Infinite Games

Lecture 15

Martin Zimmermann

Universität des Saarlandes

February 6th, 2014

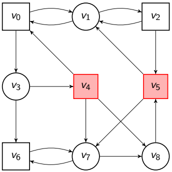
Plan for Today

- Review
- Exam
  - Organizational matters
  - Questions
- Outlook: even more games

Review

Reachability

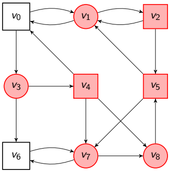
- **Name:** **Reachability Game**
- Format: $(\mathcal{A}, \textsc{reach}(R))$ with $R \subseteq V$



- Winning condition: $\mathrm{Occ}(\rho) \cap R \neq \emptyset$
- Solution complexity: linear time in $|E|$
- Algorithm: attractor
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: uniform positional
- Dual game: safety

Safety

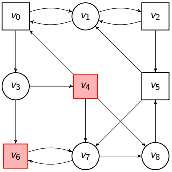
- **Name:** **Safety Game**
- Format: $(\mathcal{A}, \textsc{safe}(S))$ with $S \subseteq V$



- Winning condition: $\mathrm{Occ}(\rho) \subseteq S$
- Solution complexity: linear time in $|E|$
- Algorithm: dualize + attractor
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: uniform positional
- Dual game: reachability

Büchi

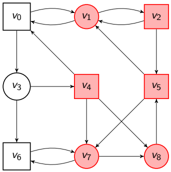
- **Name:** **Büchi Game**
- Format: $(\mathcal{A}, \textsc{Büchi}(F))$ with $F \subseteq V$



- Winning condition: $\mathrm{Inf}(\rho) \cap F \neq \emptyset$
- Solution complexity: **P**
- Algorithm: iterated attractor
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: uniform positional
- Dual game: co-Büchi

Co-Büchi

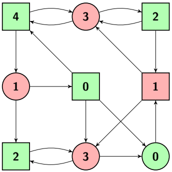
- **Name:** **Co-Büchi Game**
- Format: $(\mathcal{A}, \textsc{CoBüchi}(C))$ with $C \subseteq V$



- Winning condition: $\mathrm{Inf}(\rho) \subseteq C$
- Solution complexity: **P**
- Algorithm: dualize + iterated attractor
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: uniform positional
- Dual game: Büchi

Parity

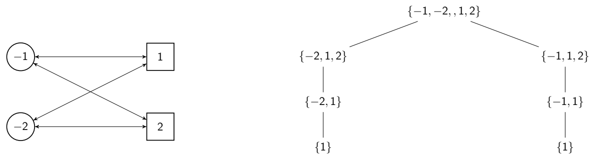
- **Name:** **Parity Game**
- Format: $(\mathcal{A}, \textsc{parity}(\Omega))$ with $\Omega \colon V \to \mathbb{N}$



- Winning condition: $\min(\mathrm{Inf}(\Omega(\rho)))$ even
- Solution complexity: $\mathbf{NP} \cap \mathbf{co\text{-}NP}$
- Algorithm: progress measures and many others
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: uniform positional
- Dual game: parity

Muller

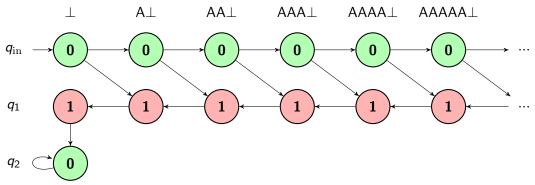
- **Name:** **Muller Game**
- Format: $(\mathcal{A}, \textsc{Muller}(\mathcal{F}))$ with $\mathcal{F} \subseteq 2^V$



- Winning condition: $\mathrm{Inf}(\rho) \in \mathcal{F}$
- Solution complexity: **P**, **NP** $\cap$ **co-NP**, **PSPACE**-complete
- Algorithm: reduction to parity and many others
- Memory requirements for Player 0: $|V|!$
- Memory requirements for Player 1: $|V|!$
- Dual game: Muller

Pushdown Parity

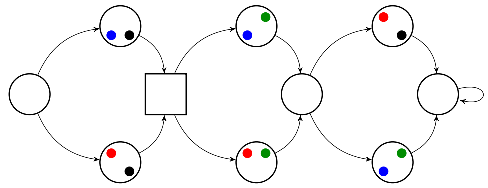
- **Name:** **Pushdown Parity Game**
- Format: $(\mathcal{A}, \textsc{parity}(\Omega))$ with $\mathcal{A}$ induced by PDS $\mathcal{P}$



- Winning condition: $\min(\mathrm{Inf}(\Omega(\rho)))$ even
- Solution complexity: **EXPTIME**-complete
- Algorithm: reduction to parity games
- Memory requirements for Player 0: infinite (pd. transducer)
- Memory requirements for Player 1: infinite (pd. transducer)
- Dual game: pushdown parity

Generalized Reachability

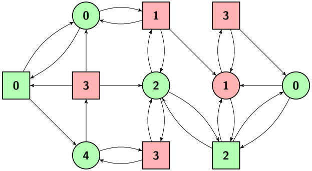
- **Name:** **Generalized Reachability Game**
- Format: $(\mathcal{A}, \textsc{ChReach}(\mathcal{R}))$ with $\mathcal{R} \subseteq 2^V$



- Winning condition: $\forall R \in \mathcal{R}.\ \mathrm{Occ}(\rho) \cap R \neq \emptyset$
- Solution complexity: **PSPACE**-complete
- Algorithm: Simulate for $|V| \cdot |\mathcal{R}|$ steps
- Memory requirements for Player 0: $2^{|\mathcal{R}|}$
- Memory requirements for Player 1: $\binom{|\mathcal{R}|}{\lfloor |\mathcal{R}|/2 \rfloor}$
- Dual game: disjunctive safety

Weak Parity

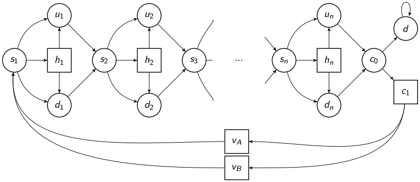
- **Name:** **Weak Parity Game**
- Format: $(\mathcal{A}, \textsc{wparity}(\Omega))$ with $\Omega \colon V \to \mathbb{N}$



- Winning condition: $\min(\mathrm{Occ}(\Omega(\rho)))$ even
- Solution complexity: **P**
- Algorithm: iterated attractor
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: uniform positional
- Dual game: weak parity

Weak Muller

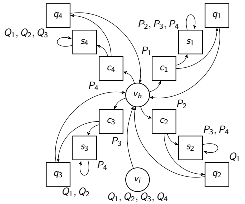
- **Name:** **Weak Muller Game**
- Format: $(\mathcal{A}, \textsc{wMuller}(\mathcal{F}))$ with $\mathcal{F} \subseteq 2^V$



- Winning condition: $\mathrm{Occ}(\rho) \in \mathcal{F}$
- Solution complexity: **PSPACE**-complete
- Algorithm: reduction to weak parity or direct one
- Memory requirements for Player 0: $2^{|V|}$
- Memory requirements for Player 1: $2^{|V|}$
- Dual game: weak Muller

