# Embedded Systems 5

# REVIEW: Petri Nets

**Def.:** $N=(C,E,F)$ is called a **Petri net**, iff the following holds

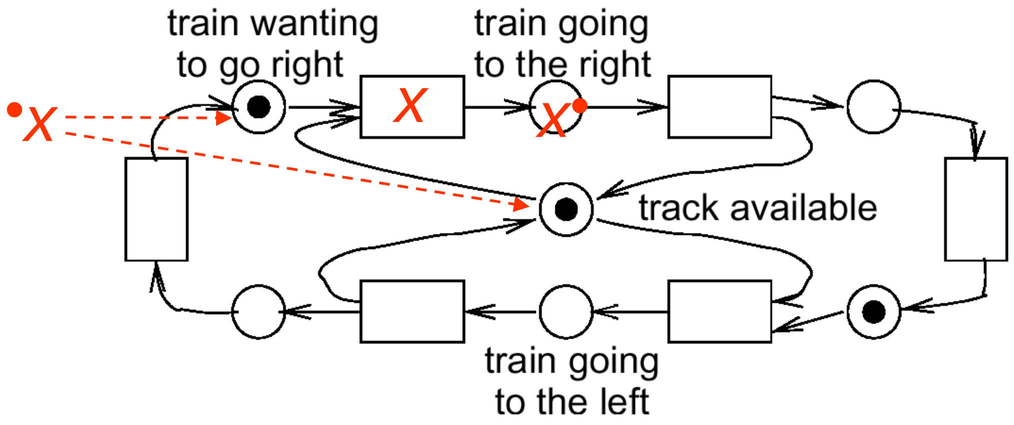
1. $C$ and $E$ are disjoint sets
2. $F \subseteq (C \times E) \cup (E \times C)$; is binary relation, („**flow relation**")

**Def.:** Let $N$ be a net and let $x \in (C \cup E)$.

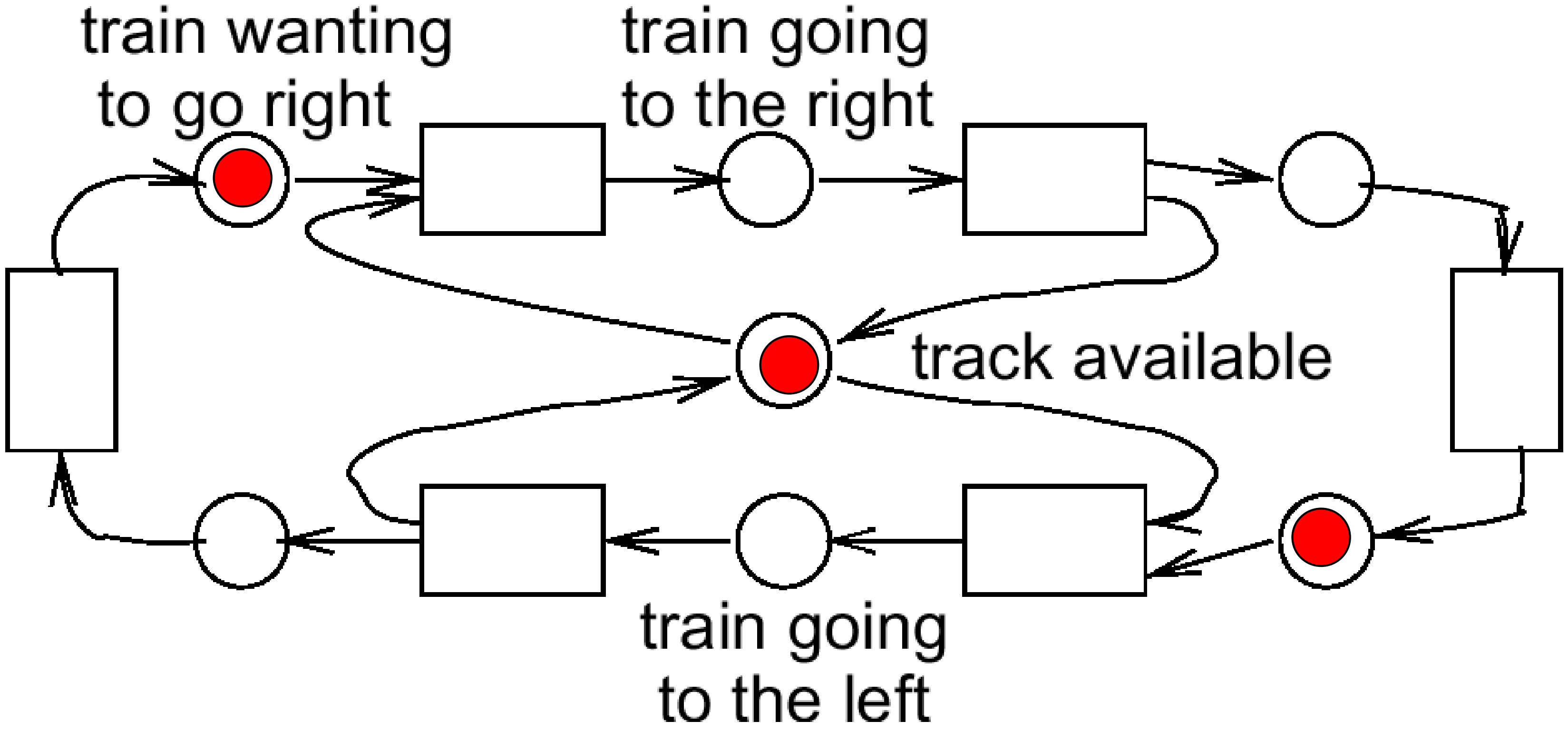
$^{\bullet}x := \{y \mid y\ F\ x\}$ is called the set of **preconditions.**

$x^{\bullet} := \{y \mid x\ F\ y\}$ is called the set of **postconditions.**

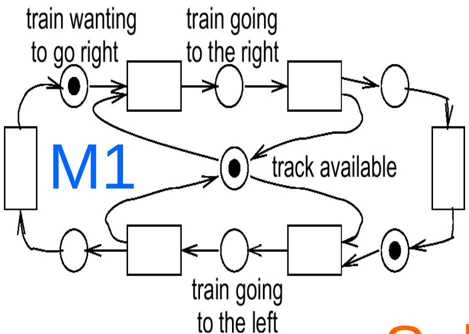
**Example:**



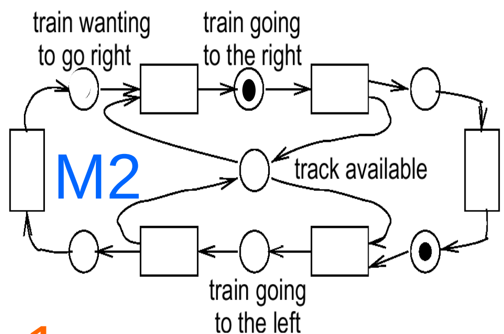
# Competing Trains Example: Conflict for resource „track“

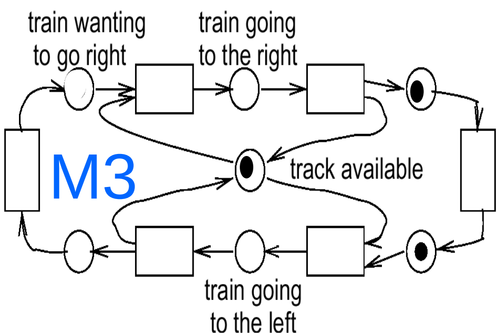


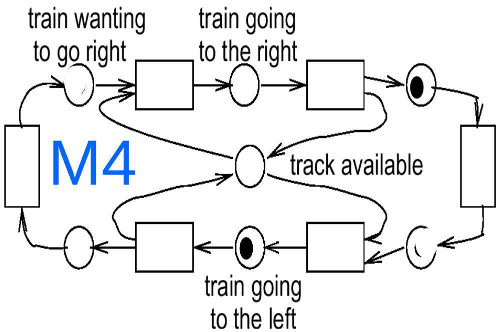
# Competing Trains Example: Boolean marking and computing changes of markings
# Reachable markings

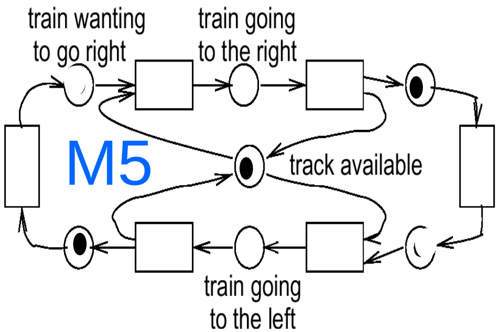


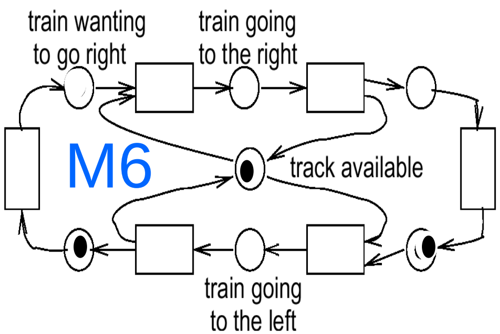

Split 1.1



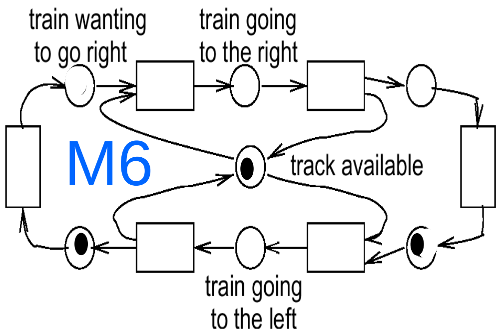



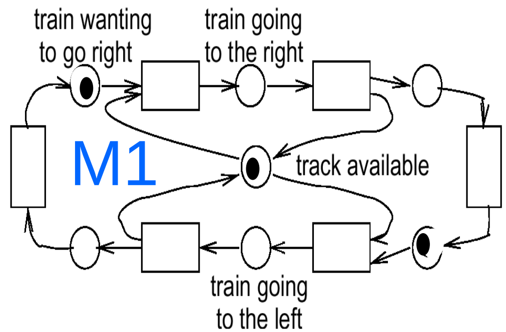

Split 4.1

Split 5.1

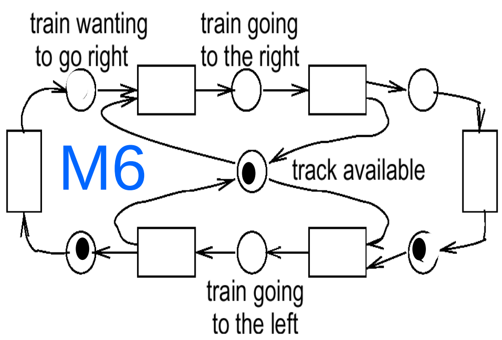
# Competing Trains Example: Boolean marking and computing changes of markings
# Reachable markings

