

Infinite Games

Deadline: July, 4th 2016

Exercise 11.1 - S2S

(1 + 3 + 2 Points)

Give S2S formulas defining the following tree languages over the alphabet $\Sigma = \{a, b, c\}$:

- The language of trees containing an a -labeled vertex whose right sub-tree contains a b -labeled vertex whose left sub-tree contains an a -labeled vertex.
- The language of trees t satisfying $t|_{1^\omega} \in (aa)^*b^\omega$.
- The language of trees containing at least one a -labeled vertex and at most one b -labeled vertex.

Exercise 11.2 - Subset and Connectedness

(1 + 2 Points)

Show that the subset-relation and “being connected” is expressible in S2S.

- Give a formula $\varphi_{\subseteq}(X, Y)$ with two free second-order variables X and Y and no free first-order variable such that $t, \mu \models \varphi_{\subseteq}$ if and only if $\mu(X) \subseteq \mu(Y)$.
- Give a formula $\varphi_c(X)$ with one free second-order variable X and no free first-order variable such that $t, \mu \models \varphi_c$ if and only if $\mu(X)$ is connected, i.e., if w and w' are in $\mu(X)$ and w' is a descendant of w , then all vertices on the path between w and w' are in $\mu(X)$, as well.

Exercise 11.3 - Syntactic Sugar

(1 + 2 Points)

Show that ε and \preceq are syntactic sugar.

- Give a formula $\varphi_\varepsilon(x)$ not containing ε with one free first-order variable x and no free second-order variable such that $t, \mu \models \varphi_\varepsilon$ if and only if $\mu(x) = \varepsilon$.
- Give a formula $\varphi_{\preceq}(x, y)$ not containing \preceq with two free first-order variables x and y and no free second-order variable such that $t, \mu \models \varphi_{\preceq}$ if and only if $\mu(x)$ is a prefix of $\mu(y)$.

Exercise 11.4 - Tournament, Round 5

(4 Points + 2 Bonus Points)

After logging onto <http://react-teach.cs.uni-saarland.de> you will find a new problem set that contains two problems, each of which asks you to solve an energy parity game constructed by some other student. You will receive two points for each energy parity game that you solve correctly.

Moreover, each time the energy parity game constructed by you for the second-to-last last exercise is not solved correctly by another student, you will receive an additional bonus point.