Request-Response

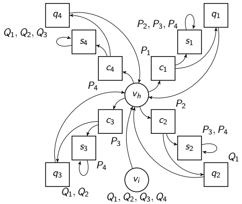
- **Name:** **Request-Response Game**
- Format: $(\mathcal{A}, \textsc{ReqRes}((Q_j, P_j)_{j \in [k]}))$ with $Q_j, P_j \subseteq V$



- Winning condition: $\forall j \, \forall n (\rho_n \in Q_j \rightarrow \exists m \geq n. \, \rho_m \in P_j)$
- Solution complexity: **EXPTIME**-complete
- Algorithm: reduction to Büchi
- Memory requirements for Player 0: $k \cdot 2^k$
- Memory requirements for Player 1: 2^k
- Dual game: n/a

Rabin

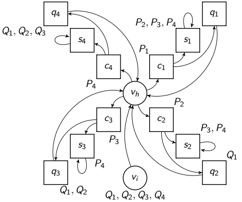
- **Name:** **Rabin Game**
- Format: $(\mathcal{A}, \textsc{rabin}((Q_j, P_j)_{j\in[k]}))$ with $Q_j, P_j \subseteq V$



- Winning condition: $\exists j(\mathrm{Inf}(\rho) \cap Q_j \neq \emptyset \wedge \mathrm{Inf}(\rho) \cap P_j = \emptyset)$
- Solution complexity: **NP**-complete
- Algorithm: reduction to parity or direct one
- Memory requirements for Player 0: uniform positional
- Memory requirements for Player 1: $k!$
- Dual game: Streett

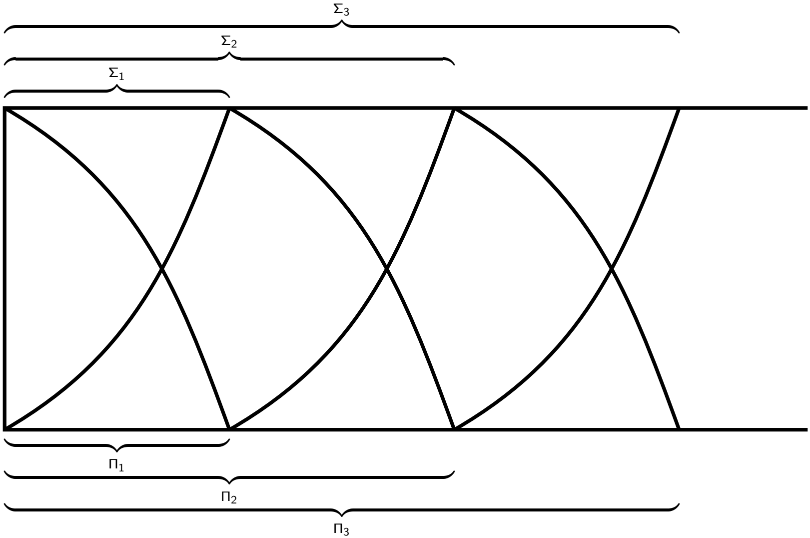
Streett

- **Name:** **Streett Game**
- Format: $(\mathcal{A}, \text{STREETT}((Q_j, P_j)_{j \in [k]}))$ with $Q_j, P_j \subseteq V$



- Winning condition: $\forall j (\text{Inf}(\rho) \cap Q_j \neq \emptyset \rightarrow \text{Inf}(\rho) \cap P_j \neq \emptyset)$
- Solution complexity: **co-NP**-complete
- Algorithm: reduction to parity or direct one
- Memory requirements for Player 0: $k!$
- Memory requirements for Player 1: uniform positional
- Dual game: Rabin

Reducibility



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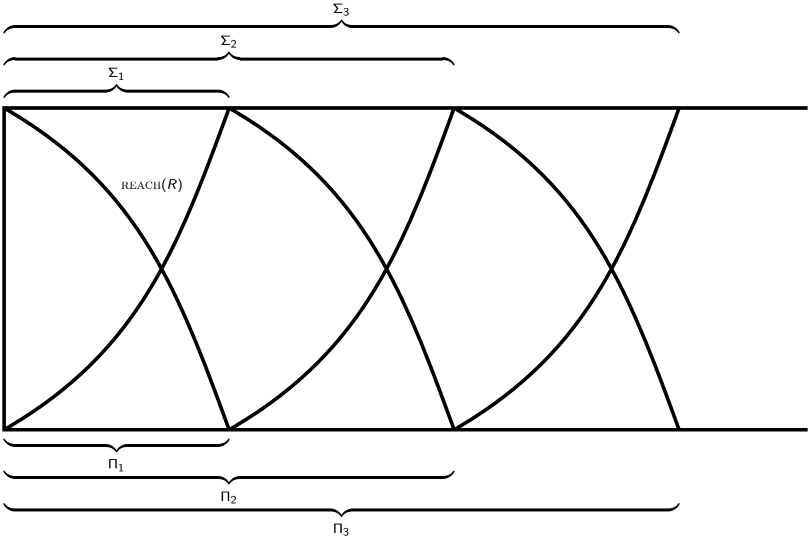

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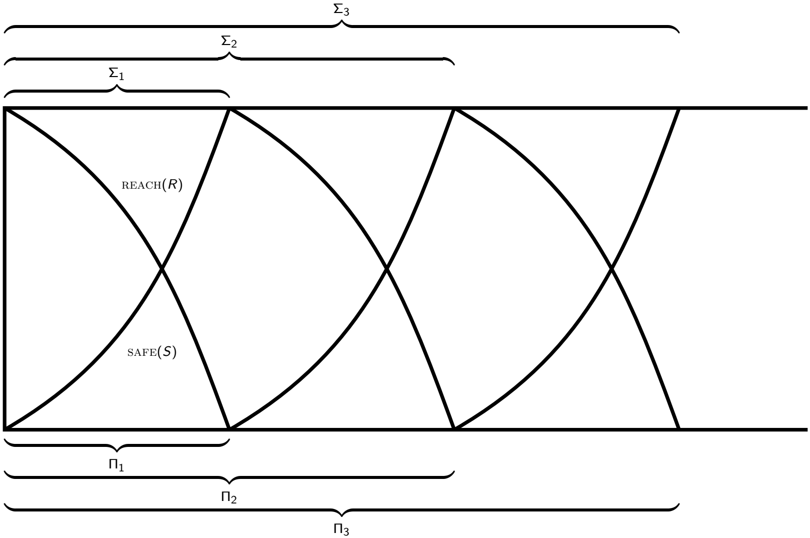