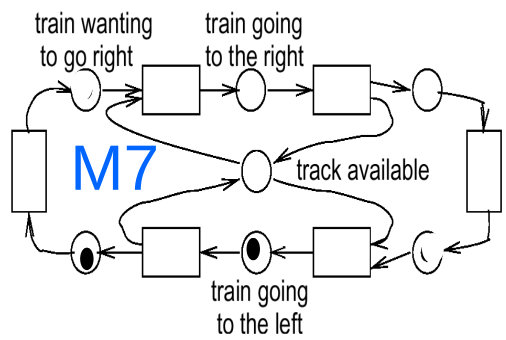


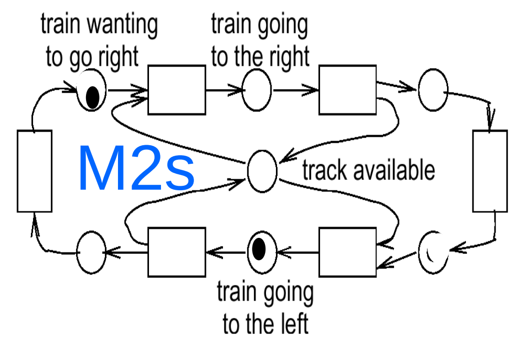

Split 6.1



# Competing Trains Example: Boolean marking and computing changes of markings
# Reachable markings

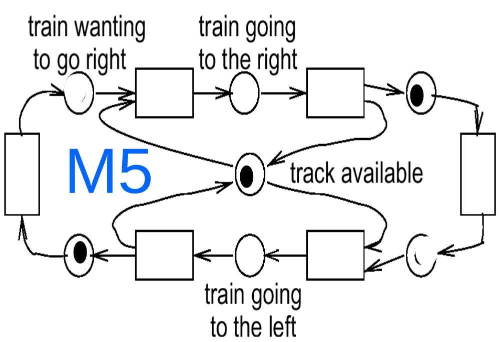


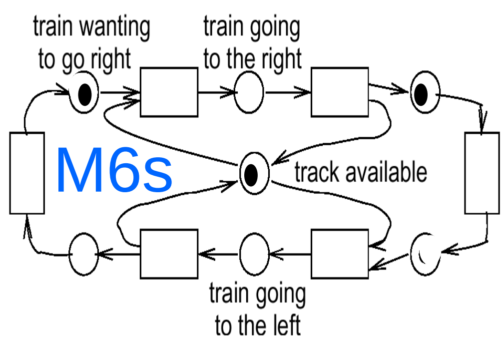




Split 6.2

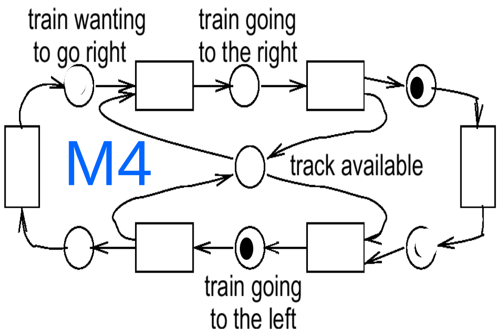
# Competing Trains Example: Boolean marking and computing changes of markings
# Reachable markings

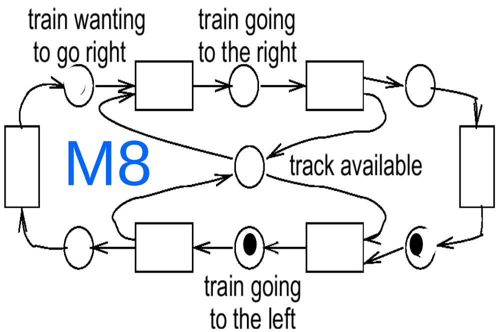


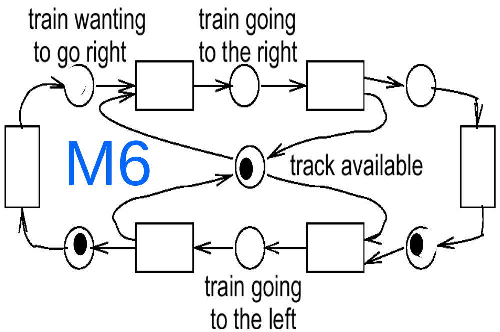




Split 5.2

# Competing Trains Example: Boolean marking and computing changes of markings
# Reachable markings

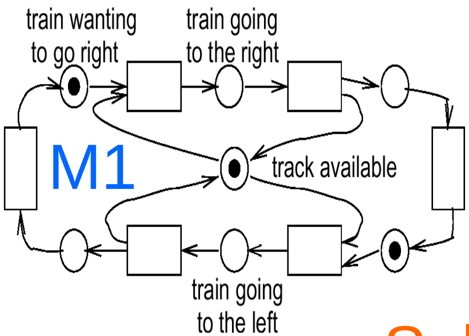


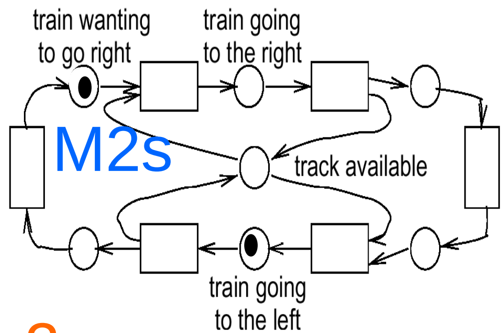




Split 4.2

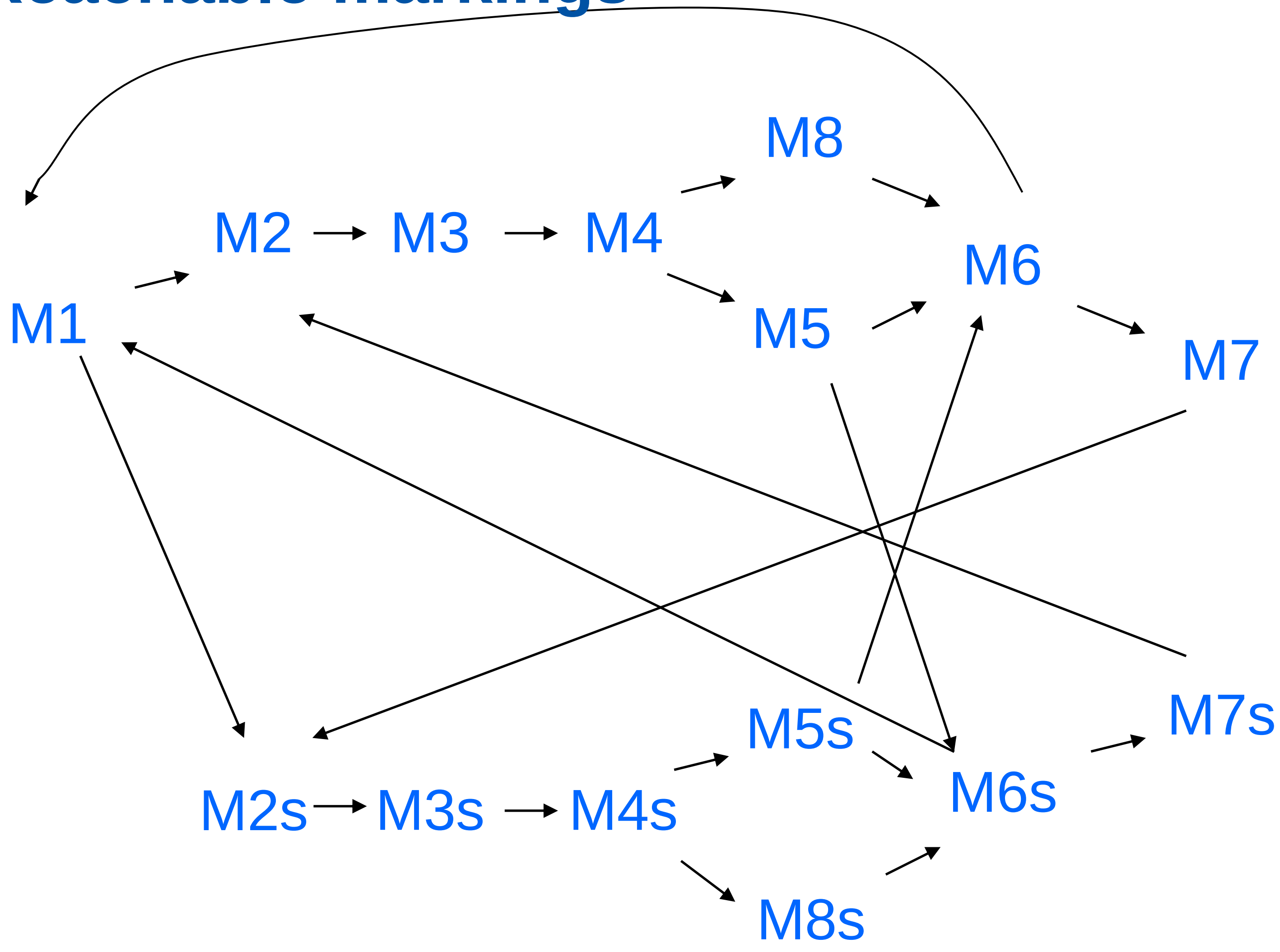
# Competing Trains Example: Boolean marking and computing changes of markings
# Reachable markings



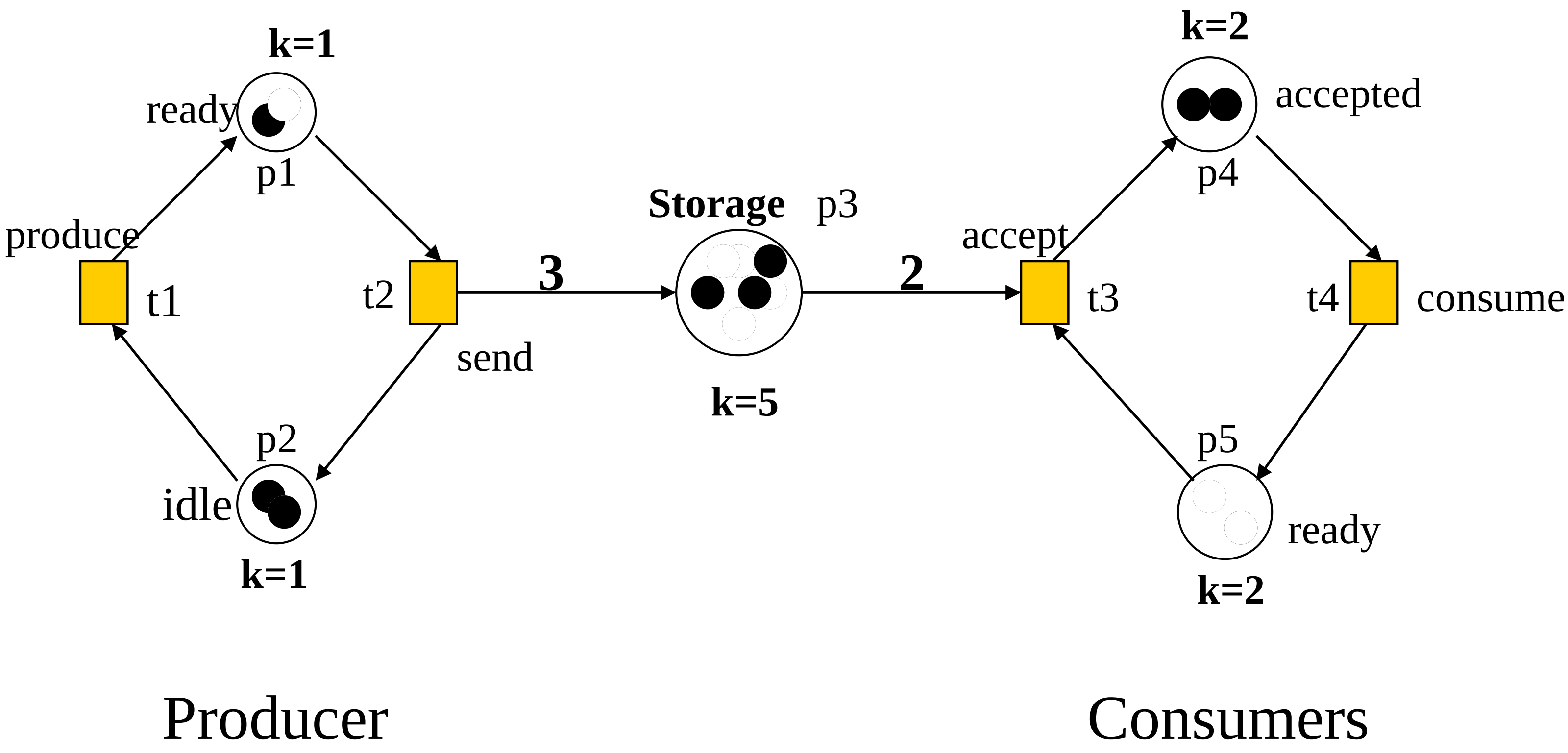

Split 1.2

# Competing Trains Example: Boolean marking and computing changes of markings Reachable markings



# Realistic scenarios need more general definitions

- More than one token per condition, capacities of places
- weights of edges
- state space of Petri nets may become infinite!



