Timed Interfaces

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Seminar Games in Verification and Synthesis
Summer Term 2008
Saarland University

original paper by

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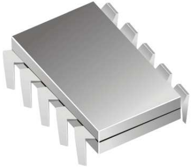
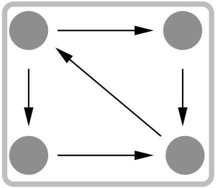
Motivation

Component



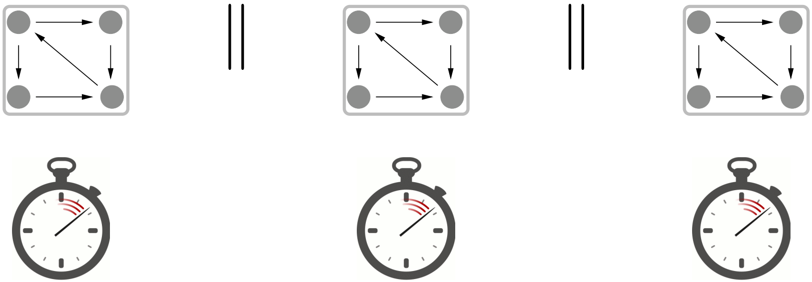
Interface



Timing
Requirements

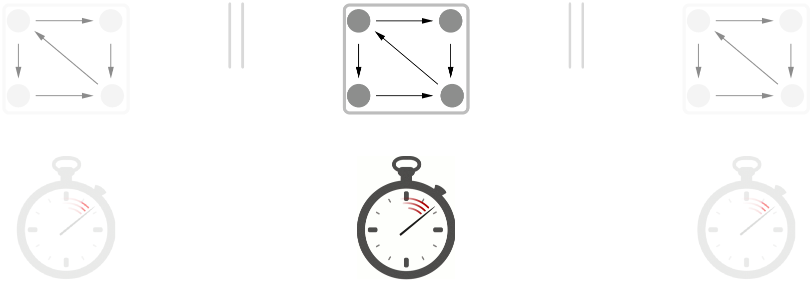
- complex real-time systems
- component based design
- interface describes component behavior

Motivation



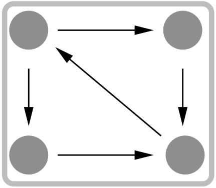
- model component interaction
- type system for interfaces

Motivation



- well-formed?
- input assumptions: expected use
- output guarantees: correct input $\Rightarrow$ correct output

Motivation



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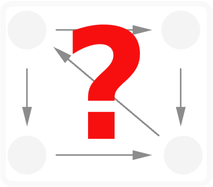


System vs. Environment

well-formed $\iff$ $\exists$ Environment that fulfils input assumptions

Timed Interface Theory

We are interested in

- Well-formedness
- Compatibility
- Composition

Talk Outline

Composition and Compatibilty

Timed Interfaces as Timed Games

Timed Interface Automata

Solving Timed Games

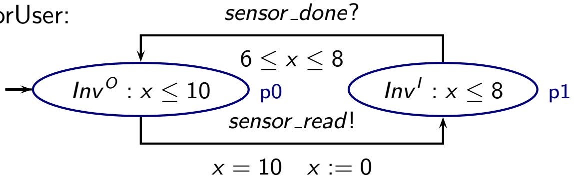
Definition: Timed Interface

Timed interface $\mathcal{P} = (S_{\mathcal{P}}, s^{init}_{\mathcal{P}}, Acts^{I}_{\mathcal{P}}, Acts^{O}_{\mathcal{P}}, \rho^{I}_{\mathcal{P}}, \rho^{O}_{\mathcal{P}})$ with:

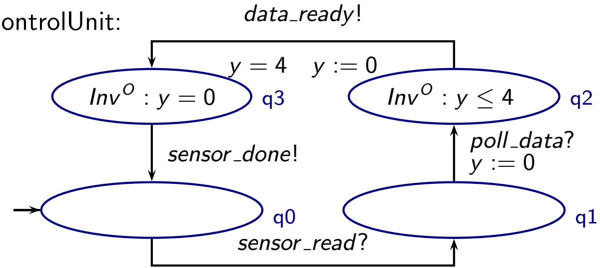
- $S_{\mathcal{P}}$: set of states
- $s^{init}_{\mathcal{P}} \in S_{\mathcal{P}}$: initial state
- $Acts^{I}_{\mathcal{P}}$ and $Acts^{O}_{\mathcal{P}}$: immediate input/output actions
- $\mathbb{T}$: set of timed actions ($\mathbb{T} = \mathbb{R}_{\geq 0}$ or $\mathbb{T} = \mathbb{N}$)
- $Acts_{\mathcal{P}} = Acts^{I}_{\mathcal{P}} \cup Acts^{O}_{\mathcal{P}}$
- $\Gamma^{I}_{\mathcal{P}} = Acts^{I}_{\mathcal{P}} \cup \mathbb{T}$: set of all input actions
- $\Gamma^{O}_{\mathcal{P}} = Acts^{O}_{\mathcal{P}} \cup \mathbb{T}$: set of all output actions
- $\rho^{I}_{\mathcal{P}} \subseteq S_{\mathcal{P}} \times \Gamma^{I}_{\mathcal{P}} \times S_{\mathcal{P}}$: input transition relation
- $\rho^{O}_{\mathcal{P}} \subseteq S_{\mathcal{P}} \times \Gamma^{O}_{\mathcal{P}} \times S_{\mathcal{P}}$: output transition relation

Example

SensorUser:



SensorControlUnit:



Composability

Timed interfaces $\mathcal{P}$ and $\mathcal{Q}$ are composable if

- $\mathcal{P}$ and $\mathcal{Q}$ are well-formed
- $Acts^O_{\mathcal{P}} \cap Acts^O_{\mathcal{Q}} = \emptyset$

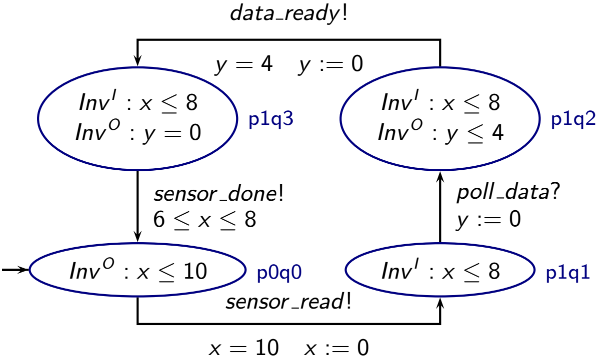
Shared actions: $shared(\mathcal{P}, \mathcal{Q}) := Acts_{\mathcal{P}} \cap Acts_{\mathcal{Q}}$

Interface Product

For $\mathcal{P}$ and $\mathcal{Q}$ composable timed interfaces

- $S_{\mathcal{P}\otimes\mathcal{Q}} = S_{\mathcal{P}} \times S_{\mathcal{Q}}$
- $s^{init}_{\mathcal{P}\otimes\mathcal{Q}} = (s^{init}_{\mathcal{P}}, s^{init}_{\mathcal{Q}})$.
- $Acts^{I}_{\mathcal{P}\otimes\mathcal{Q}} = Acts^{I}_{\mathcal{P}} \cup Acts^{I}_{\mathcal{Q}} \setminus shared(\mathcal{P},\mathcal{Q})$
- $Acts^{O}_{\mathcal{P}\otimes\mathcal{Q}} = Acts^{O}_{\mathcal{P}} \cup Acts^{O}_{\mathcal{Q}}$.
- $\rho^{I}_{\mathcal{P}\otimes\mathcal{Q}} = \{((s_1,s_2),\alpha,(s'_1,s'_2)|(s_1,\alpha,s'_1) \in \rho^{I}_{\mathcal{P}}$ *and* $(s_2,\beta,s'_2) \in \rho^{I}_{\mathcal{Q}}, \beta = \alpha$ *if* $\alpha \in Acts^{I}_{\mathcal{Q}}$ *or* $\beta = 0$ *otherwise*$\} \cup \{((s_1,s_2),\alpha,(s'_1,s'_2)|(s_2,\alpha,s'_2) \in \rho^{I}_{\mathcal{Q}}$ *and* $(s_1,\beta,s'_1) \in \rho^{I}_{\mathcal{P}}$, $\beta = \alpha$ *if* $\alpha \in Acts^{I}_{\mathcal{P}}$ *or* $\beta = 0$ *otherwise*$\}$
- $\rho^{O}_{\mathcal{P}\otimes\mathcal{Q}} = \{((s_1,s_2),\alpha,(s'_1,s'_2)|(s_1,\alpha,s'_1) \in \rho^{O}_{\mathcal{P}}$ *and* $(s_2,\beta,s'_2) \in \rho^{I}_{\mathcal{Q}}, \beta = \alpha$ *if* $\alpha \in Acts^{I}_{\mathcal{Q}}$ *or* $\beta = 0$ *otherwise*$\} \cup \{((s_1,s_2),\alpha,(s'_1,s'_2)|(s_2,\alpha,s'_2) \in \rho^{O}_{\mathcal{Q}}$ *and* $(s_1,\beta,s'_1) \in \rho^{I}_{\mathcal{P}}$, $\beta = \alpha$ *if* $\alpha \in Acts^{I}_{\mathcal{P}}$ *or* $\beta = 0$ *otherwise*$\}$

