Dr. Martin Zimmermann
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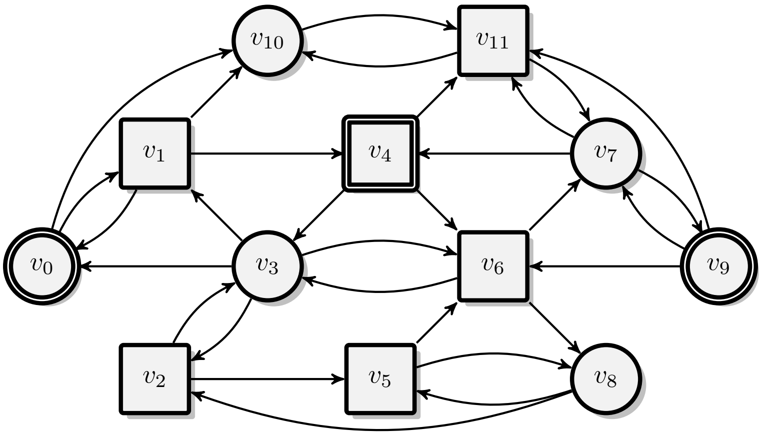
Infinite Games

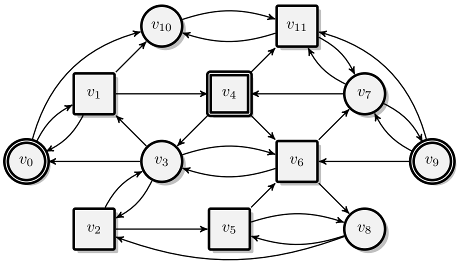
Deadline: May, 2nd 2016

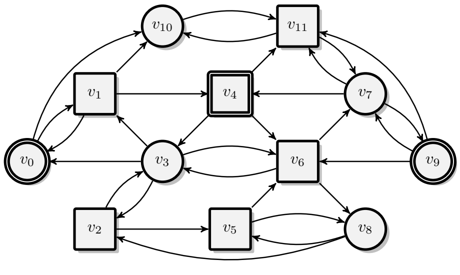
Exercise 2.1 - A Reachability Game (2 + 2 Points)

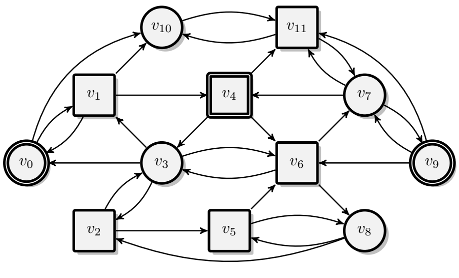


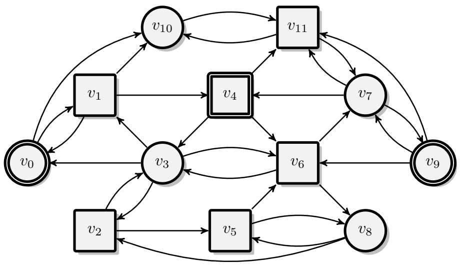
Consider the reachability game $\mathcal{G} = (\mathcal{A}, \textsc{reach}(R))$ depicted above.

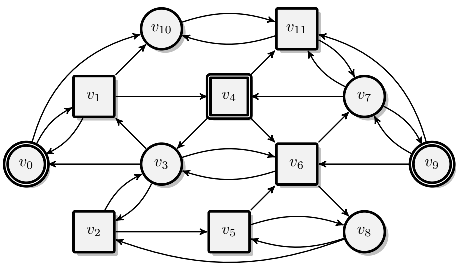
a) Determine the attractor sets $\text{Attr}_0^n(R)$ for all $n \in \mathbb{N}$. Mark the corresponding vertices in the copies below and indicate which $\text{Attr}_0^n(R)$ you have calculated.

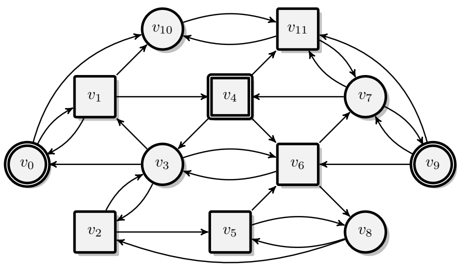












b) Give uniform positional winning strategies for both players resulting from the attractor construction.