# From conditions to resources

- c/e nets model the flow of information at a fundamental level (true/false)
- there are natural application areas for which the flow/transport of resources and the number of available resources is important (data flow, document-/workflow, production lines, communication networks, www, ..)
- place/transition nets are a generalization of c/e nets

# From conditions to resources

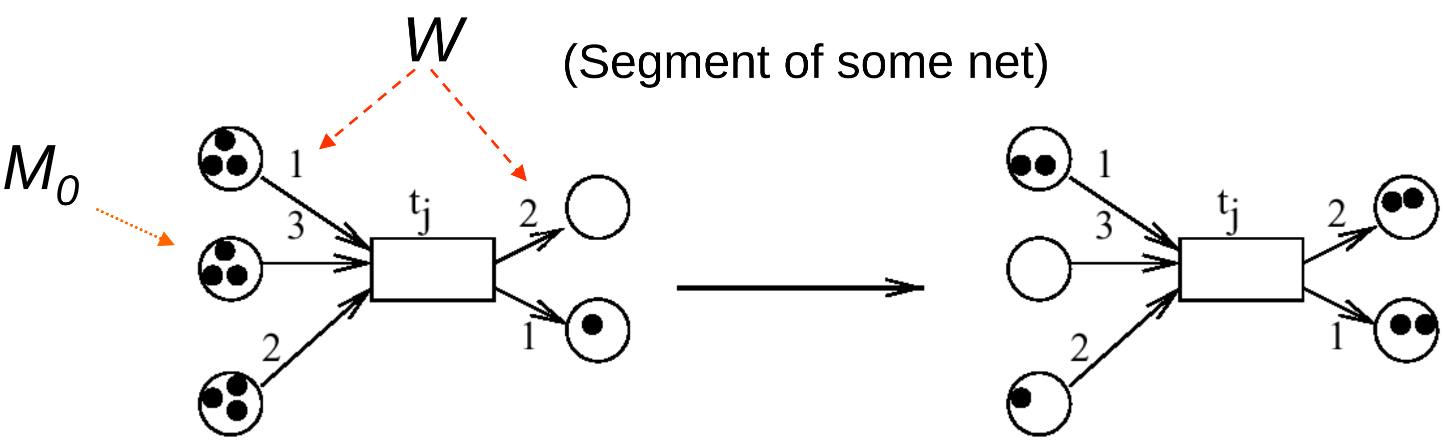
- place/transition nets are a generalization of c/e nets:
  - state elements represent places where resources (tokens) can be stored
  - transition elements represent local transitions or transport of resources
- a transition is enabled if and only if
  - sufficient resources are available on all its input places
  - sufficient capacities are available on all its output places
- a transition occurrence
  - consumes tokens from each input place and
  - produces tokens on each output place

# Place/transition nets

multiple tokens per place

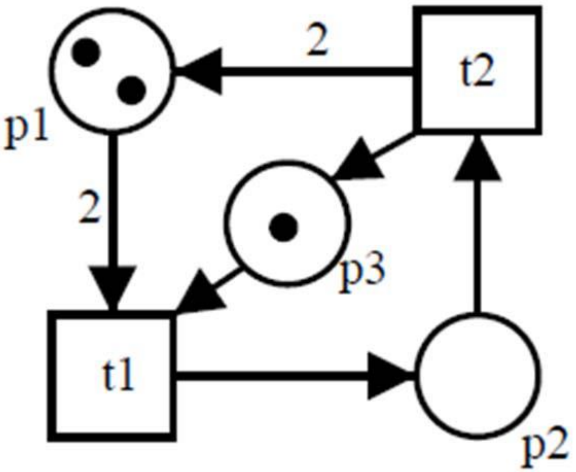
**Def.:** $(P, T, F, K, W, M_0)$ is called a **place/transition net (P/T net)** iff

1. $N=(P,T,F)$ is a **net** with places P and transitions T
2. $K: P \rightarrow (\mathbf{N}_0 \cup \{\omega\}) \setminus \{0\}$ denotes the **capacity** of places ($\omega$ symbolizes infinite capacity)
3. $W: F \rightarrow (\mathbf{N}_0 \setminus \{0\})$ denotes the **weight of graph edges**
4. $M_0: P \rightarrow \mathbf{N}_0 \cup \{\omega\}$ represents the **initial marking** of places



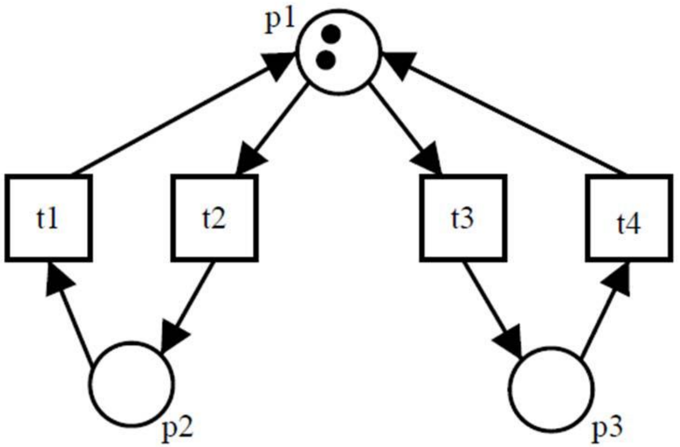

default:
K = $\omega$
W = 1

# Example

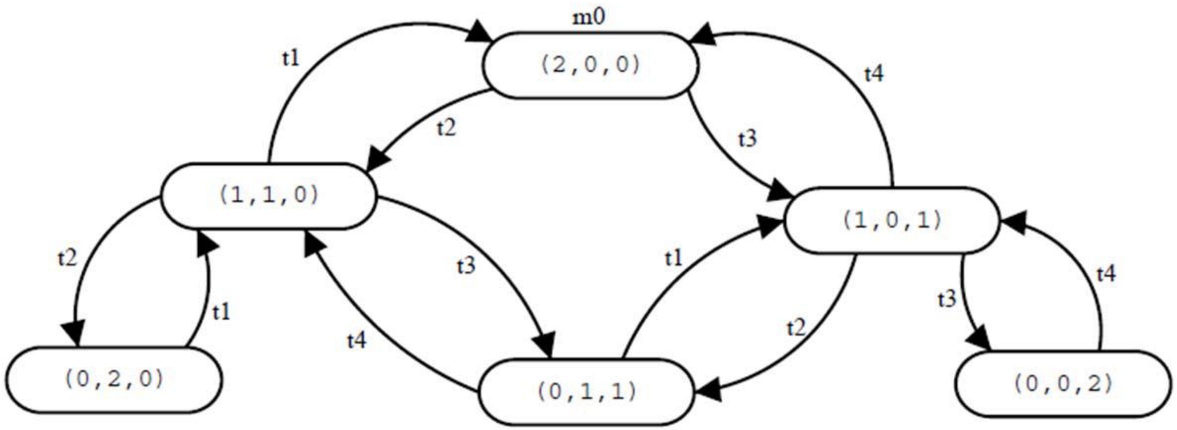



- P = (p1, p2, p3)
- T = {t1, t2}
- F = {(p1, t1), (p2, t2), (p3, t1), (t1, p2), (t2, p1), (t2, p3)}
- W = {(p1, t1) $\rightarrow$ 2, (p2, t2) $\rightarrow$ 1, (p3, t1) $\rightarrow$ 1, (t1, p2) $\rightarrow$ 1, (t2, p1) $\rightarrow$ 2, (t2, p3) $\rightarrow$ 1}
- m0 = (2, 0, 1)

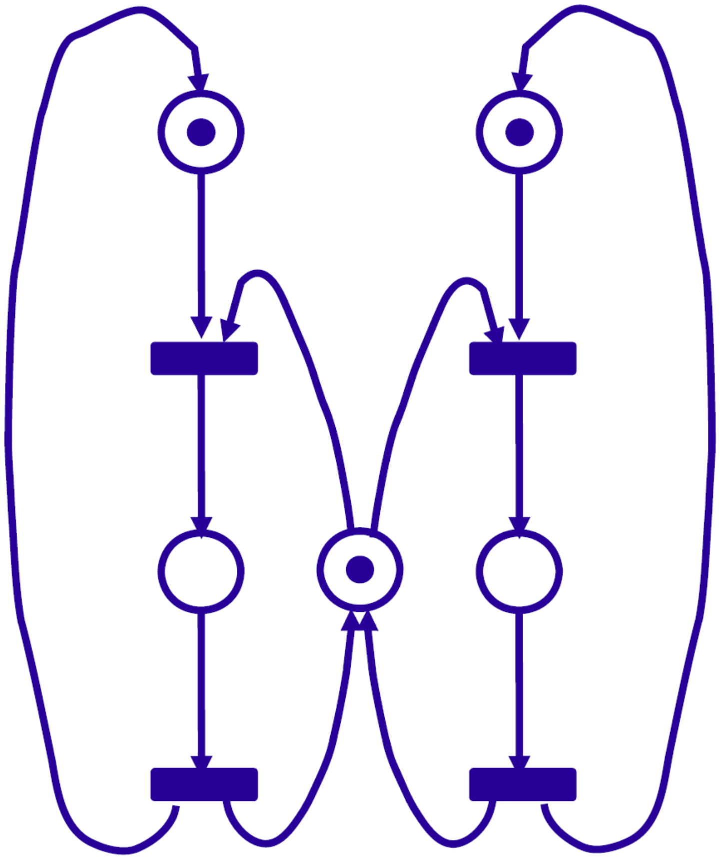
# Reachability

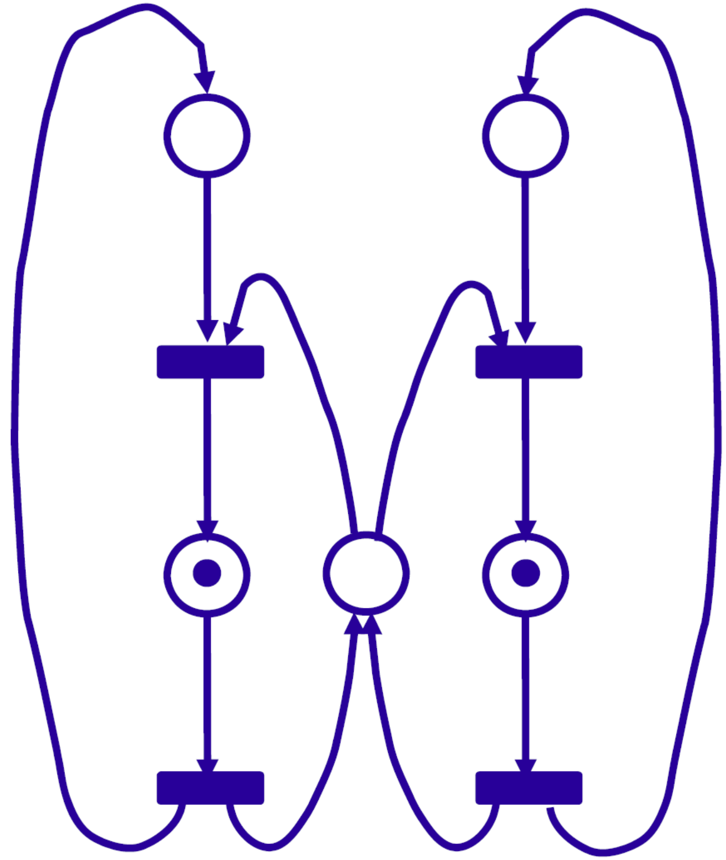



Reachability graph:



