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Summer term 2014
Problem Set 4
Due: Tuesday, 3rd June 2014

Embedded Systems

Please indicate your **name**, **group number**, and **discussion slot tutor**. Only one submission per group is necessary.

Problem 1: Analyzing a Petri Net

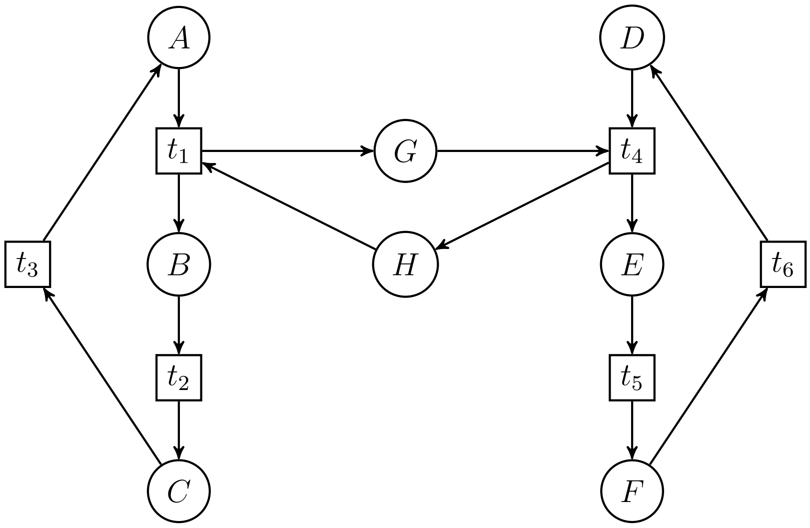

Figure 1: Petri net for Exercise 2 modeling a producer/consumer pattern.

Consider the Petri net in Fig. 1 with places $A, \ldots, H$ and transitions $t_1, \ldots t_6$. Assume an initial marking M_0 with $M_0(A) = 1$, $M_0(H) = 3$, $M_0(D) = 1$, and $M_0(p) = 0$ for every other place p.

(a) Is there a dead marking for this Petri net?

(b) Is the Petri net deadlock-free?

(c) Is this Petri net live?

(d) Give the incidence matrix for the Petri net.

(e) Use the incidence matrix to deduce all place invariants.

(f) Is the net bounded? Justify your answer.

Problem 2: Petri Net Properties

Is every live Petri net deadlock-free? Prove or disprove.

Problem 3: Modeling with Petri Nets

Consider the net in Fig. 2.

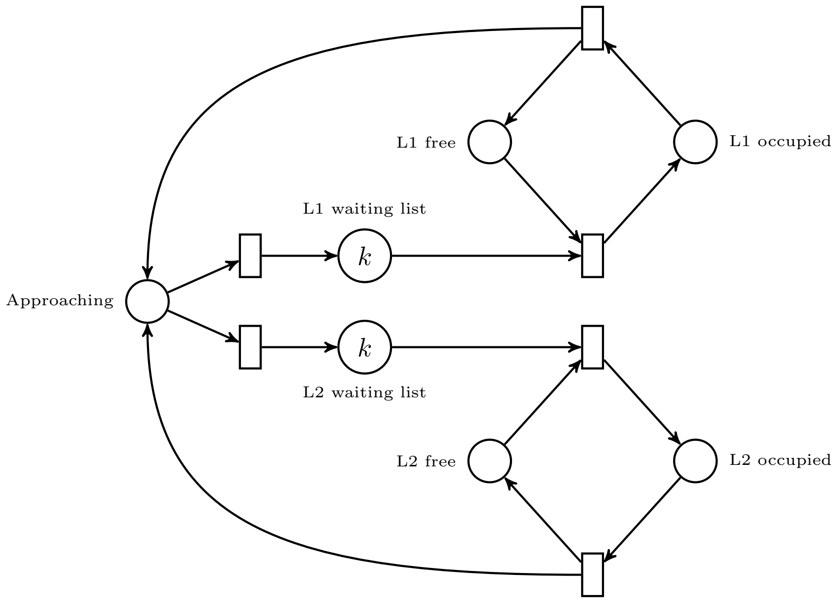

Figure 2: A Petri net modeling two landing strips.

It models the allocation of the landing strips of an airport. The airport has two strips exclusively reserved for landing airplanes. Each strip has a waiting list that should contain at most k airplanes. Furthermore, a strip can either be free or occupied. When an airplane approaches it selects a strip. It is possible to use both strips at the same time. Initially, there are n airplanes flying.

1. Complete the Petri net. Give the weights of the edges, the capacity of the places, and a meaningful initial marking. Do not rely on the default values for capacities (∞) and weights (1) but rather give all values explicitly.

2. Modify the net so that the maximal number of airplanes in one waiting list is coded not in the capacity of the places, but rather in the initial marking.

Problem 4: Arithmetic Operations with Petri Nets

For each subtask of this exercise, construct a Petri net that comprises two input places a and b, and one output place z (additionally to the internal places that you might add to do the actual computation). The input of the arithmetic operation is specified in terms of the initial markings $M_0(a)$ and $M_0(b)$. The transitions in the net are fired until some final marking is reached, where no firing is possible. Recall that, due to the non-determinism in the order of the transition firings of a Petri net, there can be multiple final markings

$$M^0_\infty, M^1_\infty, M^2_\infty, M^3_\infty, \ldots.$$

You are only allowed to use *constants* (not the actual input values) to specify the initial markings of the internal places and z. In your submission, please use the *graphical* notation for the Petri nets.

(a) Construct a Petri net such that for all reachable final markings M^i_∞, $i \geq 0$, $M^i_\infty(z) = M_0(a) + M_0(b)$.

(b) Construct a Petri net such that for all reachable final markings M^i_∞, $i \geq 0$, $M^i_\infty(z) = \max(0, M_0(a) - M_0(b))$.

(c) Construct a Petri net such that $\max_{i \geq 0} M^i_\infty(z) = \frac{M_0(a)}{2} \cdot (M_0(a) + 1)$.
Note that $M_0(b)$ can be ignored here.