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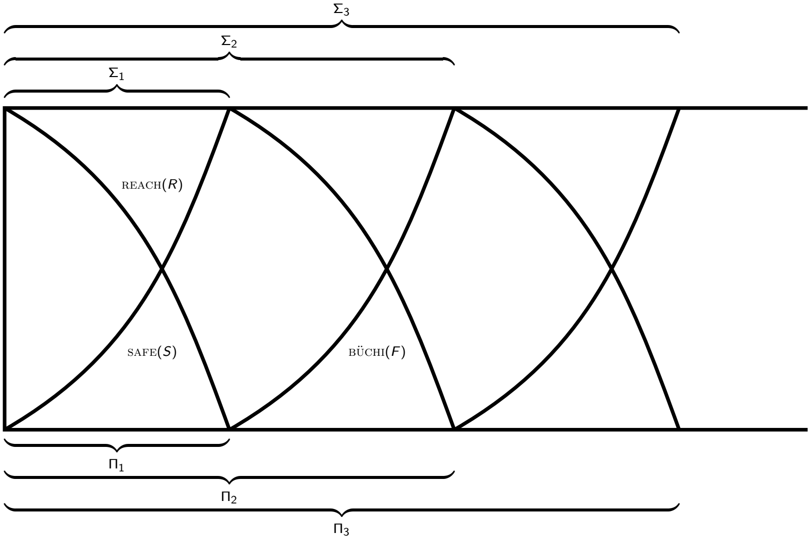

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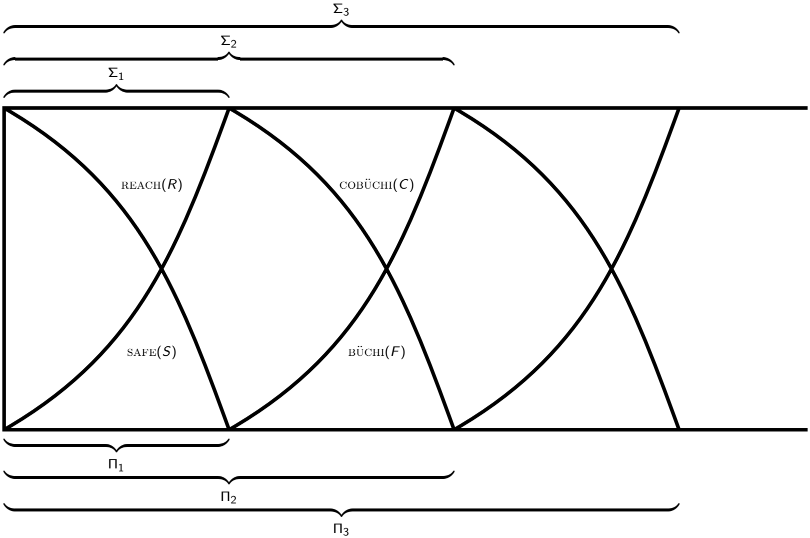

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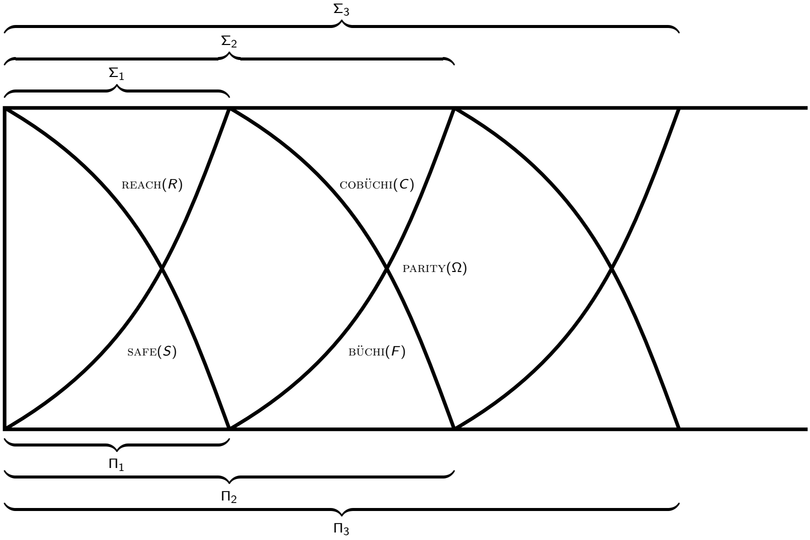

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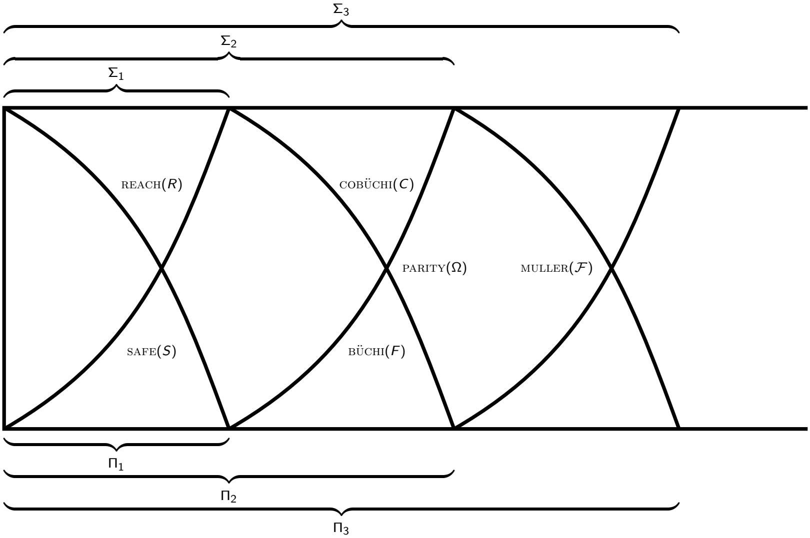

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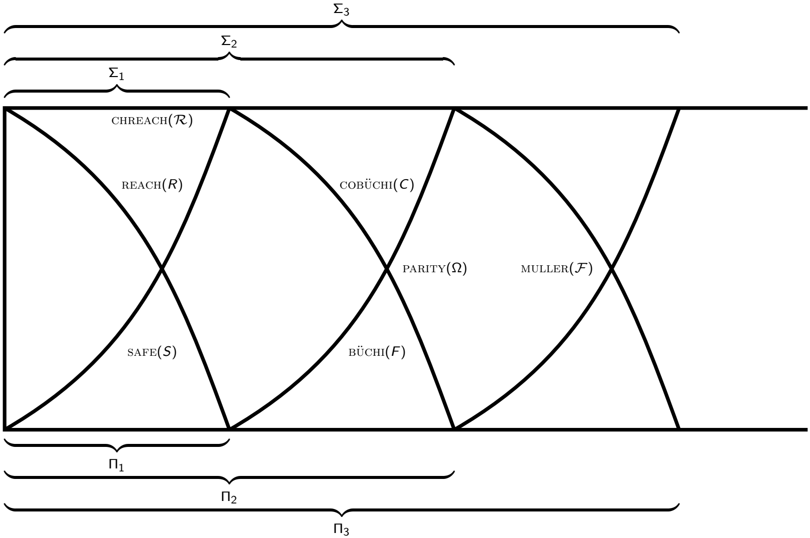

