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Exercise Sheet 11

Infinite Games

Deadline: 14 Jan. 2014

Exercise 11.1 - S2S (1+3+1)

Give S2S formulas defining the following tree languages over the alphabet $\Sigma = \{a, b, c\}$:

a) The language of trees containing an a-labeled vertex whose left subtree contains a b-labeled vertex and whose right sub-tree contains a c-labeled vertex.

b) The language of trees t satisfying $t_{|0^\omega} \in (aa)^* b^\omega$.

c) The language of trees containing at least one a-labeled vertex and at least one b-labeled vertex.

Exercise 11.2 - Syntactic Sugar (2+4)

Show that ε and $\preceq$ are syntactic sugar.

a) Give a formula $\varphi_\varepsilon(x)$ not containing ε with one free first-order variable x and no free second-order variable such that $t, \mu \vDash \varphi_\varepsilon$ if and only if $\mu(x) = \varepsilon$.

b) Give a formula $\varphi_\preceq(x, y)$ not containing $\preceq$ with two free first-order variables x and y and no free second-order variable such that $t, \mu \vDash \varphi_\preceq$ if and only if $\mu(x)$ is a prefix of $\mu(y)$.

Exercise 11.3 - Subset & Connectedness (2+2)

Show that the subset-relation and "being connected" is expressible in S2S.

a) Give a formula $\varphi_\subseteq(X, Y)$ with two free second-order variables X and Y and no free first-order variable such that $t, \mu \vDash \varphi_\subseteq$ if and only if $\mu(X) \subseteq \mu(Y)$.

b) Give a formula $\varphi_c(X)$ with one free second-order variable X and no free first-order variable such that $t, \mu \vDash \varphi_c$ if and only if $\mu(X)$ is connected, i.e., if w and w' are in $\mu(X)$ and w' is a descendant of w, then all vertices on the path between w and w' are in $\mu(X)$, as well.

Exercise 11.4 - Challenge (2 Bonus Points)

Give an S2S formula defining the tree language over the alphabet $\Sigma = \{a, b, c\}$ containing exactly the trees having only finitely many a-labeled vertices. Argue informally why your solution is correct.