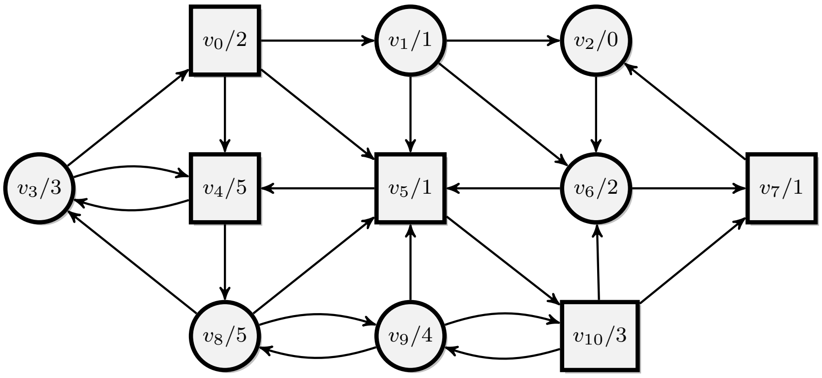
Dr. Martin Zimmermann
Alexander Weinert, M.Sc.

Infinite Games

Problem Set 4

Deadline: May 17th, 2016

Exercise 4.1 - A Parity Game (4 Points)



Consider the parity game $\mathcal{G} = (\mathcal{A}, \textsc{Parity}(\Omega))$ with arena $\mathcal{A}$ and coloring Ω depicted above. Compute the winning regions and uniform positional winning strategies for both players using the recursive construction underlying the proof of Theorem 3.5. You do not have to give intermediate steps of the attractor computations.

Exercise 4.2 - Tournament, Round 2 (2 Points)

At `http://react-teach.cs.uni-saarland.de` you will find a new task for you. Construct a parity game with anywhere between nine and 13 vertices and up to five different colors, where the winning region for both players each have at least two vertices. Also, mark the winning regions of both players.

Note that, similarly to the last round of this tournament, the games you constructed will be re-distributed among the other students for the next exercise sheet. In contrast to the last iteration, however, you will be awarded bonus points based on the difficulty of your game - the harder it is to solve, the more bonus points you will receive.

Exercise 4.3 - Variations of Parity Games (1 + 2 Points)

1. Prove Item 2 of Lemma 3.9 from the lecture notes.
2. Prove Item 3 of Lemma 3.9 from the lecture notes.

Exercise 4.4 - Weak-Parity Games (4 + 3 Points)

In a parity game, the goal for Player i is to ensure that the minimal color occuring infinitely often has parity i. By replacing "infinitely often" with "at least once" we obtain a definition for a weaker variant of parity games: the weak parity condition $\textsc{wParity}(\Omega)$ for a coloring $\Omega\colon V \to \mathbb{N}$ and an arena $\mathcal{A} = (V, V_0, V_1, E)$ is defined as

$$\textsc{wParity}(\Omega) := \{\rho \in \mathrm{Plays}(\mathcal{A}) \mid \min \mathrm{Occ}(\Omega(\rho_0)\Omega(\rho_1)\Omega(\rho_2)\cdots) \text{ is even}\}.$$

We call a game $\mathcal{G} = (\mathcal{A}, \textsc{wParity}(\Omega))$ a *weak parity game* with coloring Ω.

a) Give a polynomial-time algorithm that computes the winning regions and uniform positional winning strategies for both players in a weak parity game $\mathcal{G}$.

b) Consider the game $\mathcal{G} = (\mathcal{A}, \textsc{wParity}(\Omega))$ where $\mathcal{A}$ and Ω are defined as in Exercise 4.1. Determine the winning regions and uniform positional winning strategies of both players using your algorithm given in Part a).