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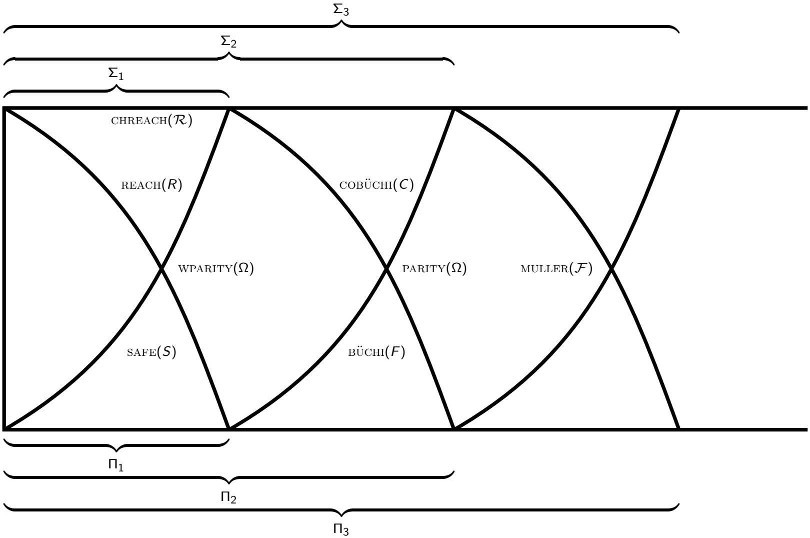

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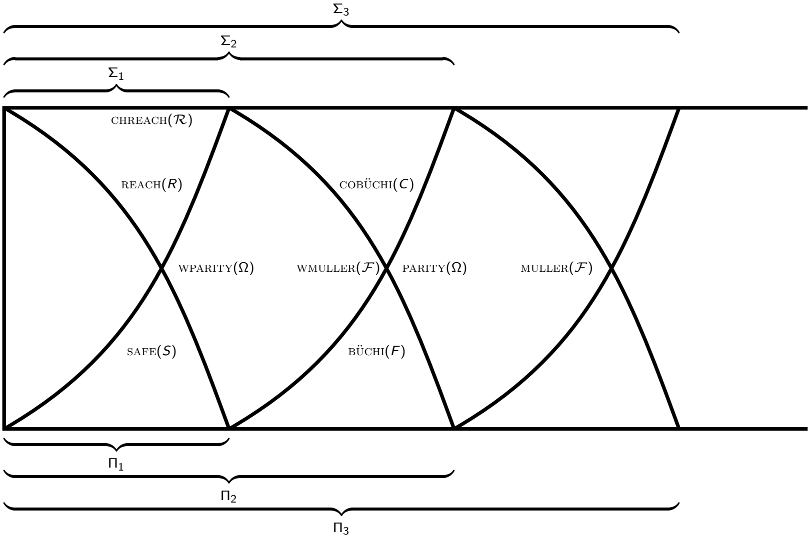

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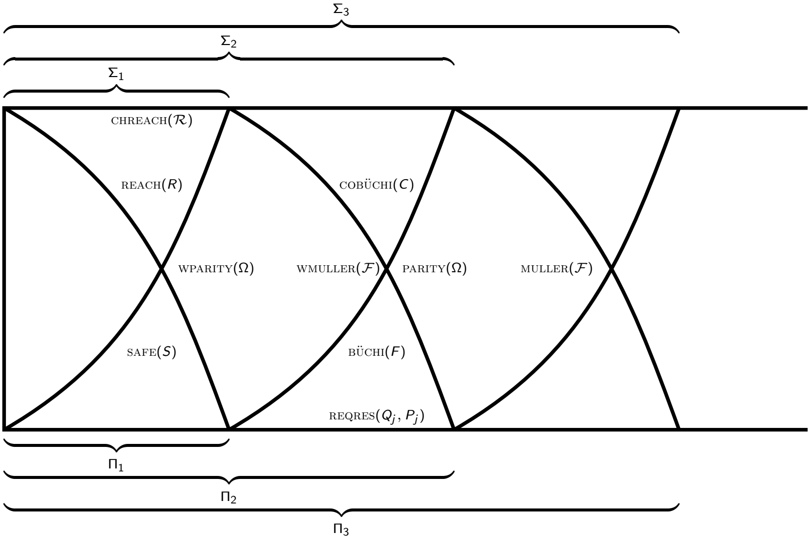

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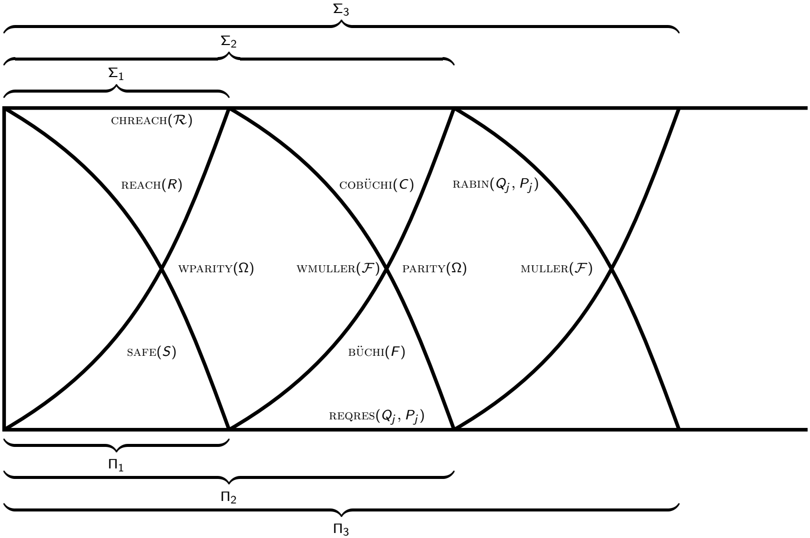

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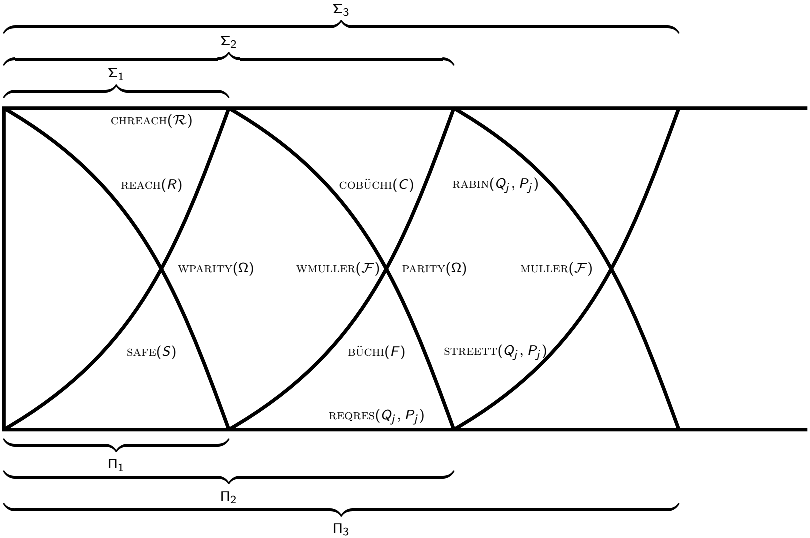