# Reachability





Marking
M

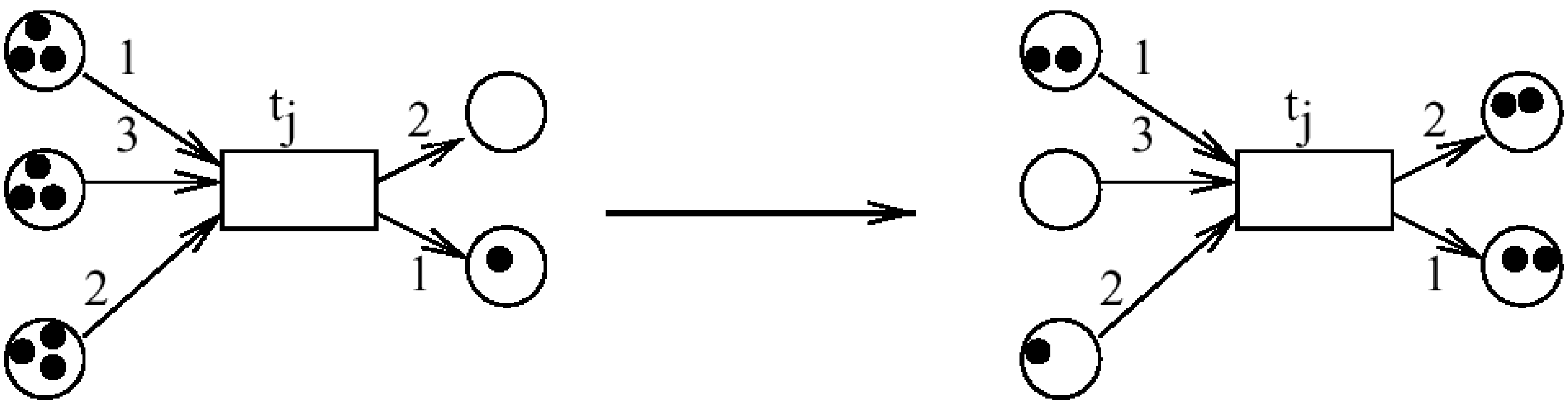
Is there a sequence of transition firings such that M ⟶ M'?

Marking
M'

# Computing changes of markings

- „Firing“ transitions *t* generate new markings on each of the places *p* according to the following rules:

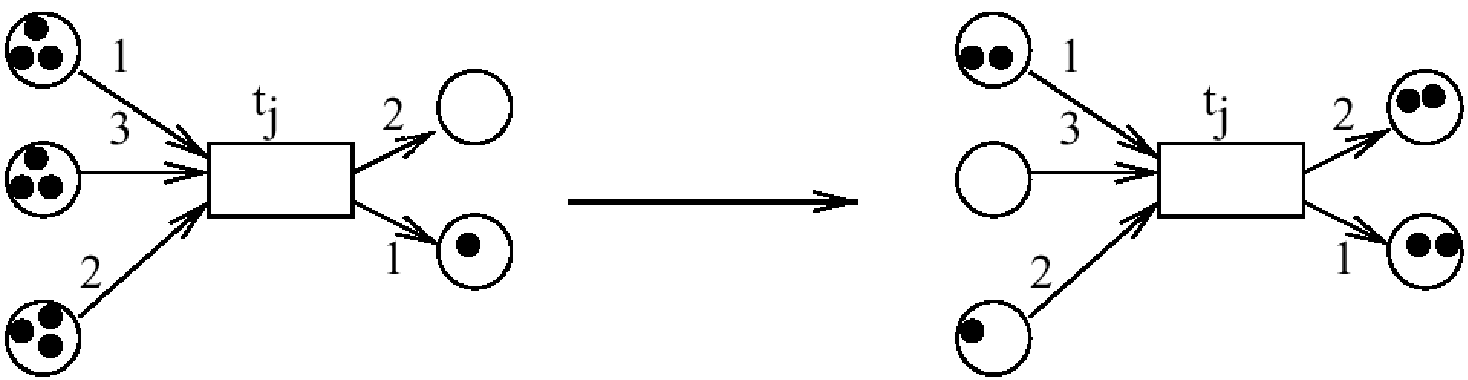
$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$



# Activated transitions

- Transition *t* is „activated“ iff

$$(\forall p \in {}^{\bullet}t : M(p) \geq W(p,t)) \wedge (\forall p \in t^{\bullet} : M(p) + W(t,p) \leq K(p))$$



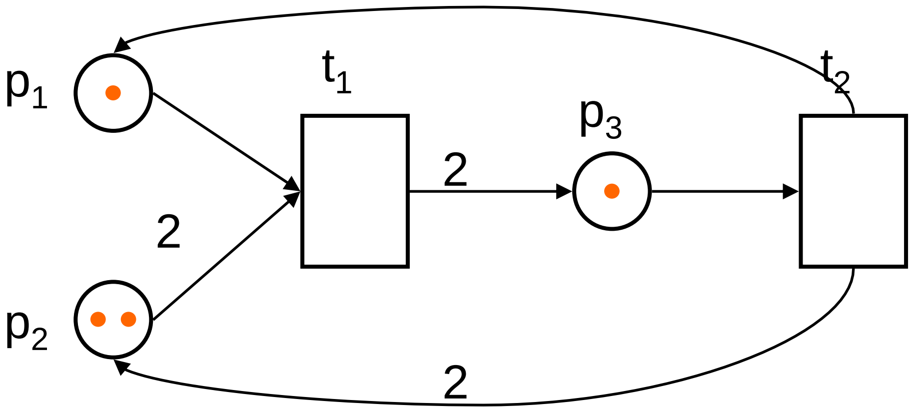

Activated transitions can „take place“ or „fire“, but don‘t have to.
The order in which activated transitions fire is not fixed (it is non-deterministic).

# Boundedness

- A place is called **k-bounded** or **k-safe** if it contains in all reachable markings at most k tokens
- A net is **bounded** if each place is bounded

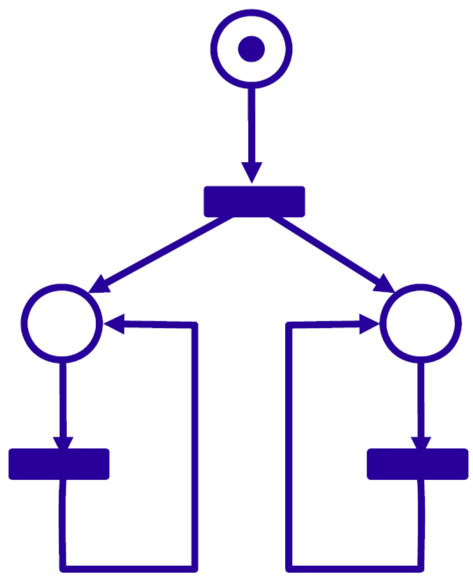
Application: places represent buffers and registers

$\rightarrow$ avoid buffer overflow

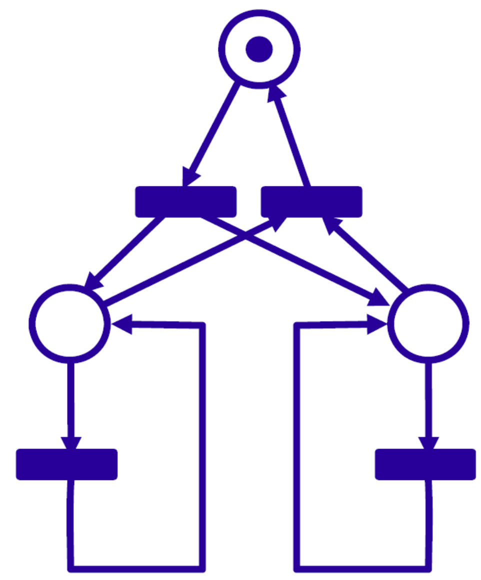


# Liveness

- A transition is **live** if in every reachable marking there exists a firing sequence such that the transition becomes enabled
- A net is **live** if all its transitions are live

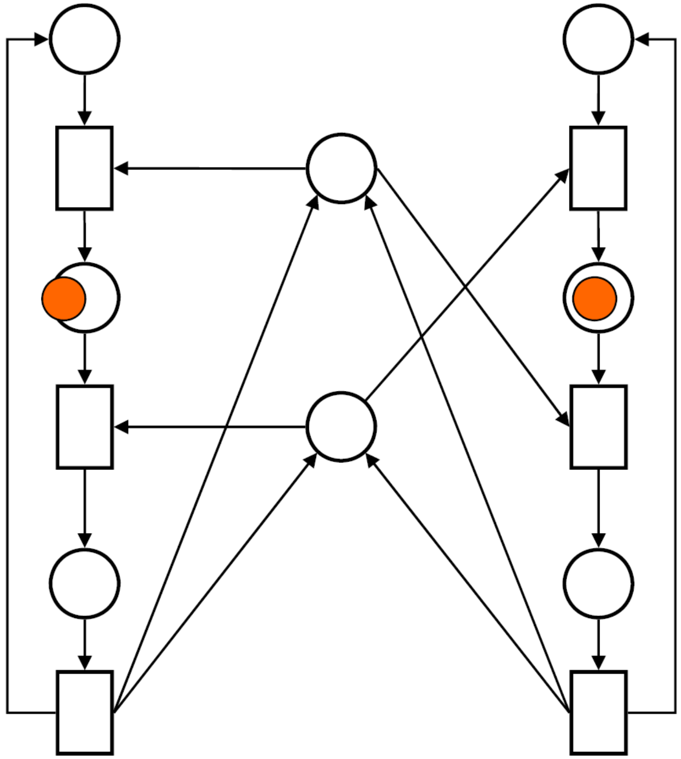


Live ?



