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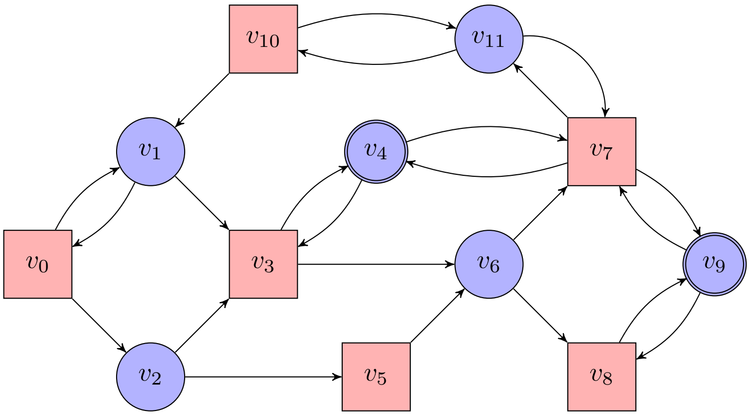
Infinite Games

Exercise Sheet 2

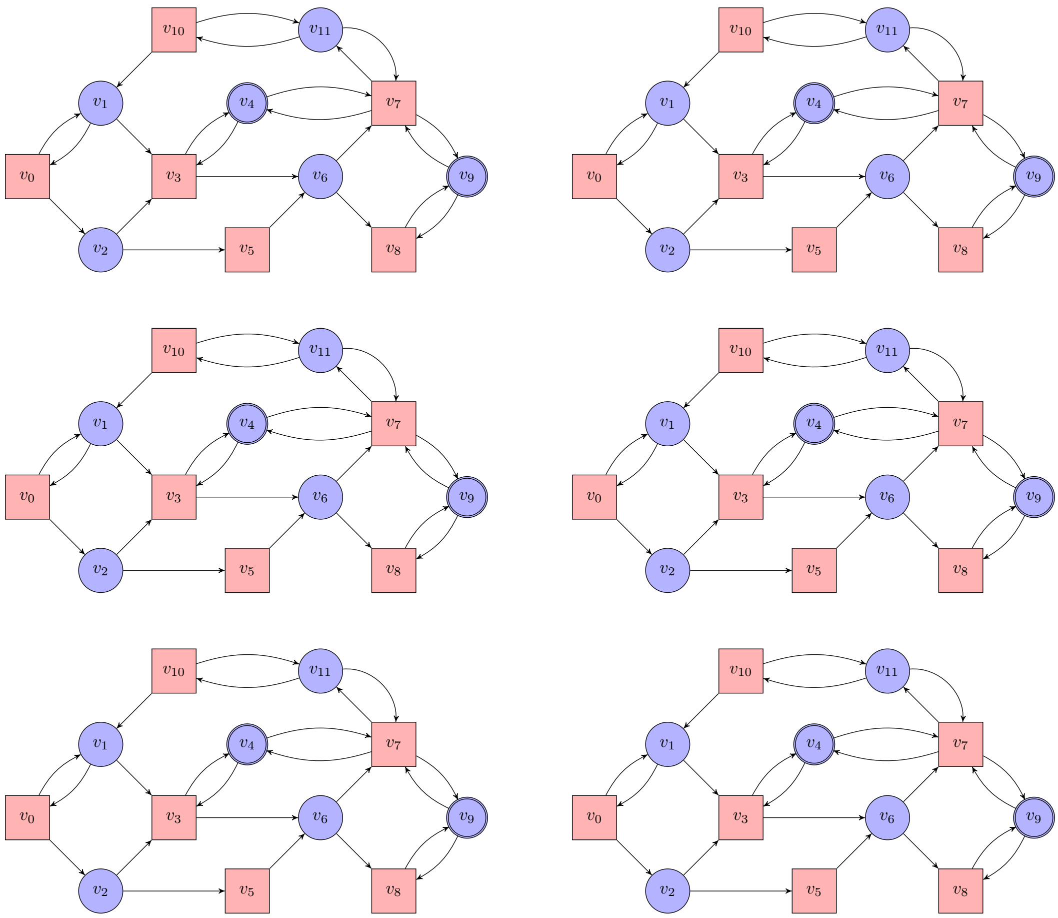
Deadline: 5 Nov. 2013

Exercise 2.1 - Reachability (1 + 2)



Consider the reachability game $\mathcal{G} = (\mathcal{A}, \text{REACH}(R))$ depicted above.

a) Determine the attractor sets $\text{Attr}_0^n(R)$ for all $n \in \mathbb{N}$. Therefore mark the corresponding vertices in the copies below and indicate which $\text{Attr}_0^n(R)$ you have calculated.



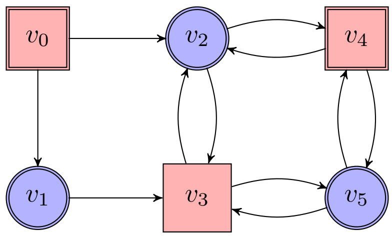
b) Give the uniform winning strategies for both players resulting from the attractor construction. Argue how you constructed them.

Exercise 2.2 - Reachability (5 + 3)

a) Prove Remark 2.3 in the lecture notes by providing a corresponding algorithm. The algorithm gets as input a reachability game $\mathcal{G}$, represented by an arena $\mathcal{A}$ and a reachability set R (with sets represented as lists), and outputs the attractor of R. Argue why your algorithm has the desired time complexity.

b) Complete the proof of Theorem 2.1 in the lecture notes. Therefore show that you also can compute the corresponding winning strategies for both players in the desired time.

Exercise 2.3 - Safety (2 + 2)



Consider the safety game $\mathcal{G} = (\mathcal{A}, \textsc{safe}(S))$ depicted above.

a) Solve the game by transforming it into an equivalent reachability game first, solving the reachability game and transforming back the results to the actual safety game.

b) Prove Lemma 2.2 and Corollary 2.1 in the lecture notes.

Exercise 2.4 - Challenge (2 Bonus Points)

Consider the definition of a challenging reachability game given below.

> **Definition E2.4.1.** Let the challenging reachability condition $\textsc{chreach}(\mathcal{R})$ on a set of sets $\mathcal{R} \subseteq 2^V$ for an arena $\mathcal{A} = (V, V_0, V_1, E)$ be defined as:
>
> $$\textsc{chreach}(\mathcal{R}) = \{\, \rho \in \text{Plays}(\mathcal{A}) \mid \forall R \in \mathcal{R}.\ \text{Occ}(\rho) \cap R \neq \emptyset \,\}$$
>
> Then we call the game $\mathcal{G} = (\mathcal{A}, \textsc{chreach}(\mathcal{R}))$ a *challenging reachability game* with challenging reachability set $\mathcal{R}$.

Prove that solving challenging reachability games is PSPACE hard. The size of a challenging reachability game $\mathcal{G} = (\mathcal{A}, \textsc{chreach}(\mathcal{R}))$ is defined to be $|\mathcal{A}| + |\mathcal{R}|$.