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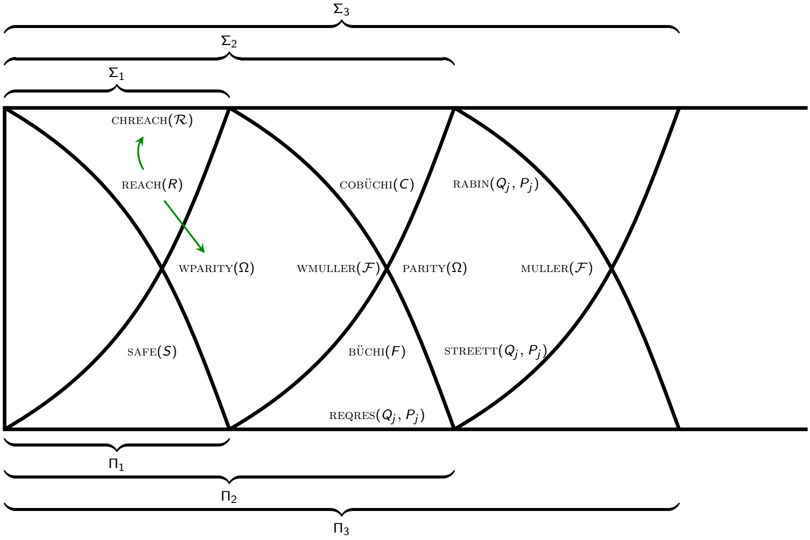

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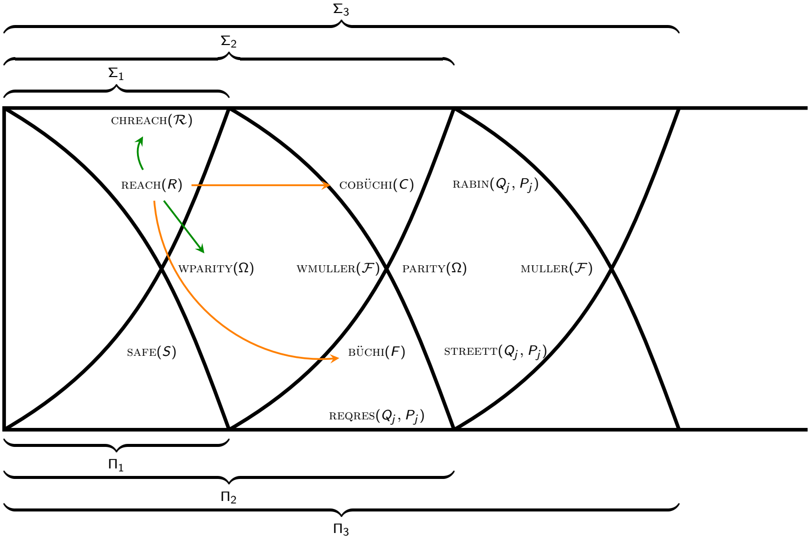

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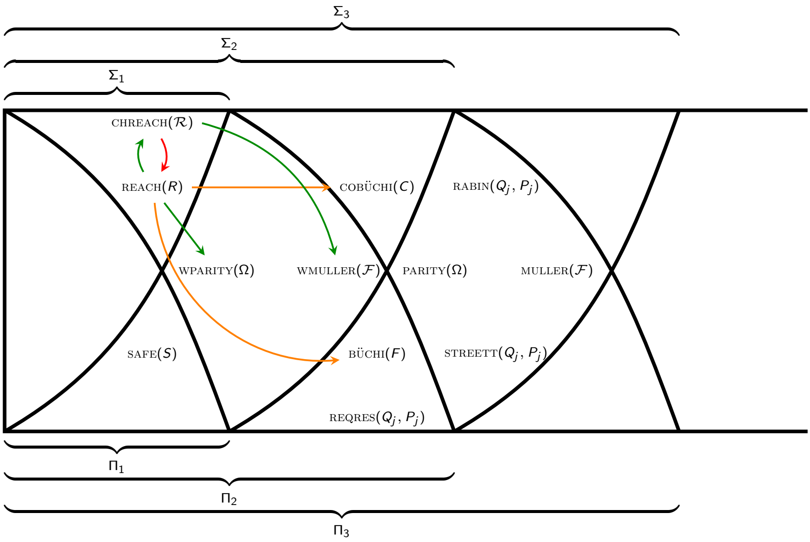

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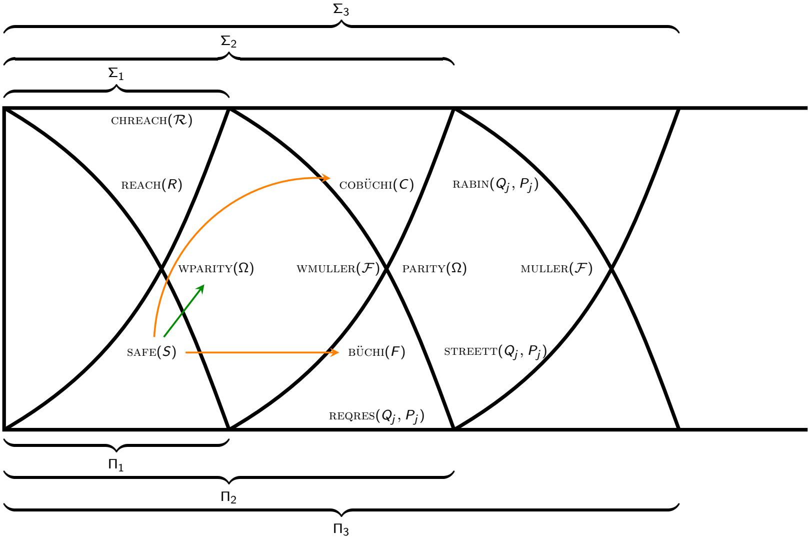

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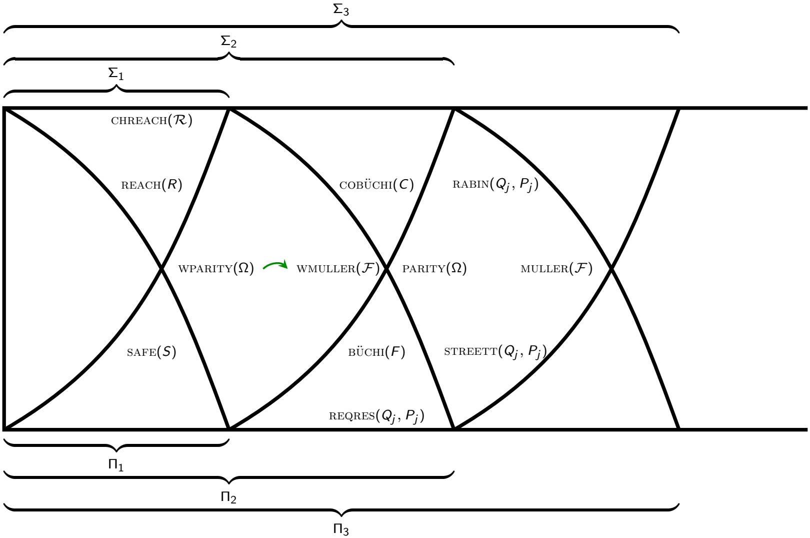