# Deadlock

- A **dead marking (deadlock)** is a marking where no transition can fire
- A net is **deadlock-free** if no dead marking is reachable

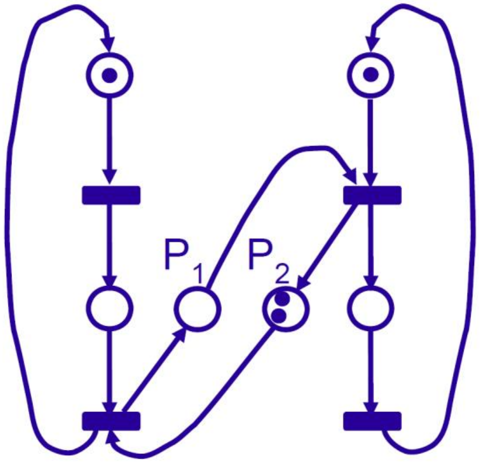


# Computation of Invariants

We are interested in subsets consisting of places whose number of tokens remain invariant under transitions,

e.g. the number of trains commuting between Amsterdam and Paris (Cologne and Paris) remains constant

Important for correctness proofs,
e.g. the proof of liveness




$P_1 + P_2 = 2$

# Shorthand for changes of markings

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

$\Rightarrow$ $\forall p \in P$: $M'(p) = M(p) + \underline{t}(p)$

$\Rightarrow$ $M' = M + \underline{t}$ +: vector add

# Matrix $\underline{N}$ describing all changes of markings

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

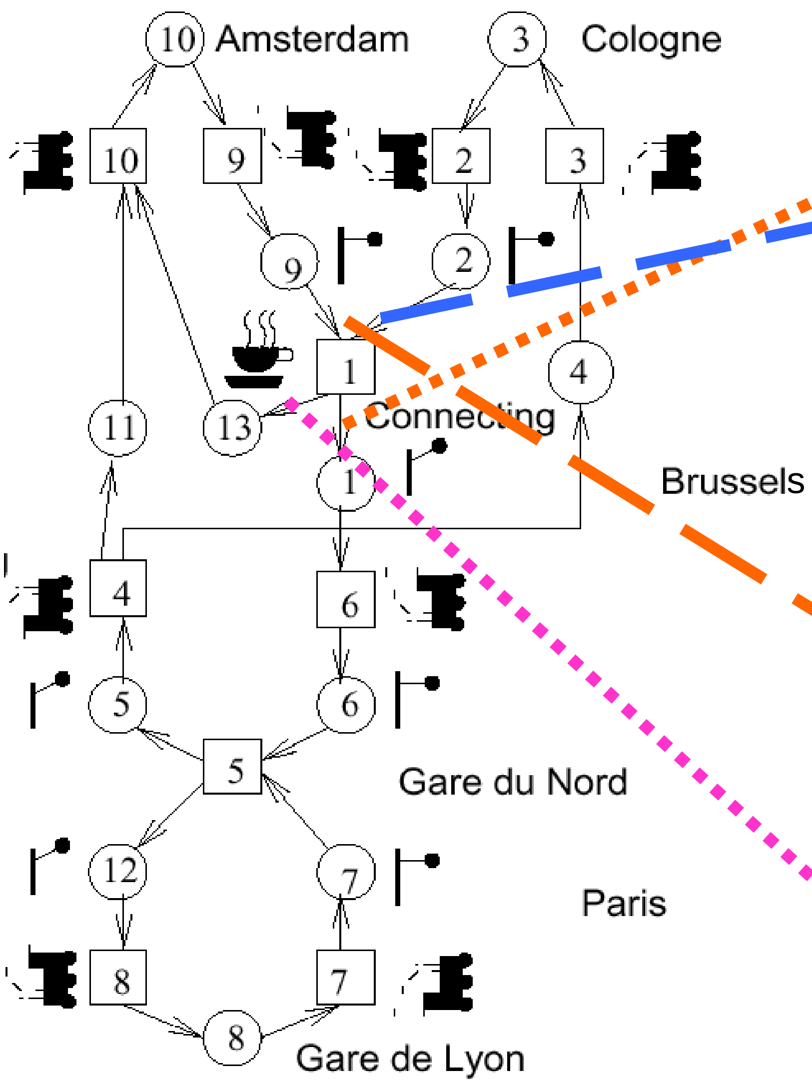
Def.: Matrix $\underline{N}$ of net $N$ is a mapping

$$\underline{N}: P \times T \rightarrow Z \text{ (integers)}$$

such that $\forall\, t \in T$: $\underline{N}(p,t) = \underline{t}(p)$

Component in column $t$ and row $p$ indicates the change of the marking of place $p$ if transition $t$ takes place.

# Example: $\underline{N}$ =



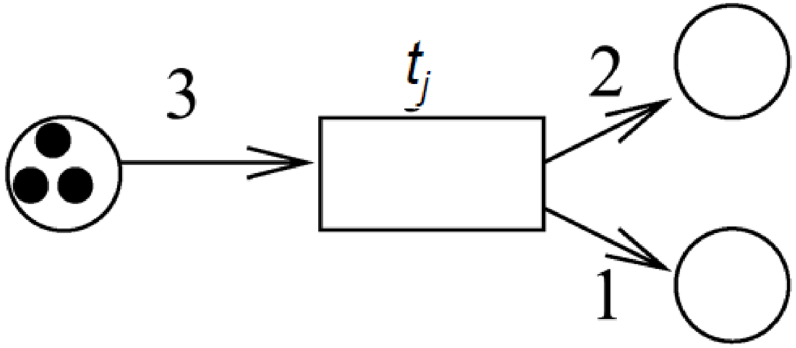

| | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_7$ | $t_8$ | $t_9$ | $t_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_1$ | 1 | | | | | –1 | | | | |
| $p_2$ | –1 | 1 | | | | | | | | |
| $p_3$ | | –1 | 1 | | | | | | | |
| $p_4$ | | | –1 | 1 | | | | | | |
| $p_5$ | | | | –1 | 1 | | | | | |
| $p_6$ | | | | | –1 | 1 | | | | |
| $p_7$ | | | | | –1 | | 1 | | | |
| $p_8$ | | | | | | | –1 | | | |
| $p_9$ | –1 | | | | | | | 1 | 1 | |
| $p_{10}$ | | | | | | | | | –1 | 1 |
| $p_{11}$ | | | | 1 | | | | | | –1 |
| $p_{12}$ | | | | | 1 | | | –1 | | |
| $p_{13}$ | 1 | | | | | | | | | –1 |

# Place invariants

For any transition $t_j \in T$ we are looking for sets $R \subseteq P$ of places for which the accumulated marking is constant:

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Example:



# Characteristic Vector

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Let:

$$\underline{c}_R(p) = \begin{cases} 1 \text{ if } & p \in R \\ 0 \text{ if } & p \notin R \end{cases}$$

$$\Rightarrow \quad \sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Scalar product

# Condition for place invariants

$$\sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Accumulated marking constant for **all** transitions if

$$\begin{aligned} \underline{t}_1 \cdot \underline{c}_R &= 0 \\ \dots \quad &\dots \quad \dots \\ \underline{t}_n \cdot \underline{c}_R &= 0 \end{aligned}$$

Equivalent to $\underline{\boldsymbol{N}}^{T}\, \underline{\boldsymbol{c}}_{\boldsymbol{R}} = \boldsymbol{0}$ where $\underline{N}^T$ is the transposed of $\underline{N}$

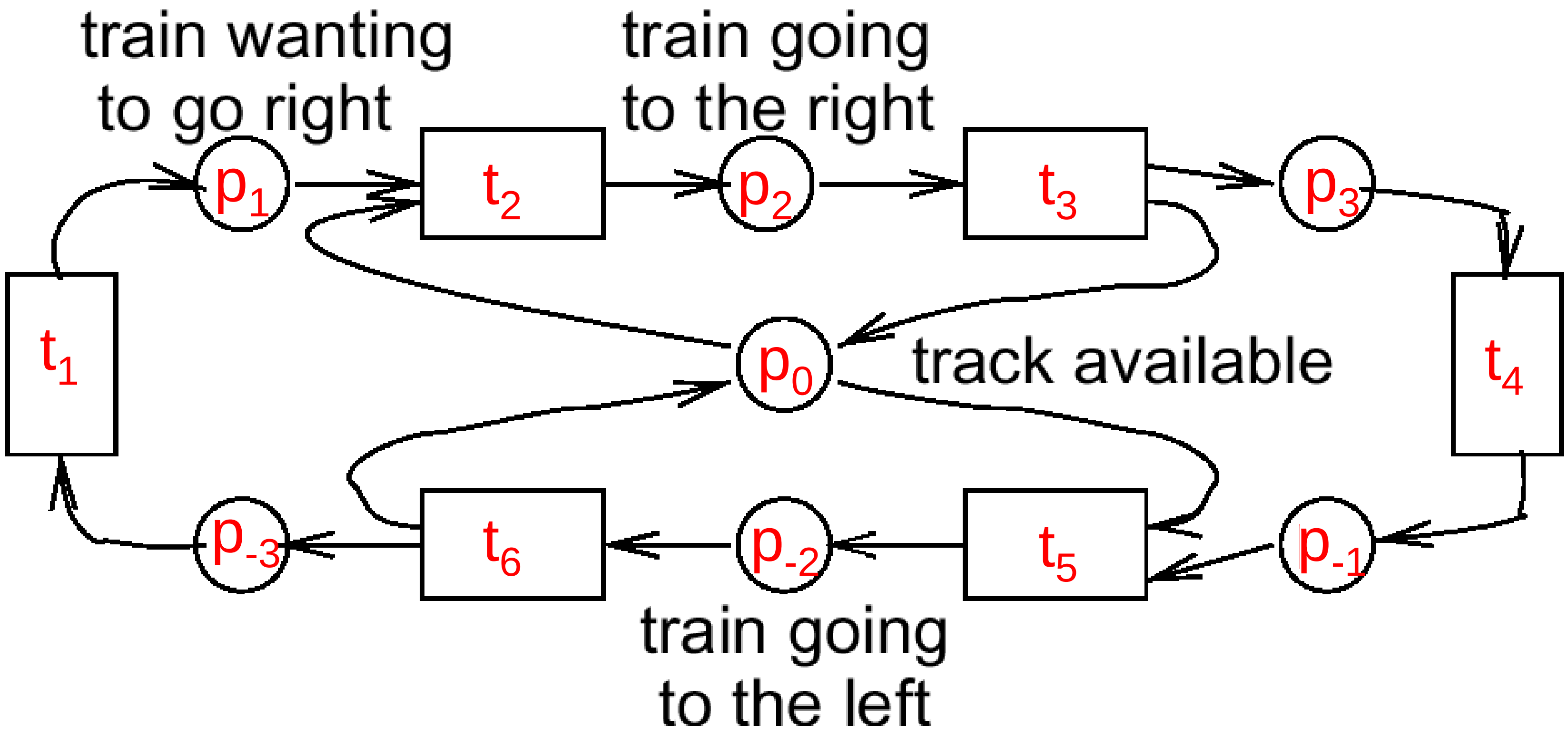
# System of linear equations

$$\begin{pmatrix} \underline{t}_1(p_1)\ldots\underline{t}_1(p_n) \\ \underline{t}_2(p_1)\ldots\underline{t}_2(p_n) \\ \ldots \\ \underline{t}_m(p_1)\ldots\underline{t}_m(p_n) \end{pmatrix} \begin{pmatrix} \underline{c}_R(p_1) \\ \underline{c}_R(p_2) \\ \ldots \\ \underline{c}_R(p_n) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

System of linear equations.

