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Problem Set 5

Infinite Games

Deadline: May 23rd, 2016

Exercise 5.1 - Tournament, Round 3 (2 + 2 + 2 Points + 3 Bonus Points)

After logging onto `http://react-teach.cs.uni-saarland.de` you will find a new problem set that contains three problems, each of which asks you to solve a parity game constructed by some other student. You will receive two points for each parity game that you solve correctly.

Moreover, each time the parity game constructed by you for the last exercise is not solved correctly by another student, you will receive an additional bonus point.

Exercise 5.2 - Single-Player Games (2 + 3 Points)

A single-player game for Player i is a game played in an arena where every vertex of Player $1 - i$ has exactly one successor.

a) Let σ be a positional strategy for Player i in a game $(\mathcal{A}, \mathrm{Win})$ and recall the definition of the arena $\mathcal{A}_\sigma$ from the lecture notes.

   Show that σ is a winning strategy for Player i from a vertex v in $\mathcal{G}$ if, and only if, v is in the winning region of Player i in the single-player game $(\mathcal{A}_\sigma, \mathrm{Win})$.

b) Show that single-player parity games can be solved in polynomial time.

Exercise 5.3 - Finitary Parity Games (1 + 2 + 2 Points)

In this exercise, we consider a variant of parity games, so-called finitary parity games, in which visits to vertices of odd and even colors are interpreted as requests and responses, respectively. As is the case with traditional parity games, after some finite prefix, each request has to be answered by a subsequent lower-priority response. In contrast to traditional parity games, however, Player 1 must not be able to delay these visits arbitrarily.

a) Read the paragraphs entitled *game graphs*, *plays*, and *strategies* in Section 2.1, the paragraphs entitled *objectives* and *winning* in Section 2.2, as well as Section 3.1 of [1].[1] Draw the game shown in Figure 1 of that work using the notation from the lecture. In particular, note that [1] uses Players 1 and 2 and denotes their vertices as diamonds and rectangles. There is a unique correct way of mapping this notation to the model used in the lecture.

b) Show formally that Player 1 (as defined in the lecture, i.e., the adversarial player) does not have a positional winning strategy from s_0 in $\mathcal{G}$, where $\mathcal{G}$ is the finitary parity game shown in Figure 1 of [1].

c) At `http://react-teach.cs.uni-saarland.de` you will find a new problem which asks you to construct a finitary parity game and specify the winning regions. Solve this problem using between six and nine vertices and at most four colors. Again, your finitary parity game will be given to other students to be solved next week. You will receive bonus points the harder your problem is for other students to solve. Moreover, you will only receive bonus points if you were able to solve your own game correctly.

References

[1] K. Chatterjee, T. Henzinger, F. Horn, *Finitary Winning in ω-Regular Games.* ACM Transactions on Computational Logic 11(1), pp. 1-27 (2009).

[1]The work [1] is available from the UdS-network via ACM. If you are unable to obtain a copy, please send a mail to `infinitegames16@react.uni-saarland.de`.