Exercise 2.2 - Set Operations on Attractors (1 + 1 + 1 + 1 Points)

Let $\mathcal{A} = (V, V_0, V_1, E)$ be an arena, let F and F' be subsets of V, and let $i \in \{0, 1\}$. Prove or disprove the following statements:

a) $\mathrm{Attr}_i(F) \cup \mathrm{Attr}_i(F') = \mathrm{Attr}_i(F \cup F')$

b) $\mathrm{Attr}_i(F) \cap \mathrm{Attr}_i(F') = \mathrm{Attr}_i(F \cap F')$

c) $\mathrm{Attr}_i(F) \setminus \mathrm{Attr}_i(F') = \mathrm{Attr}_i(F \setminus F')$

d) $V \setminus \mathrm{Attr}_i(F) = \mathrm{Attr}_i(V \setminus F)$

Exercise 2.3 - Tournament, Round 0 (Individual Task) (1 + 1 + 1 Points)

At `http://react-teach.cs.uni-saarland.de` you will find the starting page of Automata Tutor. We will use this tool throughout the course to allow you to construct different kinds of games and solve each other's games.

a) Go to `http://react-teach.cs.uni-saarland.de` and register for an account. Use the email-address that you used to register for the lecture!

b) On the left-hand side you can find the menu item "Courses". Register for the course with the course ID **1INFINITE** and the password **JLTIPEA2**.

c) In the course overview for this course you will find a problem set containing a single task, which prompts you to construct a reachability game. Solve this task. Construct an arena with between nine and 13 states and solve the resulting game as demonstrated in the tutorial. The arena must be connected. Also, the winning region for either player should contain at least two states.

Participation in the tournament happens on an individual basis. Each student has to register for their own account and solve the tournament-tasks on their own. There may be no accounts for groups.

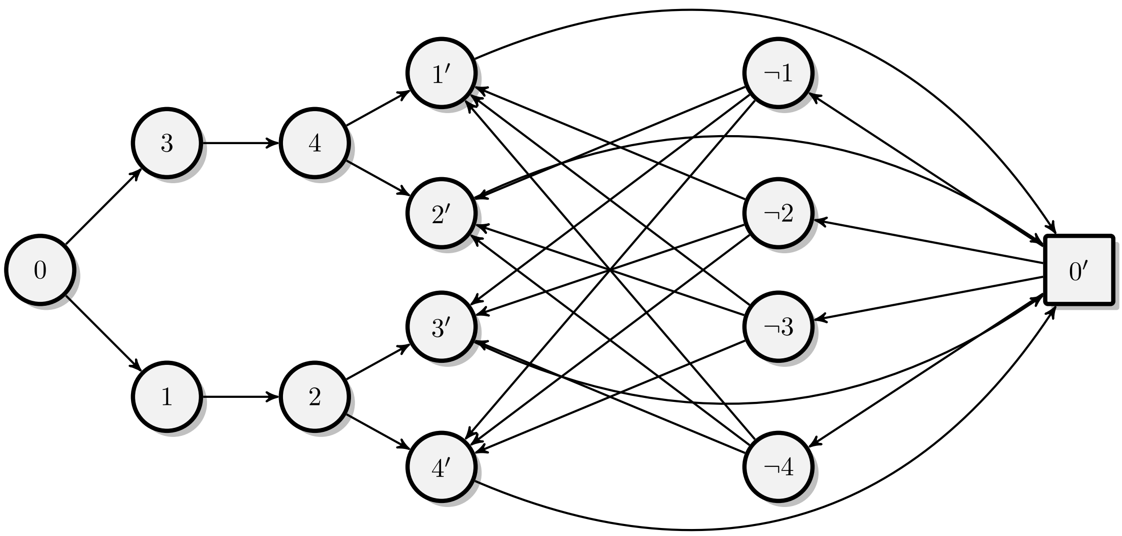
Note that the web application does not perform any checks on the arena that you hand in, so please make sure that it is a valid arena, i.e., all vertices have outgoing edges, and that the winning areas are correct.

Exercise 2.4 - Generalized Reachability (2 + 3 Points)

Let $\mathcal{A} = (V, V_0, V_1, E)$ be an arena. We define the generalized reachability condition $\textsc{genreach}(\mathcal{R})$ over a family of sets $\mathcal{R} \subseteq 2^V$ as follows:

$$\textsc{genreach}(\mathcal{R}) := \{\rho \in V^\omega \mid \forall R \in \mathcal{R}.\ R \cap \mathrm{Occ}(V) \neq \emptyset\}$$

Let $\mathcal{A}$ be defined as follows:



Consider the generalized reachability game $\mathcal{G} = (\mathcal{A}, \textsc{GenReach}(\mathcal{R}))$ with

$$\mathcal{R} := \{\{1, 1'\}, \{2, 2'\}, \{3, 3'\}, \{4, 4'\}\},$$

i.e., in order for a play to be winning for Player 0, it has to visit vertex j or vertex j' for each $j \in \{1, 2, 3, 4\}$.

a) Show that Player 1 has a winning strategy from vertex 0.

b) Show that Player 1 does not have a positional winning strategy from vertex 0.