Solution vectors must consist of zeros and ones.

# Competing trains example



# Application to Thalys example

$\underline{N}^T \underline{c}_R = 0$, with $\underline{N}^T =$

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ | $p_{11}$ | $p_{12}$ | $p_{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_1$ | 1 | -1 | | | | | | | -1 | | | | 1 |
| $t_2$ | | 1 | -1 | | | | | | | | | | |
| $t_3$ | | | 1 | -1 | | | | | | | | | |
| $t_4$ | | | | 1 | -1 | | | | | | 1 | | |
| $t_5$ | | | | | 1 | -1 | -1 | | | | | 1 | |
| $t_6$ | -1 | | | | | 1 | | | | | | | |
| $t_7$ | | | | | | | 1 | -1 | | | | | |
| $t_8$ | | | | | | | | 1 | | | | -1 | |
| $t_9$ | | | | | | | | | 1 | -1 | | | |
| $t_{10}$ | | | | | | | | | | 1 | -1 | | -1 |

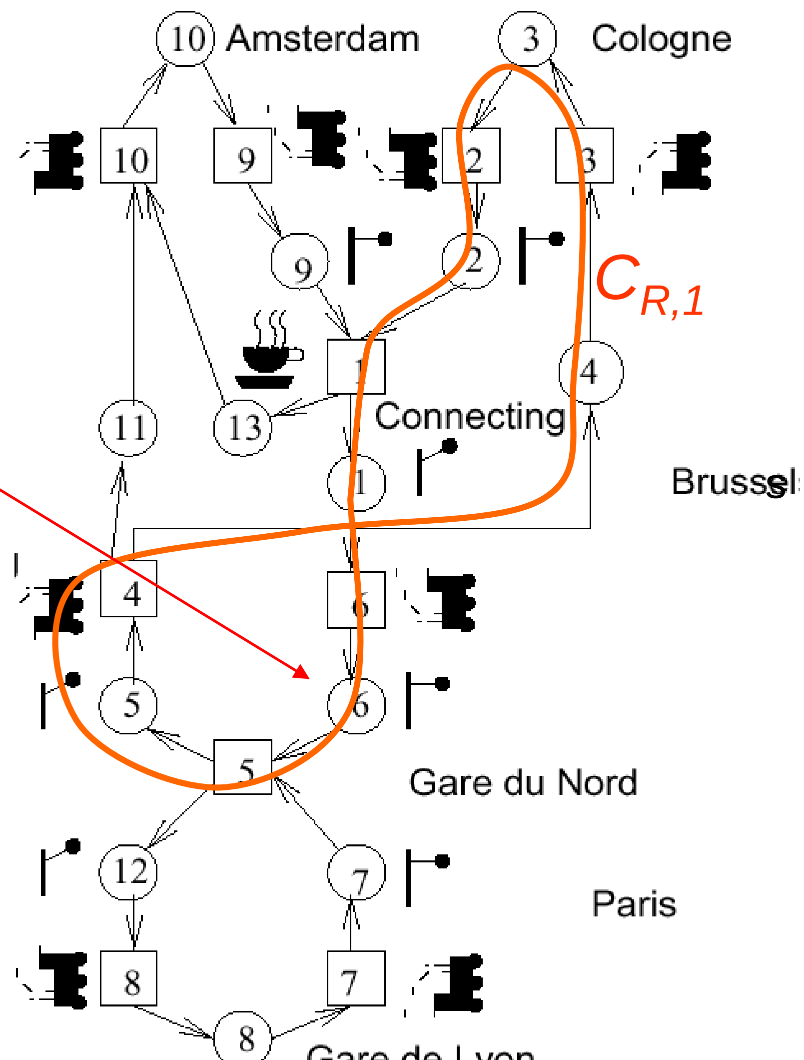
$$c_{R,1} = \left(1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right)$$

# Interpretation of the 1st invariant

$$c_{R,1} = (1\,1\,1\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0)$$

Characteristic vector describes places for Cologne train.

We proved that: the number of trains along the path remains constant.



# Application to Thalys example

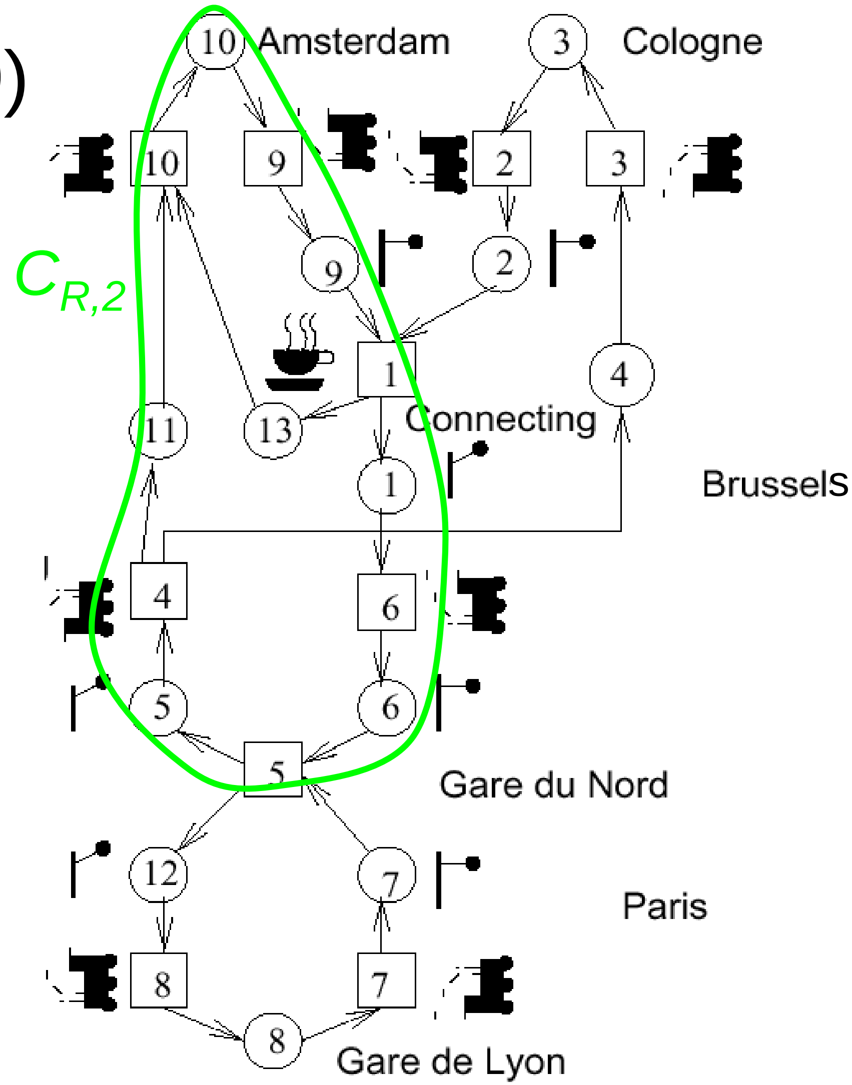
$\underline{N}^T \underline{c}_R = \mathbf{0}$, with $\underline{N}^T$=

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ | $p_{11}$ | $p_{12}$ | $p_{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_1$ | 1 | -1 | | | | | | | -1 | | | | 1 |
| $t_2$ | | 1 | -1 | | | | | | | | | | |
| $t_3$ | | | 1 | -1 | | | | | | | | | |
| $t_4$ | | | | 1 | -1 | | | | | | 1 | | |
| $t_5$ | | | | | 1 | -1 | -1 | | | | | 1 | |
| $t_6$ | -1 | | | | | 1 | | | | | | | |
| $t_7$ | | | | | | | 1 | -1 | | | | | |
| $t_8$ | | | | | | | | 1 | | | | -1 | |
| $t_9$ | | | | | | | | | 1 | -1 | | | |
| $t_{10}$ | | | | | | | | | | 1 | -1 | | -1 |

$$c_{R,2} = (1,0,0,0,1,1,0,0,1,1,1,0,0)$$

# Interpretation of the 2nd invariant

$$c_{R,2} = (1,0,0,0,1,1,0,0,1,1,1,0,0)$$




We proved that:

None of the Amsterdam trains gets lost.

# Application to Thalys example

$\underline{N}^T \, \underline{c}_R = 0$, with $\underline{N}^T$=

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ | $p_{11}$ | $p_{12}$ | $p_{13}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t_1$ | 1 | -1 | | | | | | | -1 | | | | 1 |
| $t_2$ | | 1 | -1 | | | | | | | | | | |
| $t_3$ | | | 1 | -1 | | | | | | | | | |
| $t_4$ | | | | 1 | -1 | | | | | | 1 | | |
| $t_5$ | | | | | 1 | -1 | -1 | | | | | 1 | |
| $t_6$ | -1 | | | | | 1 | | | | | | | |
| $t_7$ | | | | | | | 1 | -1 | | | | | |
| $t_8$ | | | | | | | | 1 | | | | -1 | |
| $t_9$ | | | | | | | | | 1 | -1 | | | |
| $t_{10}$ | | | | | | | | | | 1 | -1 | | -1 |

$$c_{R,2} = \left(0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0\right)$$

