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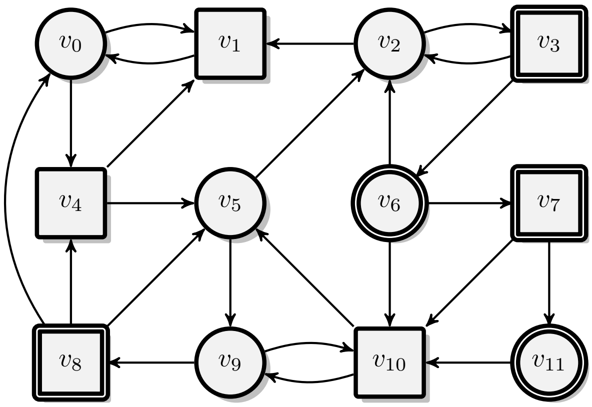
Infinite Games

Problem Set 3

Deadline: May 9th, 2016

Exercise 3.1 - A Büchi Game (4 Points)



Consider the Büchi game $\mathcal{G} = (\mathcal{A}, \textsc{Büchi}(F))$ depicted above. Compute the winning regions using the recurrence constructions and give corresponding uniform positional winning strategies for both players.

Exercise 3.2 - Duality (3 Points)

Recall that $\overline{\mathcal{A}}$ is the dual arena of $\mathcal{A}$. Two games $\mathcal{G} = (\mathcal{A}, \mathrm{Win})$ and $\mathcal{G}' = (\overline{\mathcal{A}}, \mathrm{Win}')$ are dual if Win' is the complement of Win (w.r.t. the set of vertices of $\mathcal{A}$ and $\overline{\mathcal{A}}$).

Show for $i \in \{0, 1\}$: every winning strategy σ for Player i from a vertex $v \in V$ in the game $\mathcal{G}$ is also a winning strategy for Player $1 - i$ from v in the game $\mathcal{G}'$.

Note: This is a generalization of Lemma 3.3 in the lecture.

Exercise 3.3 - Tournament, Round 1 (Individual Task) (2 Points)

At `http://react-teach.cs.uni-saarland.de` in the course "Infinite Games 16" you will find a new problem set that contains a single problem. This problem asks you to solve a reachability game constructed by another student. Solve that game, i.e., denote the winning regions for both players.

Exercise 3.4 - Traps (2 + 1 + 1 + 3 Points)

A winning condition $\mathrm{Win} \subseteq V^\omega$ is prefix-independent, if we have for every $\rho \in V^\omega$ and every $w \in V^*$:

$$\rho \in \mathrm{Win} \text{ if and only if } w\rho \in \mathrm{Win}.$$

A set $T \subseteq V$ of vertices is a trap for Player i if every successor of every vertex in $V_i \cap T$ is in T and at least one successor of every vertex in $V_{1-i} \cap T$ is in T.

a) Show: $V \setminus \mathrm{Attr}_i(R)$ is a trap for Player i for every set R

b) Prove or disprove: $\textsc{Reach}(R)$ is prefix-independent

c) Prove or disprove: $\textsc{Büchi}(F)$ is prefix-independent

d) Let $\mathcal{G} = (\mathcal{A}, \mathrm{Win})$. Show: if Win is prefix-independent, then $W_0(\mathcal{G})$ and $W_1(\mathcal{G})$ are traps for Player 1 and Player 0, respectively