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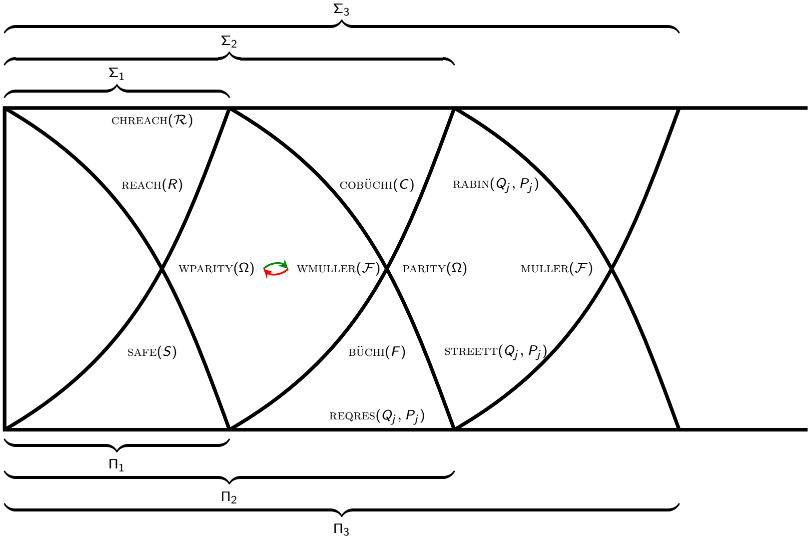

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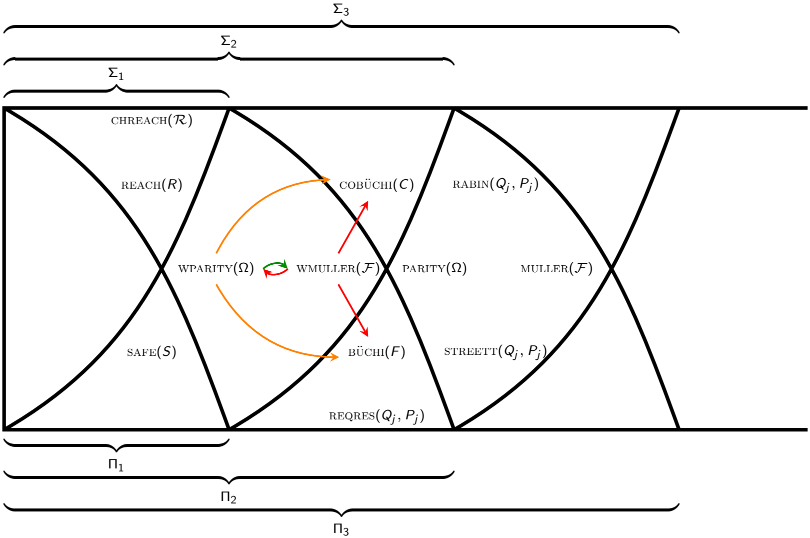

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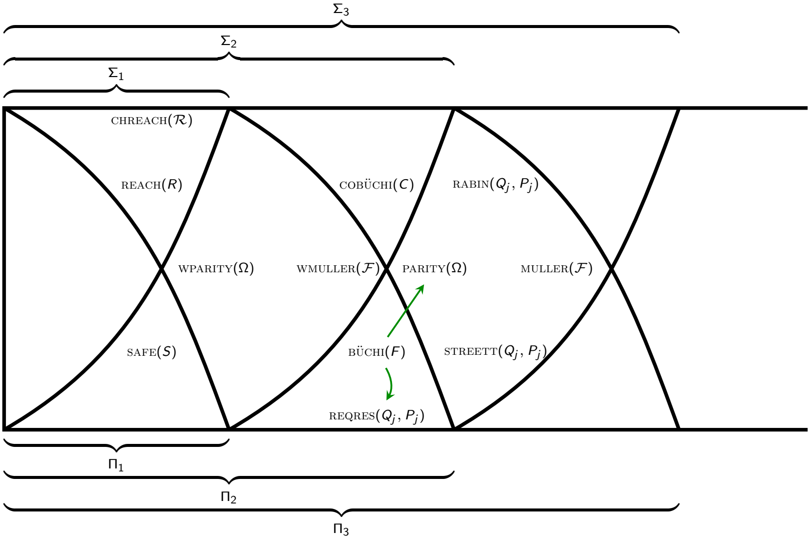

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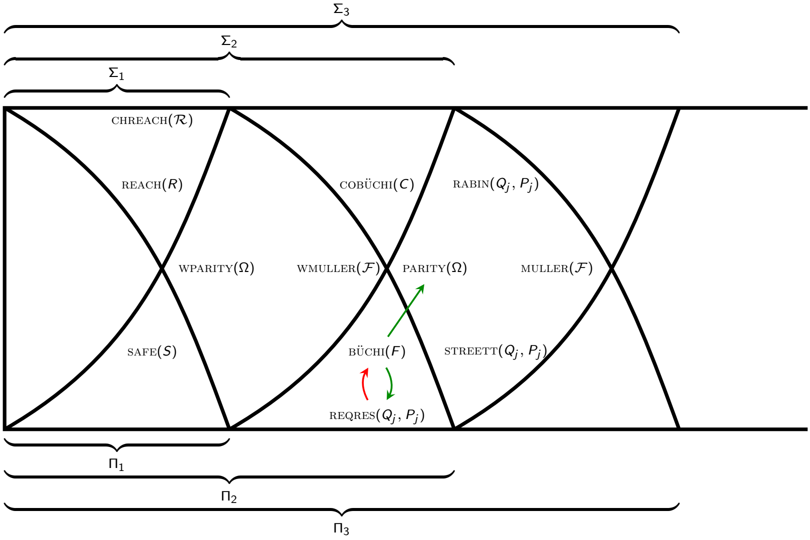

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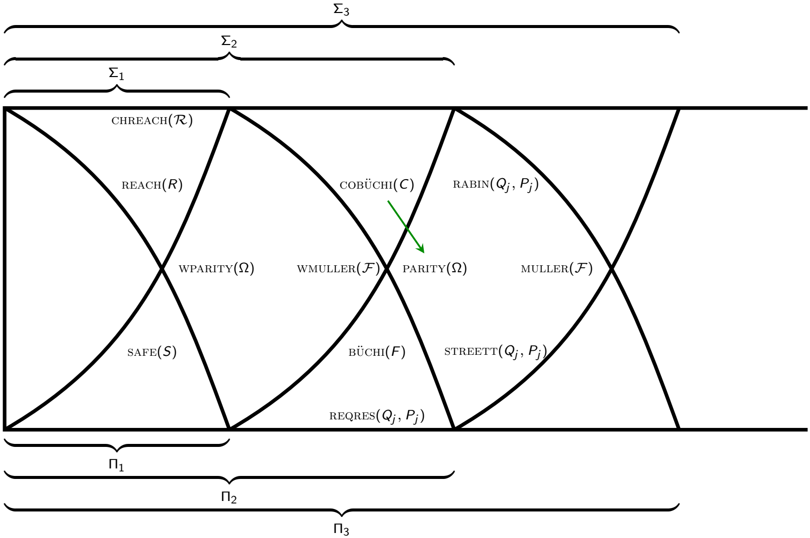