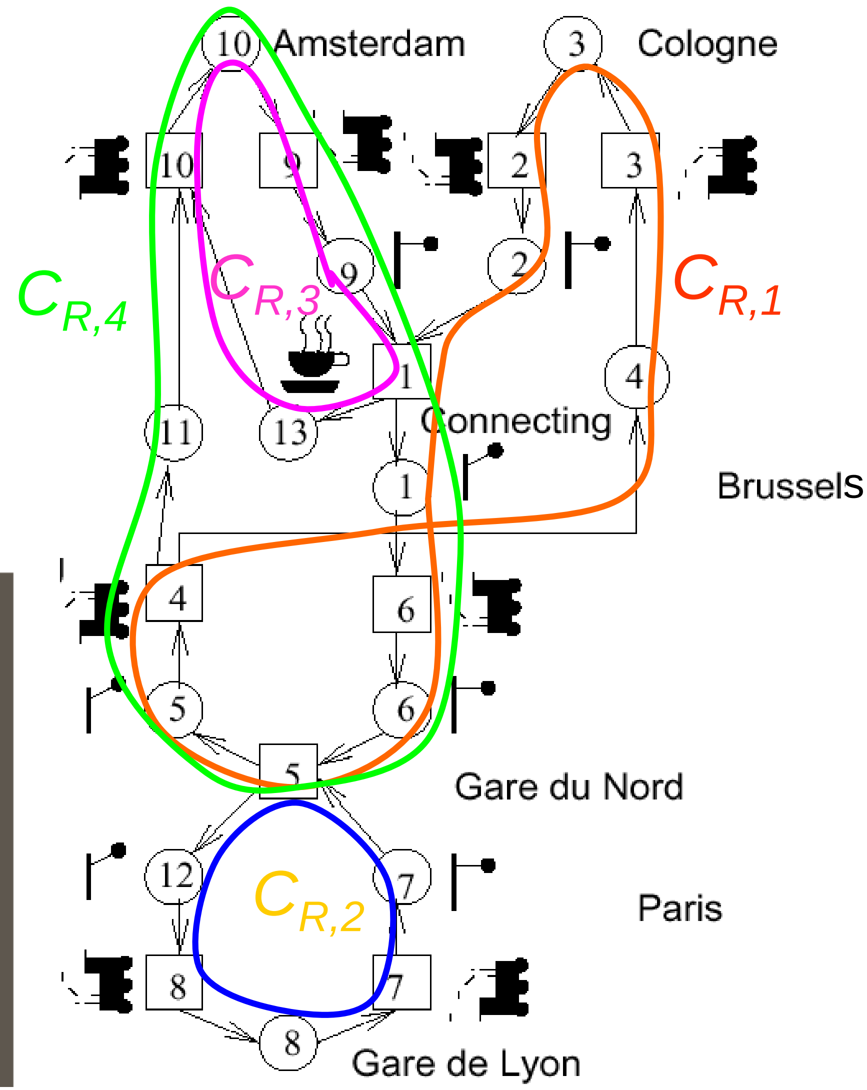
# Solution vectors for Thalys example

$$c_{R,1} = (1\;1\;1\;1\;1\;1\;0\;0\;0\;0\;0\;0\;0)$$

$$c_{R,2} = (0\;0\;0\;0\;0\;0\;1\;1\;0\;0\;0\;1\;0)$$

$$c_{R,3} = (0\;0\;0\;0\;0\;0\;0\;0\;1\;1\;0\;0\;1)$$

$$c_{R,4} = (1\;0\;0\;0\;1\;1\;0\;0\;1\;1\;1\;0\;0)$$



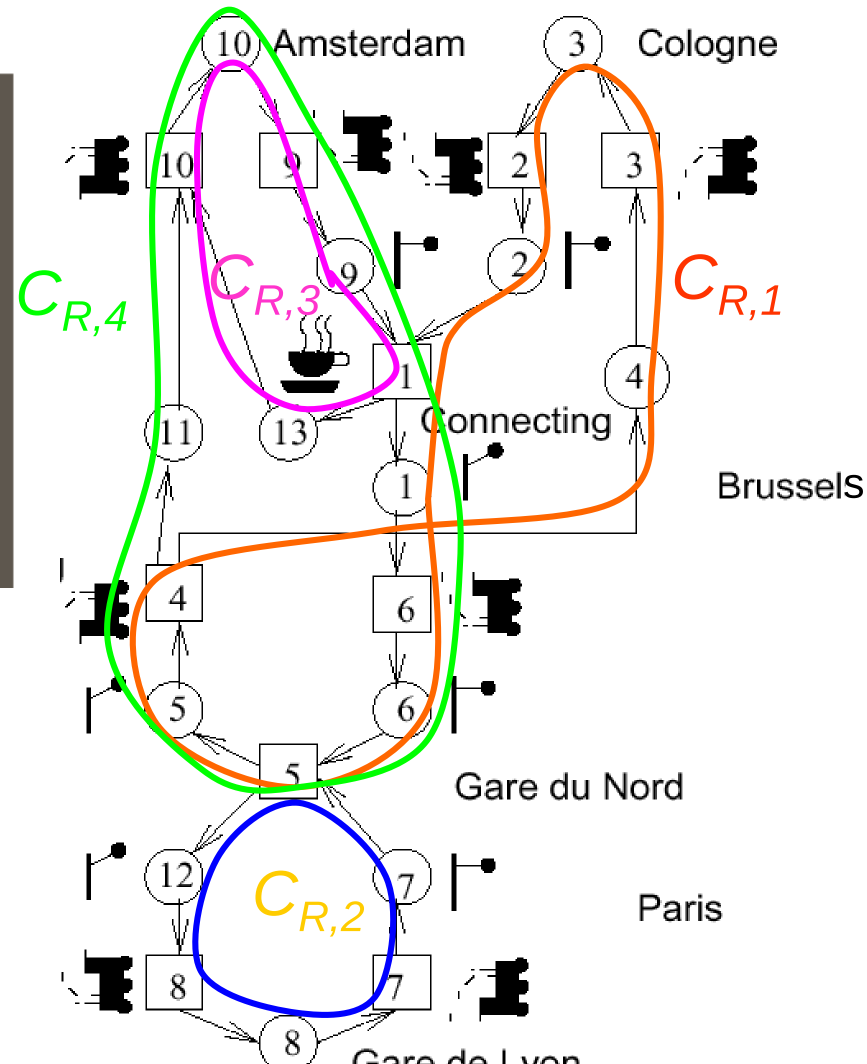

We proved that:

- the number of trains serving Amsterdam, Cologne and Paris remains constant.
- the number of train drivers remains constant.

# Solution vectors for Thalys example

It follows:

- each place invariant must have at least one label at the beginning, otherwise "dead"
- at least three labels are necessary in the example



# Invariants & boundedness

- A net is **covered** by place invariants
  iff every place is contained in some invariant.

**Theorem 1:**

**a)** If R is a place invariant and $p \in R$, then p is bounded.

**b)** If a net is covered by place invariants then it is bounded.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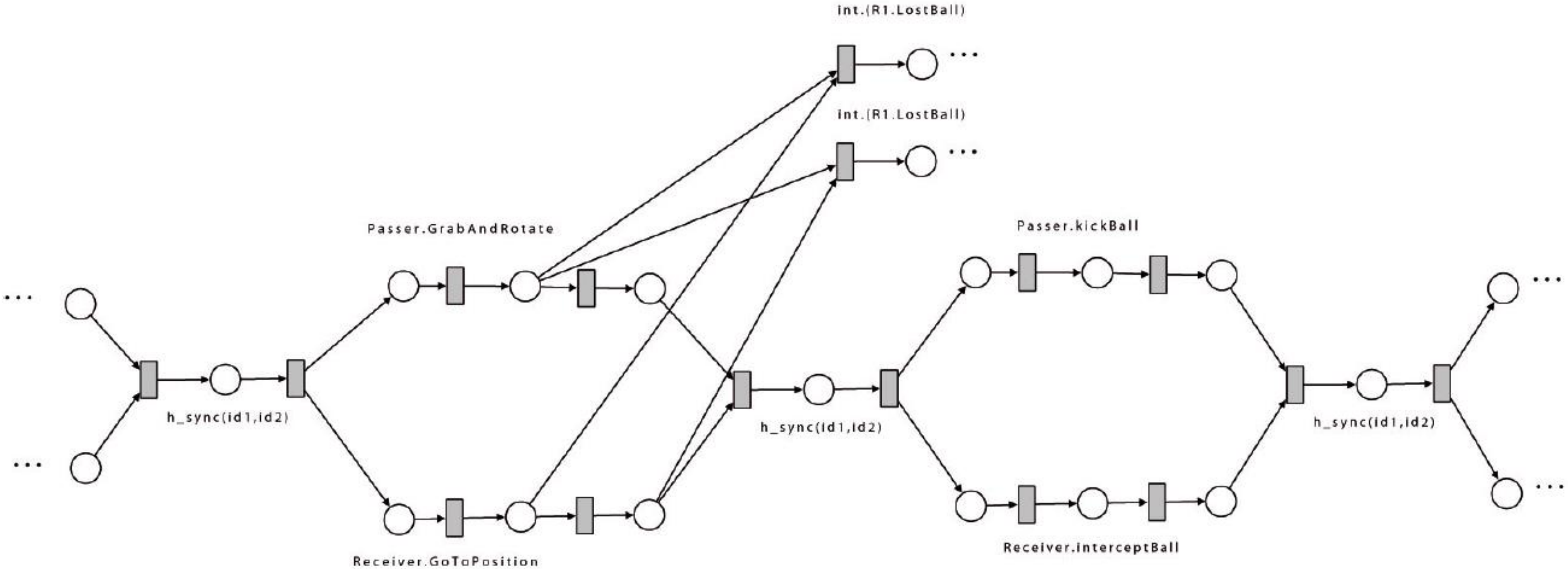
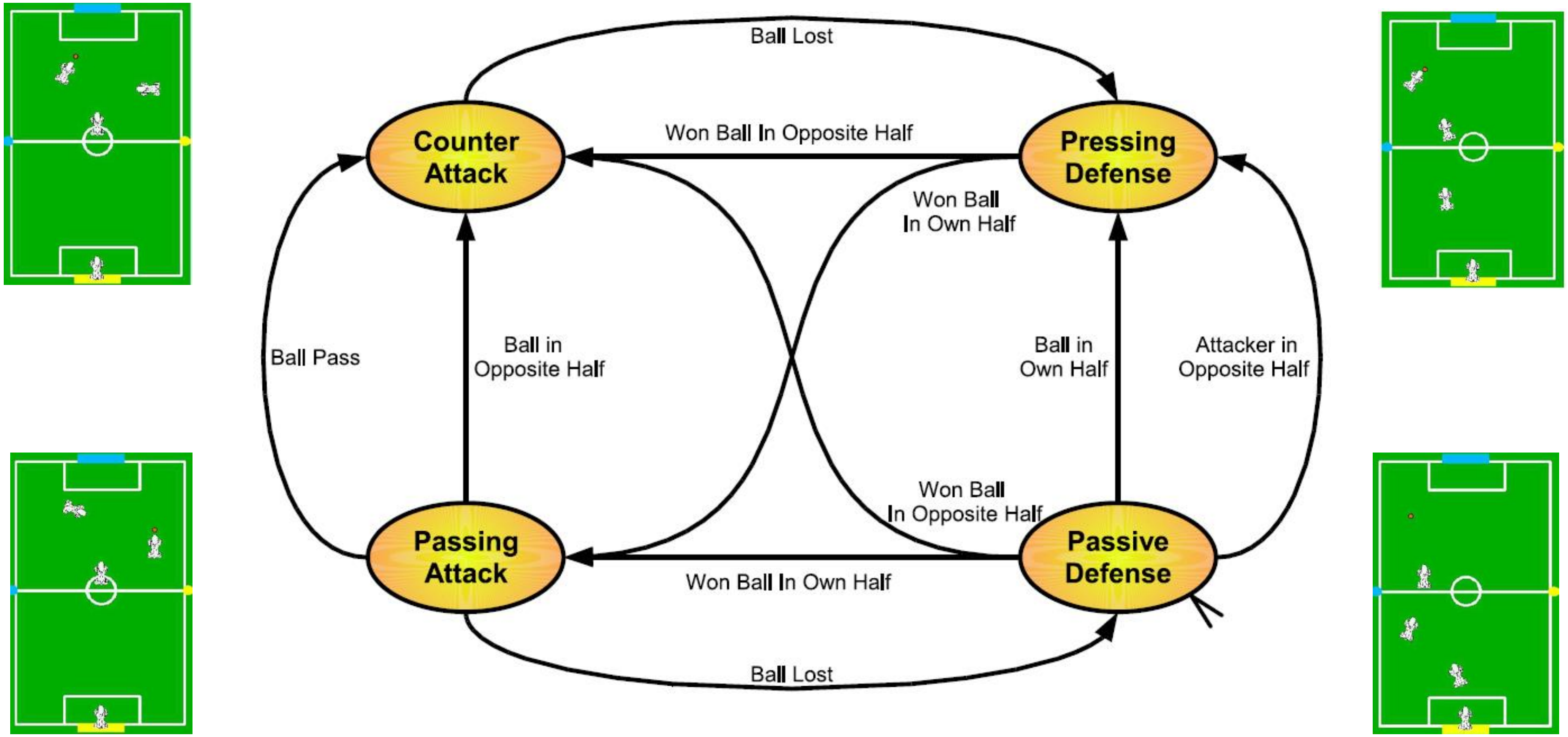
# Passing Maneuver