Example II



SensorUser $\otimes$ SensorControlUnit

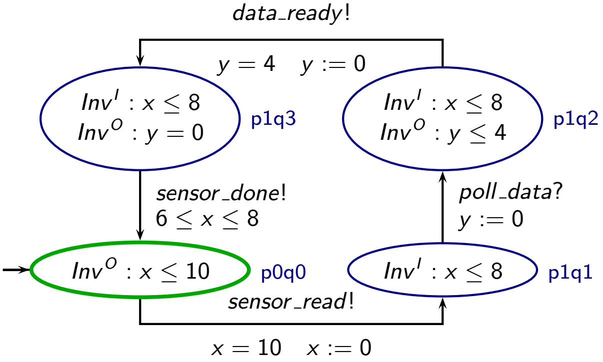
Error States

Problems with interface product:

- timing requirements of components not synchronized
- one component could create output that cannot be accepted

  $\Rightarrow$ error state

Example III

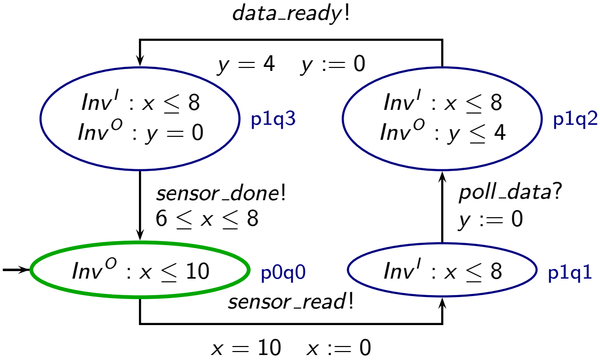


$x = 0$

$y = 0$

SensorUser $\otimes$ SensorControlUnit

Example III



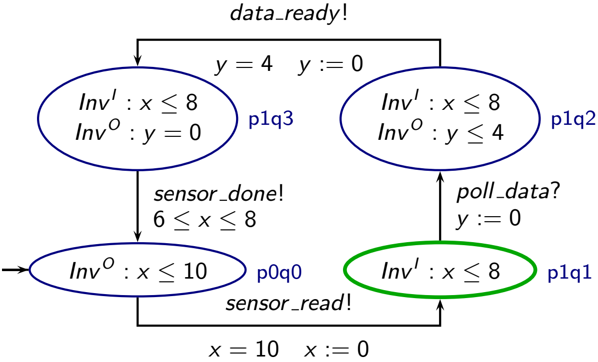
SensorUser $\otimes$ SensorControlUnit

$x = 10$

$y = 10$

$sensor\_read!$

Example III

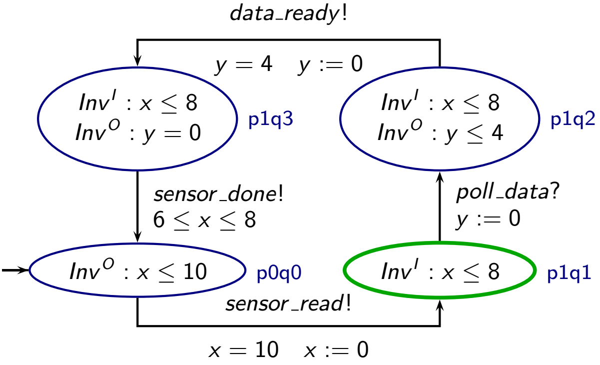


SensorUser $\otimes$ SensorControlUnit

$x = 0$

$y = 10$

Example III



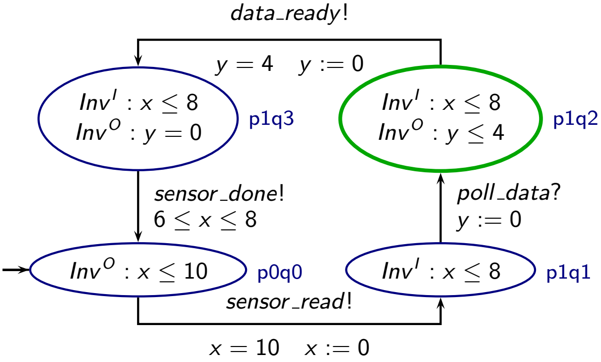
SensorUser $\otimes$ SensorControlUnit

$x = 8$

$y = 18$

$poll\_data?$

Example III



$x = 8$

$y = 0$

SensorUser $\otimes$ SensorControlUnit

Timed Interfaces as Timed Games [dAHS02]

- model interface as two-player game
- Player Input: Environment
- Player Output: Component
- Moves:

timed actions	wait for event
immediate actions	trigger event

- goal: time diverges or other player blocks

Definition: Moves and Outcome

Possible moves for player $\gamma \in \{I, O\}$ in state $s \in S_{\mathcal{P}}$

- $\Gamma^{\gamma}_{\mathcal{P}}(s) = \{\alpha \in \Gamma^{\gamma}_{\mathcal{P}} \mid \exists s' \in S_{\mathcal{P}}.(s, \alpha, s') \in \rho^{\gamma}_{\mathcal{P}}\}$
