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Infinite Games

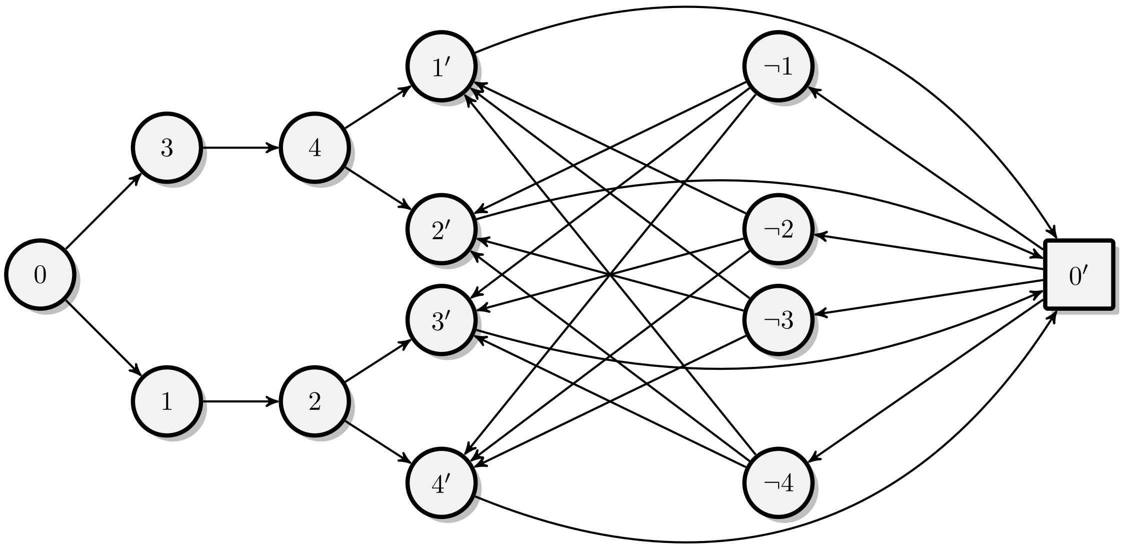
Deadline: June, 6th 2016

Exercise 7.1 - Generalized Reachability Revisited (3 Points)

Recall that, for some arena $\mathcal{A} = (V, V_0, V_1, E)$ and a family of sets $\mathcal{R} \subseteq 2^V$ we defined the generalized reachability condition $\textsc{GenReach}(\mathcal{R})$ as follows:

$$\textsc{GenReach}(\mathcal{R}) := \{\rho \in V^\omega \mid \forall R \in \mathcal{R}.\ R \cap \mathrm{Occ}(V) \neq \emptyset\}$$

Moreover, we considered the generalized reachability game in the following arena $\mathcal{A}$:



We define $\mathcal{R} := \{\{1, 1'\}, \{2, 2'\}, \{3, 3'\}, \{4, 4'\}\}$ and $\mathcal{G} = (\mathcal{A}, \textsc{GenReach}(\mathcal{R}))$.

Give a formal definition of a finite-state winning strategy of size at most 5 for Player 1 in $\mathcal{G}$ from vertex 0. Use the graphical automaton-notation from the lecture notes for this.

Exercise 7.2 - Reductions (3 Points)

Show that generalized reachability games (see Exercise 7.1) are reducible to reachability games.

Exercise 7.3 - Request-Response Games (4 + 4 Points)

Let $\mathcal{A} = (V, V_0, V_1, E)$ be an arena. Given a finite family $(Q_j, P_j)_{j=1,\dots,k}$ of subsets $Q_j, P_j \subseteq V$, we define the request-response condition by

$$\textsc{ReqRes}((Q_j, P_j)_{j=1,\dots,k}) = \{\rho \in \mathrm{Plays}(\mathcal{A}) \mid \text{for all } j = 1, \dots, k \text{ and all } n \in \mathbb{N}\text{: } \rho_n \in Q_j \text{ implies } \rho_{n'} \in P_j \text{ for some } n' \geq n\}.$$

Intuitively, a visit to Q_j is a request that has to be answered by a later response, i.e., a visit to P_j. Note that the condition demands that *every* request is answered, not only those after some finite prefix.

A game $\mathcal{G} = (\mathcal{A}, \textsc{ReqRes}((Q_j, P_j)_{j=1,\dots,k}))$ is a request-response game.

1. Show that request-response games are reducible to Büchi games.

2. Show that Player 0 needs exponential memory to win request-response games. To this end, construct a family $\mathcal{G}_n$ of request-response games of polynomial size in n, each with a designated vertex v, such that Player 0 wins $\mathcal{G}_n$ from v, but only with finite-state strategies of size 2^n. Here, the size of a request-response game is measured in the number of vertices of the arena *and* in the number of request-response pairs (Q_j, P_j).

   Hint: You may argue along the lines of the proof of Theorem 4.1 from the lecture notes

Exercise 7.4 - Uniform Finite State Strategies (2 Points)

Prove or disprove: If Player i has a finite-state winning strategy from each vertex $v \in W_i(\mathcal{G})$, in an arbitrary game $\mathcal{G}$, then Player i has a uniform finite-state winning strategy for $\mathcal{G}$.