Teamwork Design Based on Petri Net Plan

P. F. Palamara, V. A. Ziparo, L. Iocchi, D. Nardi, and P. Lima

# Team strategy

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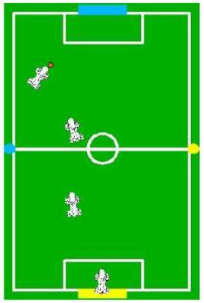
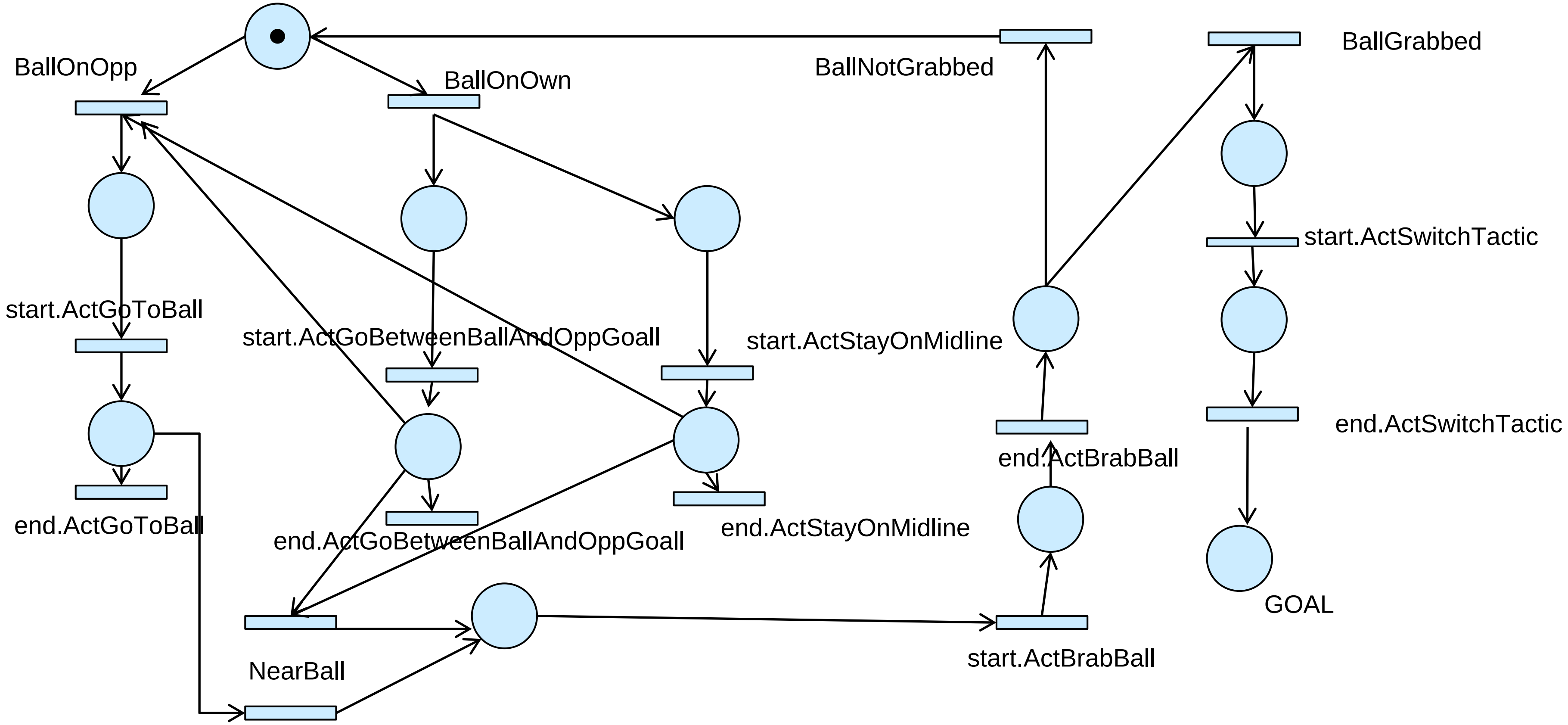
# Attacker role
# in the pressing defense tactic



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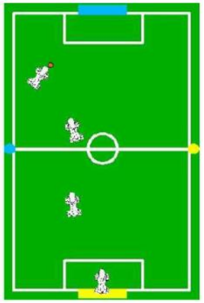
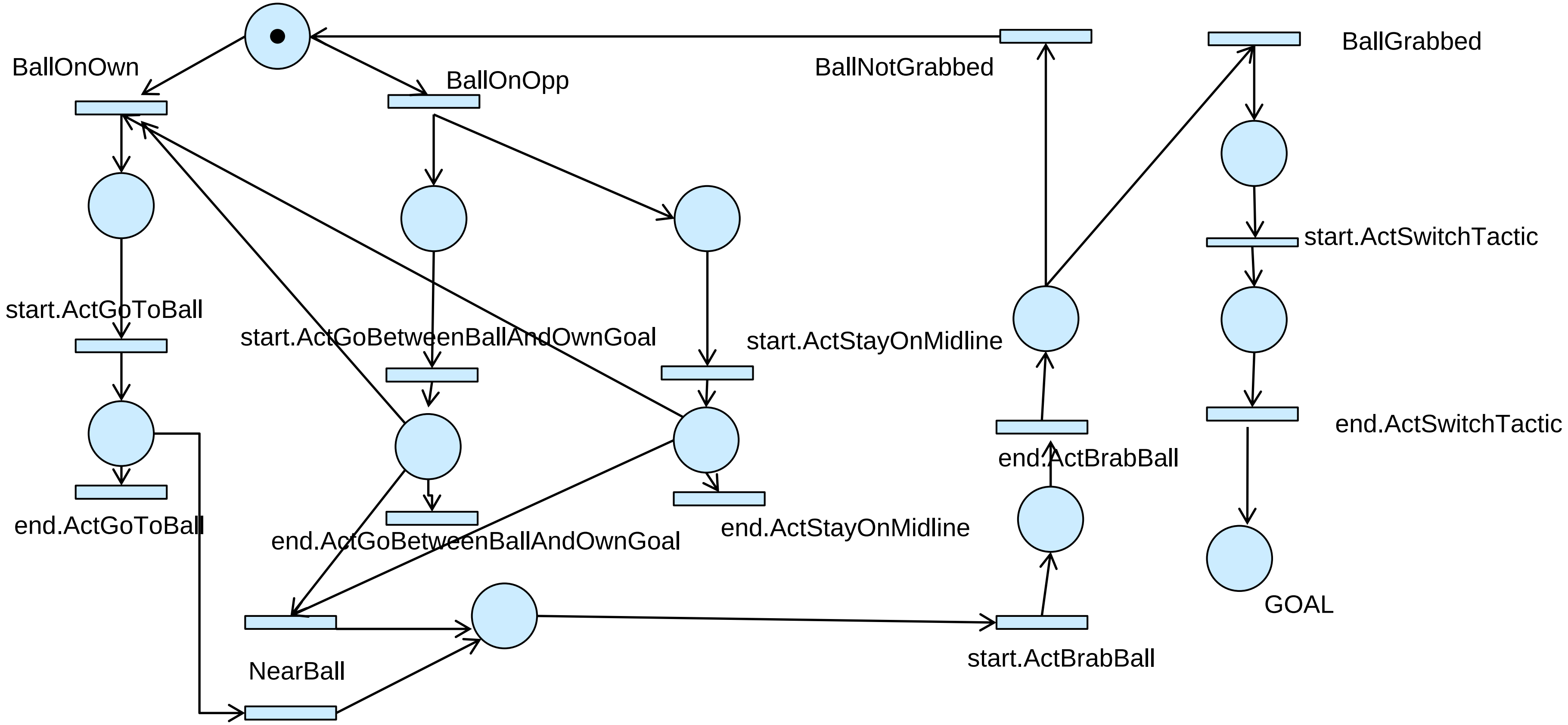
# Midfielder role
# in the pressing defense tactic



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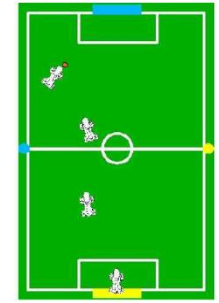
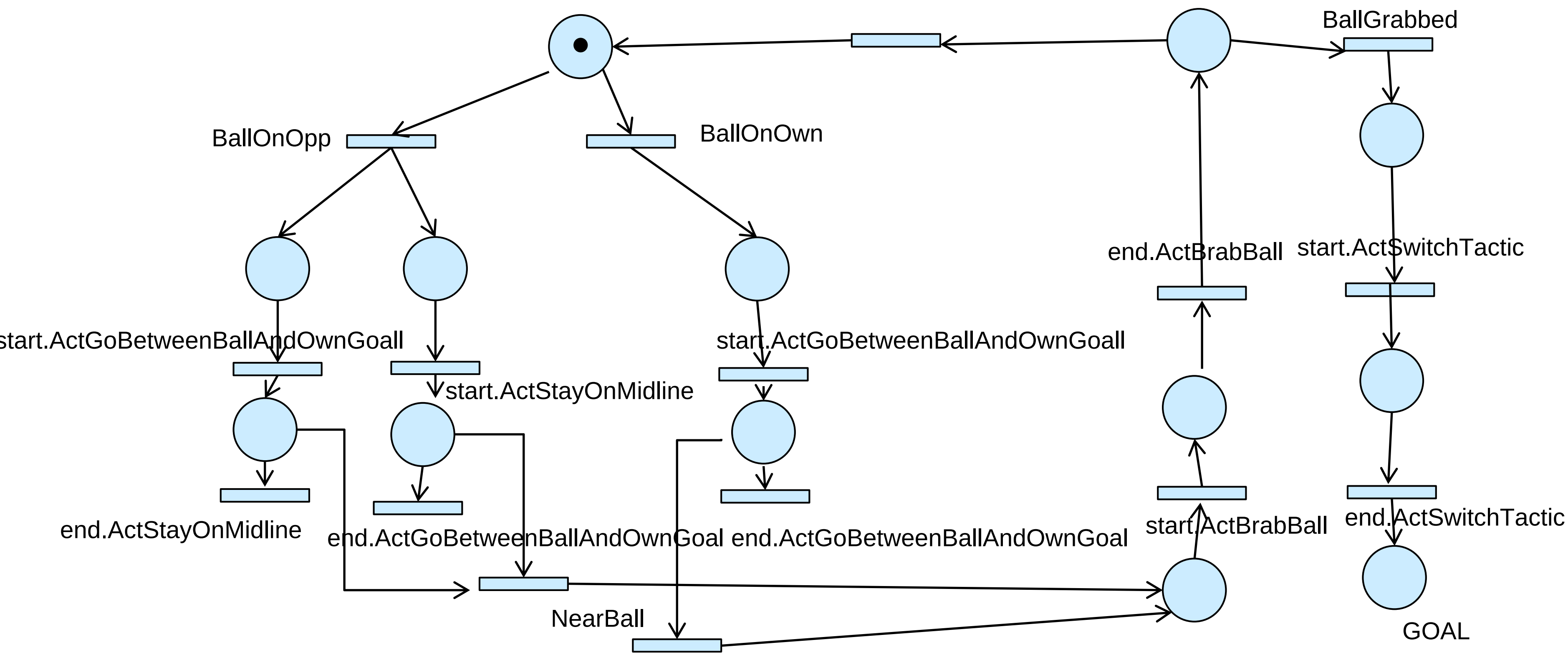
# Defender role
# in the pressing defense tactic



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