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Infinite Games

Problem Set 1

Deadline: April 25, 2016

Please register by sending an email to `infinitegames16@react.uni-saarland.de`

Note: All definitions and notations that you need and should use can be found in the lecture notes available on our course web page.

Exercise 1.1 - ω-Regular Expressions (1 + 1 + 1 + 1 Points)

This exercise should help you to recapitulate some basics that we will use during the lecture. Therefore, have a look at the appendix of the lecture notes where you can find the corresponding notions and definitions.

Let $\Sigma = \{a, b, c\}$ be an alphabet. Define the following languages using ω-regular expressions.

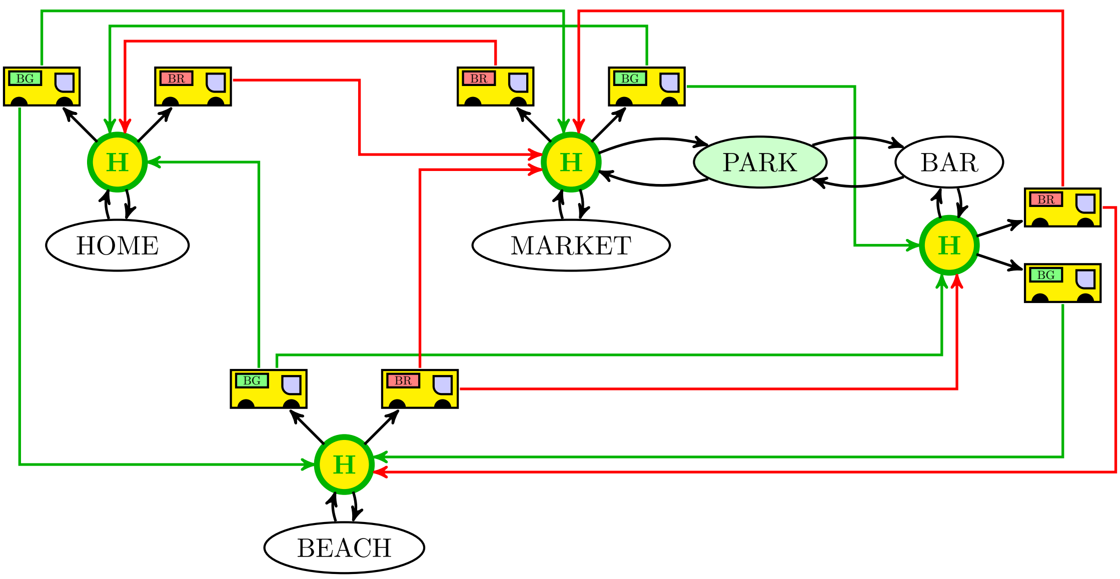
a) $L_1 = \{\, w \in \Sigma^\omega \mid$ there exist only finitely many c in $w\,\}$

b) $L_2 = \{\, w \in \Sigma^\omega \mid$ w contains only finitely many c that are directly followed by an $a\,\}$

c) $L_3 = \{\, w \in \Sigma^\omega \mid$ if there are finitely many ab in w, then there are infinitely many c in $w\,\}$

d) $L_4 = \{\, w \in \Sigma^\omega \mid$ every c in w that is eventually followed by an a is directly followed by an $a\,\}$

Exercise 1.2 - Andrew's Journey (1 + 1 + 2 + 2 Points)



Andrew wakes up at home and, seeing the beautiful weather outside, decides to spend his day at the beach. As he has a bus ticket for the whole day he decides to travel by bus and therefore has a look at the bus plan above. Unfortunately, he does not have a watch and the buses drive to different locations at different times. Consequently, he cannot exactly determine the destination a specific bus drives to. Further, it may be useful to know that Andrew is very forgetful, meaning he cannot remember the directions he already has taken[1].

a) Can you help Andrew by providing a strategy bringing him to the beach? Andrew may use the opportunity to take a shortcut by walking trough the park. Describe your solution formally and argue why it is correct.

[1] or even more precisely: he only understands positional strategies

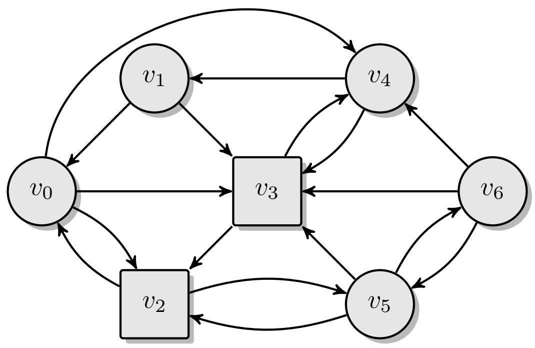
b) After spending a long and relaxing day at the beach, Andrew decides to meet some friends in the bar. How many possibilities (strategies) can he use to get there? How do you get to this answer?

c) Andrew wants to buy some presents for his friends at the market before he arrives at the bar from the beach. Is this possible such that his friends do not see him at the bar before? Argue formally.

d) Formalize the above problems as games. More precisely, give a formal representation the corresponding winning conditions. You can assume the arena to be given by $\mathcal{A}$. You also do not have to solve the games again.

(Hint: It may be useful to consider the bus plan as a directed graph with round and rectangular vertices.)

Exercise 1.3 - A Simple Game (2 + 1 + 1 Points)



Consider the game $\mathcal{G} = (\mathcal{A}, \mathrm{Win})$ with the arena $\mathcal{A} = (V, V_0, V_1, E)$ depicted above and the winning condition Win defined as $\mathrm{Win} = \{\, \rho \in \mathrm{Plays}(\mathcal{A}) \mid \mathrm{Occ}(\rho) = V \,\}$. A play is winning for Player 0 in this game iff all vertices are visited during the play.

a) Give at least one winning strategy from some vertex for each player. Argue why they are winning.

b) Determine the winning regions of the game. You do not have to give a justification.

c) Is the game positionally determined? Argue formally.

Exercise 1.4 - Uniform Strategies (2 Points)

Let $\mathcal{G}$ be a game. Prove or disprove: If Player i has a positional winning strategy from each vertex $v \in W_i(\mathcal{G})$, then Player i has a uniform positional winning strategy.