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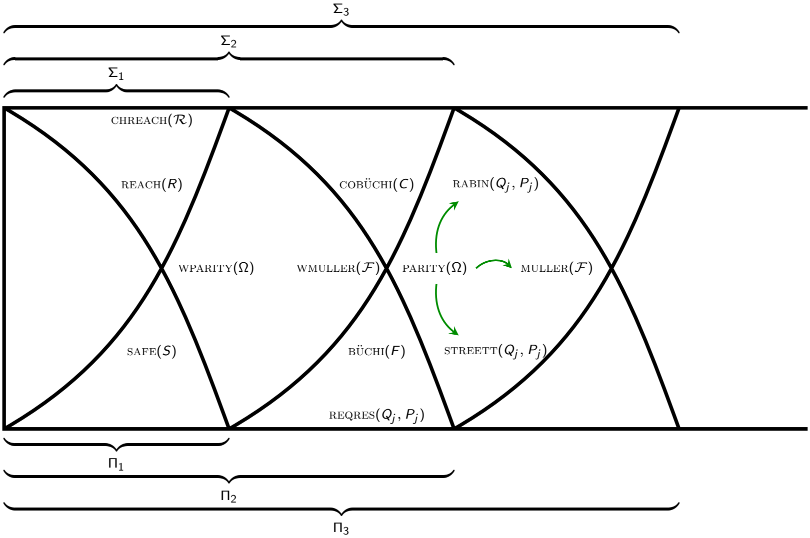

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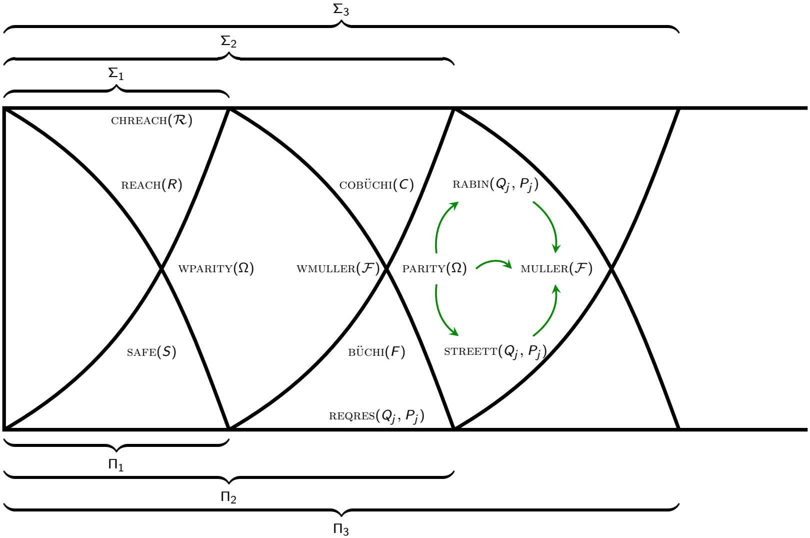

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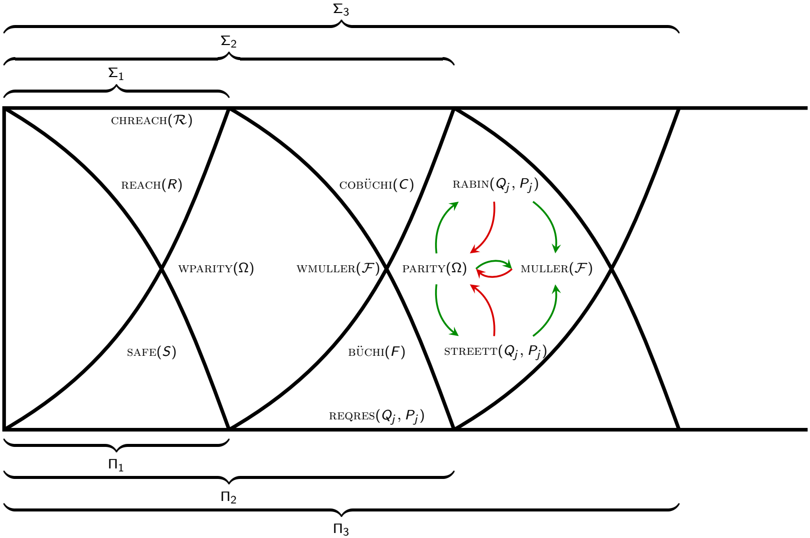

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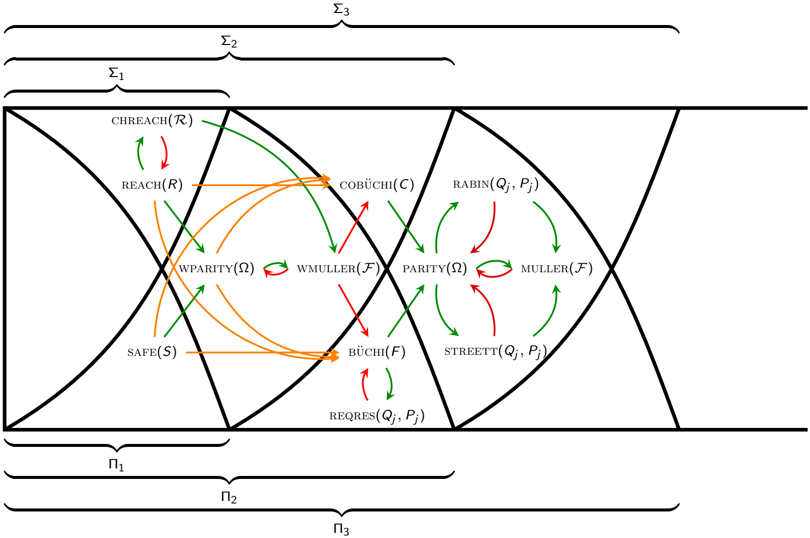

S2S and Parity Tree Automata

- S2S: Monadic Second-order logic over two successors
- PTA: Parity tree automata

Both formalisms are equivalent:

- For every $\mathscr{A}$ exists $\varphi_{\mathscr{A}}$ s.t. $t \in \mathcal{L}(\mathscr{A}) \Leftrightarrow t \models \varphi_{\mathscr{A}}$
- For every φ exists $\mathscr{A}_\varphi$ s.t. $t \models \varphi \Leftrightarrow t \in \mathcal{L}(\mathscr{A}_\varphi)$

Consequence: Satisfiability of S2S reduces to PTA emptiness

(Parity) Games everywhere:

- Acceptance game $\mathcal{G}(\mathscr{A}, t)$ for complement closure of PTA
- Emptiness game $\mathcal{G}(\mathscr{A})$ for emptiness check of PTA

"The mother of all decidability results"

Exam

Organizational Matters

End-of-term exam

- When: February 13th, 2014, 09:30 - 11:30
- Where: HS 003, Building E1.3
- Mode: Open-book
- What to bring: Student ID
- Exam inspection: Feb. 14th, 2014, 15:00 - 16:00 (Room 328?)

Organizational Matters

End-of-term exam

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End-of-semester exam: March 20th, 2014 (more information after first exam)

Questions

Challenge us before we challenge you in the exam.

Questions

Challenge us before we challenge you in the exam.

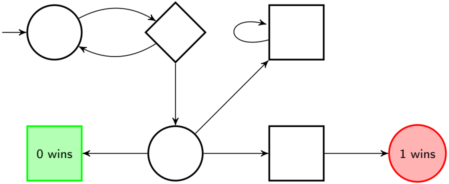
There will also be a tutorial where you can ask further questions!