- $\Gamma^{\gamma}_{\mathcal{P}}(s) \neq \emptyset \implies (s, 0, s) \in \rho^{\gamma}_{\mathcal{P}}$
- $\Gamma^{\gamma}_{\mathcal{P}}(s) = \emptyset \implies$ player γ loses because he blocks

For $s \in S_{\mathcal{P}}$, $\alpha_I \in \Gamma^{I}_{\mathcal{P}}(s)$, $\alpha_O \in \Gamma^{O}_{\mathcal{P}}(s)$ and $bl \in \{I, O\}$,
define outcome $\delta_{\mathcal{P}}(s, \alpha_I, \alpha_O) := (\alpha, s', bl)$

- α_I, $\alpha_O \in \mathbb{T} \implies \alpha = min\{\alpha_I, \alpha_O\}$
  $bl = I$ if $\alpha_I < \alpha_O$, $bl = O$ otherwise (asymmetric!).
- If $\alpha_I \in Acts_{\mathcal{P}}$ and $\alpha_O \in \mathbb{T}$, then $\alpha = \alpha_I$ and $bl = I$.
- If $\alpha_I \in \mathbb{T}$ and $\alpha_O \in Acts_{\mathcal{P}}$, then $\alpha = \alpha_O$ and $bl = O$.
- Ifα_I, $\alpha_O \in Acts_{\mathcal{P}}$, choose $\alpha = \alpha_I$ and $bl = I$ or $\alpha = \alpha_O$ and $bl = O$ nondeterministically.

Definition: Strategies and Reachable States

Strategy $\pi^\gamma : S^*_{\mathcal{P}} \rightarrow \mathit{\Gamma}^\gamma_{\mathcal{P}}$ for player $\gamma \in \{I, O\}$ assigns move $\pi^\gamma(\bar{s}) \in \mathit{\Gamma}^\gamma_{\mathcal{P}}(s)$ to every $\bar{s} \in S^*_{\mathcal{P}}$ whose final state is s, if $\mathit{\Gamma}^\gamma_{\mathcal{P}} \neq \emptyset$. Otherwise, $\pi^\gamma(\bar{s})$ is undefined.

