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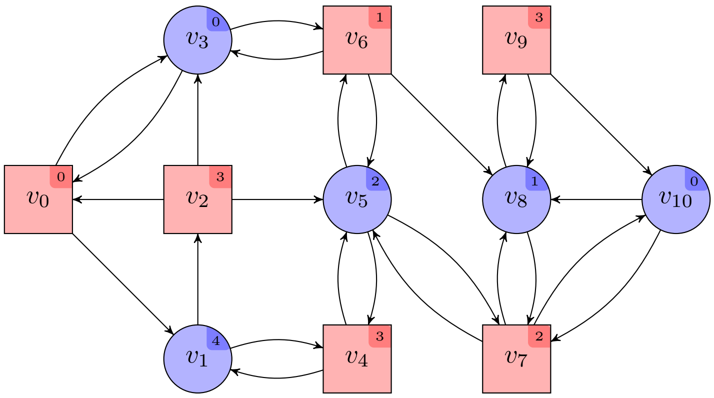
Infinite Games

Deadline: 12 Nov. 2013

Exercise 4.1 - Parity (3 + 2)



Consider the parity game $\mathcal{G} = (\mathcal{A}, \text{PARITY}(\Omega : V \to [5]))$ with arena $\mathcal{A} = (V, V_0, V_1, E)$ depicted above. Compute the winning regions and uniform positional winning strategies for both players using the idea underlying the proof of Theorem 2.5. *(Hint: You do not have to give intermediate steps for the attractor computations.)*

Exercise 4.2 - Weak Parity (4 + 3)

In a parity game, the goal for Player i is to ensure that the minimal color that occurs infinitely often has parity i. By replacing "infinitely often" by "at least once" we get a definition for a weaker variant of parity games.

Definition E3.2.1. Let the weak parity condition $\text{WPARITY}(\Omega)$ on a color function $\Omega : V \to [k]$ for some arena $\mathcal{A} = (V, V_0, V_1, E)$ and some $k \in \mathbb{N}$ be defined as:

$$\text{WPARITY}(\Omega) := \{ \rho \in \text{Plays}(\mathcal{A}) \mid \text{Par}(min(\Omega(\text{Occ}(\rho)))) = 0 \}$$

Then we call the game $\mathcal{G} = (\mathcal{A}, \text{WPARITY}(\Omega))$ a *weak parity game* with color function Ω.

a) Give a polynomial time algorithm that computes the winning regions and uniform positional winning strategies for both players for a weak parity game $\mathcal{G}$.

b) Consider the game $\mathcal{G} = (\mathcal{A}, \text{WPARITY}(\Omega))$ where $\mathcal{A}$ and Ω are defined as in Exercise 4.1. Determine the winning regions and uniform positional winning strategies of both players using your algorithm given in a).

Exercise 4.3 - Solitary Parity (3)

A game $\mathcal{G} = (\mathcal{A}, \text{Win})$ with $\mathcal{A} = (V, V_0, V_1, E)$ is called a solitary game for Player i if all vertices owned by Player $1 - i$ have exactly one outgoing edge. Formally:

$$\forall v \in V_{1-i}.\ |\{v\} \times V \cap E| = 1$$

Accordingly, the game is only played by Player i since Player $1-i$ never has a choice. Prove that solitary parity games can be solved in polynomial time in the number of edges of the arena.

Exercise 4.4 - Challenge (2 Bonus Points)

A game $\mathcal{G} = (\mathcal{A}, \text{Win})$ with $\mathcal{A} = (V, V_0, V_1, E)$ is called *undirected* iff $\forall (v, v') \in E.\ (v', v) \in E$. Prove that undirected parity games can be solved in polynomial time in the number of edges of the arena.