- When: March 11th, 2014, 16:00 - 18:00
- Where: SR U.11, Building E2.5

Outlook

(Simple) Stochastic Games

- Enter a new player ($\Diamond$), it flips a coin to pick a successor.



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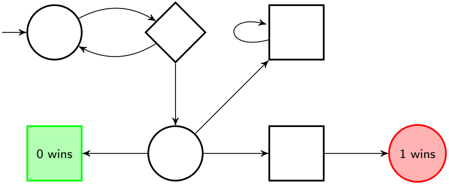

- No (sure) winning strategy...
- ...but one with probability 1.

(Simple) Stochastic Games

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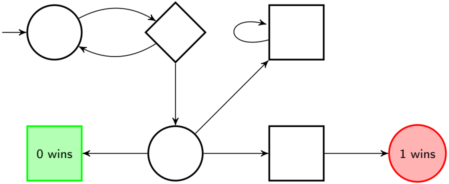


- No (sure) winning strategy...
- ...but one with probability 1.

More formally: Value of the game

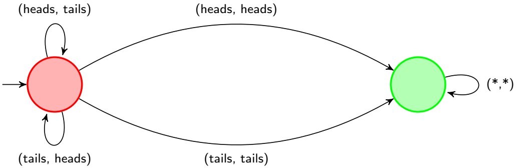
$$\max_{\sigma} \min_{\tau} p_{\sigma,\tau}$$

where $p_{\sigma,\tau}$ is the probability that Player 0 wins when using strategy σ and Player 1 uses strategy τ.

Concurrent Games

- Both players choose their moves simultaneously

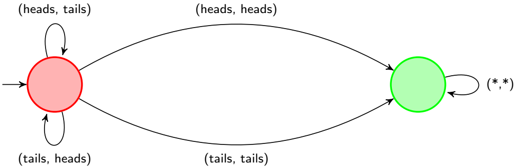
Matching pennies:



Concurrent Games

- Both players choose their moves simultaneously

Matching pennies: randomized strategy winning with probability 1.



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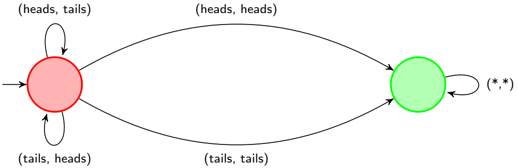
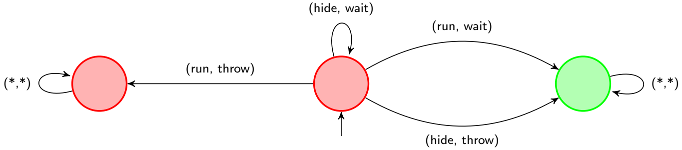


The "Snowball Game":



Concurrent Games

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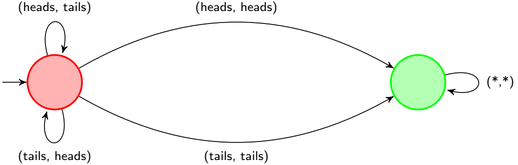
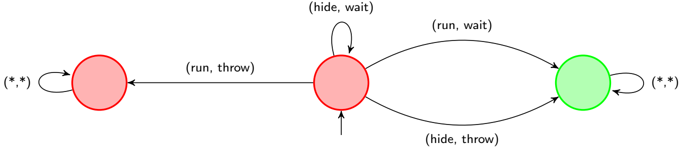
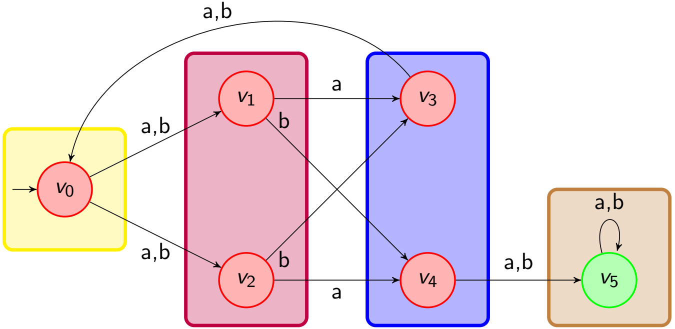


The "Snowball Game": for every ε, randomized strategy winning with probability $1 - \varepsilon$.



Games of Imperfect Information

- Players do not observe sequence of states, but sequence of non-unique observations (yellow, purple, blue, brown).
- Player 0 picks action (a,b), Player 1 resolves non-determinism.



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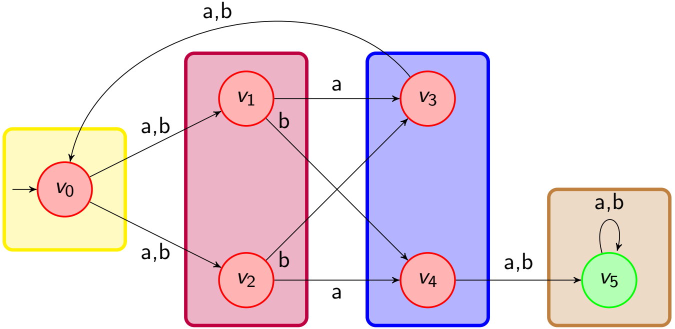
- Players do not observe sequence of states, but sequence of non-unique observations (yellow, purple, blue, brown).
- Player 0 picks action (a,b), Player 1 resolves non-determinism.



No winning strategy for Player 0: every fixed choice of actions to pick at ( )*( ) can be countered by going to v_1 or v_2.

Higher-order Pushdown Automata

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Example: on the blackboard

Theorem

Parity games on configuration graphs of higher-order pushdown automata can be solved algorithmically.

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$\mathrm{Sc}_{\{0\}}$								
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$\mathrm{Acc}_{\{0\}}$	$\emptyset$							
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Theorem

Player i has strategy to bound the opponent's scores by two when starting in $W_i(\mathcal{G})$.

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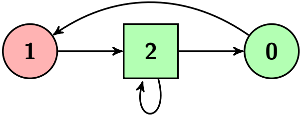
Corollary: Stopping play after first score reaches value three preserves winning regions (at most exponential play length)

Games with Costs

- Parity game: Player 0 wins from everywhere, but it takes arbitrarily long two "answer" 1 by 0.



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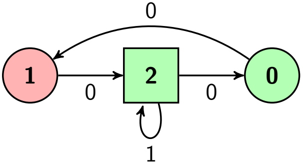

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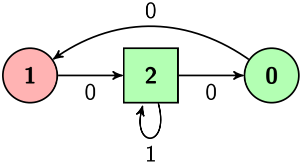

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Theorem

Parity games with costs are determined, Player 0 has positional winning strategies, and they can be solved in $\mathbf{NP} \cap \mathbf{co\text{-}NP}$.

Many other variants

- More winning conditions: various quantitative conditions (parity with costs, waiting times for RR games, and many more)

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And: any combination of extensions discussed above.

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- Even pushdown games can be played in finite time. What about higher-order pushdown games?

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- ...
- Your own idea?
- Generalized reachability games with sets of size two: **P**, **NP**, or **PSPACE**?
- Exact complexity of parity games.

Thank You

&

Good luck for the exam