State $s \in S_{\mathcal{P}}$ is reachable if there are strategies π^I and π^O for player I and O s.t s is visited during game starting from $s^{init}_{\mathcal{P}}$ that is played according to π^I and π^O.

Well-formedness

Liveness

- sum of timed actions must not converge
  (no Zeno behavior [MPS95])
- player must not block game

Blocking

- player runs out of moves
- one player always plays, but time does not converge

A timed interface is well-formed if there is strategy for both players to let time diverge or blame the other player for blocking the game.

Combining Interfaces

- one component might produce output that cannot be accepted by others $\Rightarrow$ error state
- optimistic approach: restrict interface to make it work
- can't change components $\Rightarrow$ change use (environment)
- guarantee safety by avoiding error states

Error States

Immediate error state:
$(s,t) \in S_{\mathcal{P}\otimes\mathcal{Q}}$ with $\alpha \in \mathit{shared}(\mathcal{P},\mathcal{Q})$ such that
$\exists s' : (s,\alpha,s') \in \rho^O_{\mathcal{P}}$ and $\forall t' : (t,\alpha,t') \notin \rho^I_{\mathcal{Q}}$ or
$\exists t' : (t,\alpha,t') \in \rho^O_{\mathcal{Q}}$ and $\forall s' : (s,\alpha,s') \notin \rho^I_{\mathcal{P}}$.
set of all immediate error states: $\mathit{i\text{-}errors}(\mathcal{P},\mathcal{Q}) \subseteq S_{\mathcal{P}\otimes\mathcal{Q}}$.

Time error state:
$(s,t) \in S_{\mathcal{P}\otimes\mathcal{Q}}$ reachable in $\mathcal{P}\otimes\mathcal{Q}$, but there is no strategy to win the game for player I in $S_{\mathcal{P}\otimes\mathcal{Q}} \setminus \mathit{i\text{-}errors}(\mathcal{P},\mathcal{Q})$.
set of all time error states: $\mathit{t\text{-}errors}(\mathcal{P},\mathcal{Q})$

Interface Composition

well-formed, composable interfaces $\mathcal{P}$ and $\mathcal{Q}$ are compatible if $(s_{\mathcal{P}}^{init}, s_{\mathcal{Q}}^{init}) \notin \textit{t-errors}(\mathcal{P}, \mathcal{Q})$

composition $\mathcal{P} \parallel \mathcal{Q}$ defined like $\mathcal{P} \otimes \mathcal{Q}$

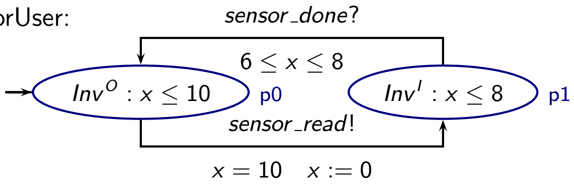
except for input transition relation:

$U = S_{\mathcal{P} \otimes \mathcal{Q}} \setminus \textit{t-errors}(\mathcal{P}, \mathcal{Q})$

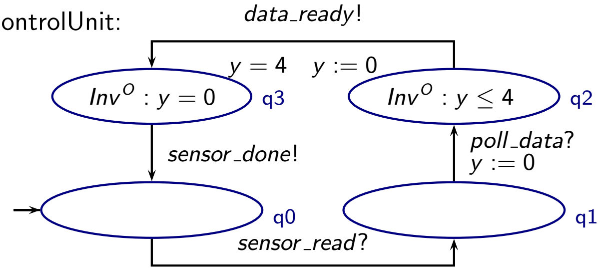
$\rho^I_{\mathcal{P} \parallel \mathcal{Q}} = \rho^I_{\mathcal{P} \otimes \mathcal{Q}} \cap (U \times \textit{Acts}^I_{\mathcal{P} \otimes \mathcal{Q}} \times U)$

