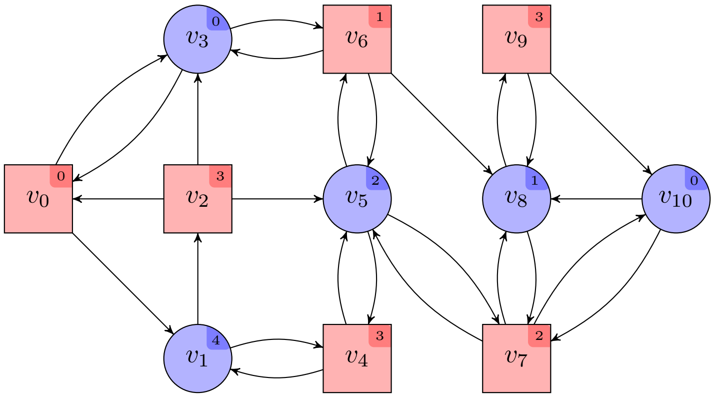
Dr. Martin Zimmermann
Felix Klein, B.Sc.

Infinite Games

Exercise Sheet 5

Deadline: 19 Nov. 2013

Exercise 5.1 - Progress Measure (6)



Consider the parity game $\mathcal{G} = (\mathcal{A}, \textsc{parity}(\Omega : V \to [5]))$ with arena $\mathcal{A} = (V, V_0, V_1, E)$ depicted above. Use the progress measure algorithm to compute the winning regions of the game. For this exercise it is sufficient to fill in the corresponding score sheets into the arenas given on the additional paper and to mark in each iteration the node that has been lifted. Consider that the score sheets are the tuples sketched inside the corresponding vertices.

Hint: This is the same parity game as on Exercise Sheet 4.

Exercise 5.2 - Progress Measure (3 + 1)

Let $\mathcal{G} = (\mathcal{A}, \textsc{parity}(\Omega))$ be a parity game with $\mathcal{A} = (V, V_0, V_1, E)$ and let $\wp \colon V \to \mathrm{Sh}(\mathcal{G})$ be a progress measure for $\mathcal{G}$.

a) Complete the proof of Lemma 2.7 in the lecture notes by showing that $\|\wp\| \subseteq W_0(\mathcal{G})$. To this end, show how to construct a positional winning strategy for Player 0 that is winning from $\|\wp\|$.

b) Compute a winning strategy for Player 0 in the game given in Exercise 5.1.

Exercise 5.3 - Positional Strategies in Prefix-independent Games (2 + 3)

Recall that we defined prefix-independent winning conditions on Exercise Sheet 3. Let $\mathcal{G} = (\mathcal{A}, \mathrm{Win})$ be a game with $\mathcal{A} = (V, V_0, V_1, E)$ and let $\mathrm{Win} \subseteq V^\omega$ be prefix-independent.

1. Let σ be a positional winning strategy for Player 0 from some vertex v. Furthermore, let v' be reachable from v via some play $\rho \in \mathrm{Plays}(\mathcal{A}, \sigma, v)$. Show that σ is winning for Player 0 from v' in $\mathcal{G}$.
2. Let W be a set of vertices such that Player 0 has a positional winning strategy σ_w in $\mathcal{G}$ from every vertex $w \in W$. Show that Player 0 has a winning strategy σ in $\mathcal{G}$ that is winning from every $w \in W$.

   Note: The order of quantification changes from

$$\forall w.\ \exists \sigma_w.\ \mathrm{Plays}(\mathcal{A}, \sigma_w, w) \subseteq \mathrm{Win} \quad \textit{to} \quad \exists \sigma.\ \forall w.\ \mathrm{Plays}(\mathcal{A}, \sigma, w) \subseteq \mathrm{Win}.$$

Exercise 5.4 - Challenge (2 Bonus Points)

Show that the progress measure algorithm has exponential running time in the worst case. To this end, construct for every $n \in \mathbb{N}$ a parity game $\mathcal{G}_n$ of polynomial size in n such that the progress measure algorithm performs an exponential number of lifts, irregardless of the order in which the Lift_v-operators are applied.