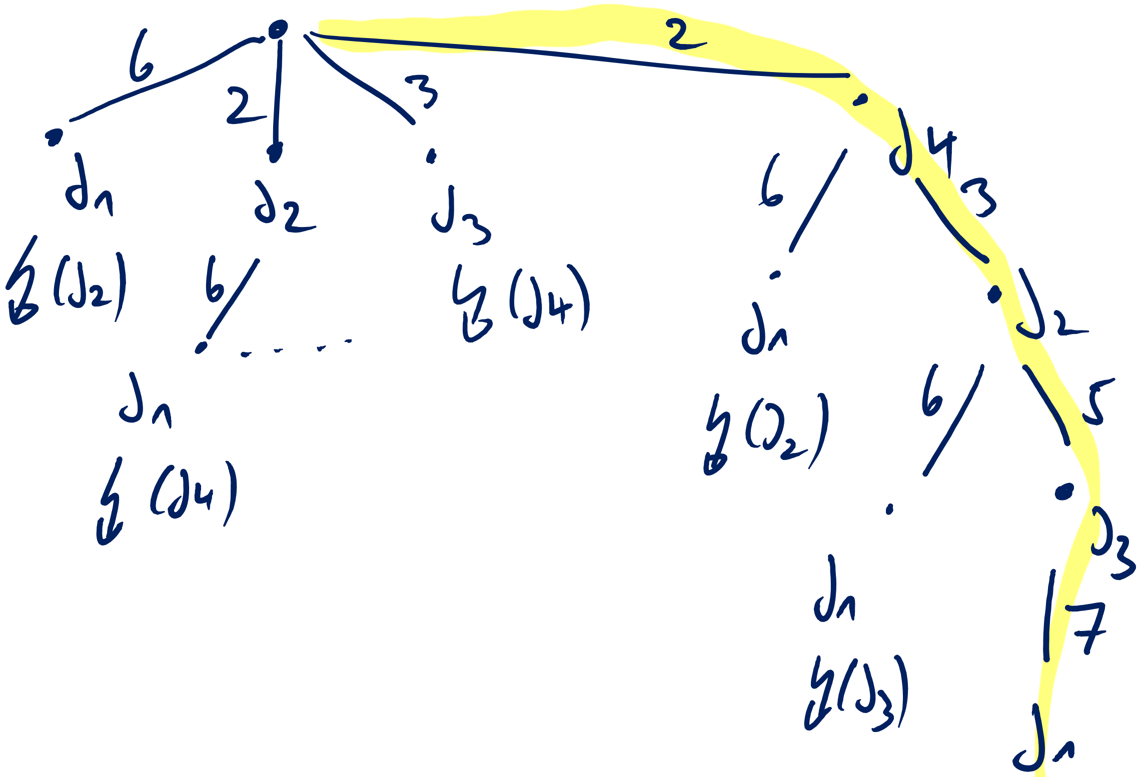
- Example:

	J_1	J_2	J_3	J_4
a_i	4	1	1	0
C_i	2	1	2	2
d_i	7	5	6	4

Bratley's algorithm

- Example:

	J_1	J_2	J_3	J_4
a_i	4	1	1	0
C_i	2	1	2	2
d_i	7	5	6	4



Bratley's algorithm

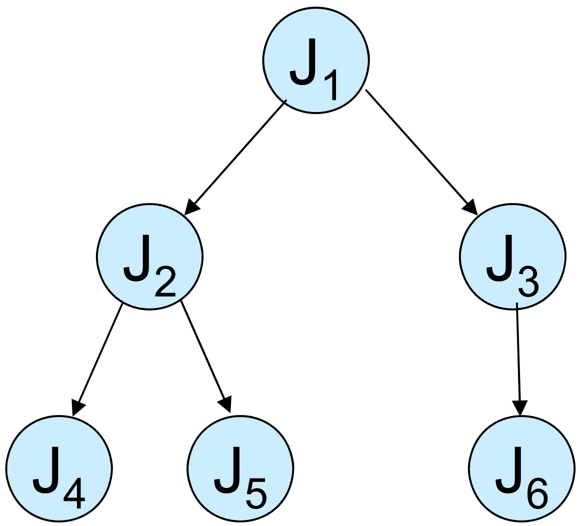
- Due to exponential worst-case complexity only applicable as off-line algorithm.
- Resulting schedule stored in task activation list.
- At runtime: dispatcher simply extracts next task from activation list.

Case 3: Scheduling with precedence constraints

- Non-preemptive scheduling with non-synchronous arrival times, deadlines and precedence constraints is **NP-hard.**
- Here restriction: synchronous arrival times (all tasks arrive at 0)

Example

	J_1	J_2	J_3	J_4	J_5	J_6
a_i	0	0	0	0	0	0
C_i	1	1	1	1	1	1
d_i	2	5	4	3	5	6



Example

One of the following algorithms is optimal. Which one?

Algorithm 1:

1. Among all **sources** in the precedence graph select the task T with **earliest** deadline. Schedule T **first**.
2. Remove T from G.
3. Repeat.

Algorithm 2:

1. Among all **sinks** in the precedence graph select the task T with **latest** deadline. Schedule T **last**.
2. Remove T from G.
3. Repeat.