Examples revisited

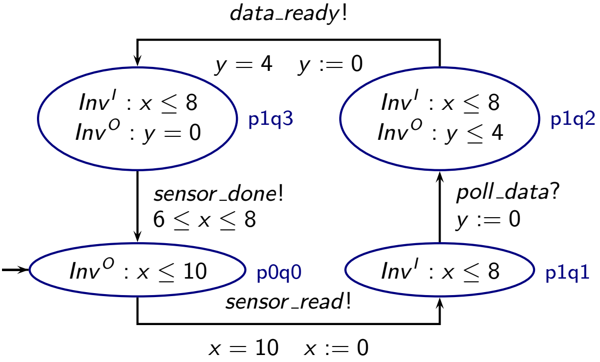
SensorUser:



SensorControlUnit:

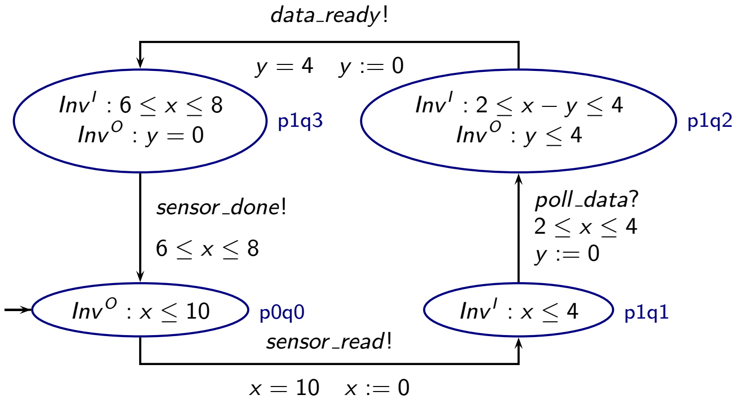


Examples revisited



SensorUser $\otimes$ SensorControlUnit

Examples revisited



SensorUser $\|$ SensorControlUnit

Timed Interface Automata

- finite representation for timed interfaces
- similiar to timed automata ([AD94])
- reuse existing algorithms for calculating live states, composition and checking well-formedness

Definition: Timed Interface Automata

Timed interface automaton
$\mathcal{A} = (Q_{\mathcal{A}}, q_{\mathcal{A}}^{init}, \mathcal{X}_{\mathcal{A}}, Acts_{\mathcal{A}}^{I}, Acts_{\mathcal{A}}^{O}, Inv_{\mathcal{A}}^{I}, Inv_{\mathcal{A}}^{O}, \rho_{\mathcal{A}})$ with:

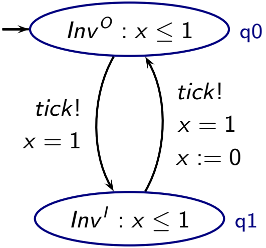
- $Q_{\mathcal{A}}$: set of *locations*
- $q_{\mathcal{A}}^{init} \in Q_{\mathcal{A}}$: initial location
- $\mathcal{X}_{\mathcal{A}}$: set of clocks
- $Acts_{\mathcal{A}}^{I}$ and $Acts_{\mathcal{A}}^{O}$: sets of input and output actions
- $Inv_{\mathcal{A}}^{I} : Q_{\mathcal{A}} \mapsto \Xi[\mathcal{X}_{\mathcal{A}}]$ and $Inv_{\mathcal{A}}^{O} : Q_{\mathcal{A}} \mapsto \Xi[\mathcal{X}_{\mathcal{A}}]$ map an input/output invariant to each location
- $\rho_{\mathcal{A}} \subseteq Q_{\mathcal{A}} \times \Xi[\mathcal{X}_{\mathcal{A}}] \times Acts_{\mathcal{A}} \times 2^{\mathcal{X}_{\mathcal{A}}} \times Q_{\mathcal{A}}$ transition relation

Solving Timed Games

Checking for winning strategy of I

- compose automaton with Tick automaton
- check if there is strategy for $\Box\Diamond q_1 \vee \Diamond\Box bl = O$
- use algorithm for untimed games
- similar for player Output

Tick automaton



Solving Timed Games

Reachable states

- definable by clock conditions
- use algorithms for timed automata [AD94]

Well-formedness

Check: *reachable* $\implies$ *both players have winning strategy*

Summary

- new approach: model interface as asymmetric game
- restrict moves of input to guarantee safety and liveness
- optimistic
- automata representation allows use of existing algorithms

$\Rightarrow$ better model for interaction of real-time components

Thank you for your attention!

Questions?

References

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