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Problem Set 12

Infinite Games

Deadline: 11th July, 2016

Exercise 12.1 - Parity Tree Automata (1 + 1 + 2 Points)

Give parity tree automata defining the following tree languages over the alphabet $\Sigma = \{a, b, c\}$:

a) The language of trees containing an a-labeled vertex whose right sub-tree contains a b-labeled vertex whose left sub-tree contains an a-labeled vertex.

b) The language of trees t satisfying $t_{|1^\omega} \in (aa)^* b^\omega$.

c) The language of trees containing at least one a-labeled vertex and at most one b-labeled vertex.

You do not need to show the correctness of your automata.

Exercise 12.2 - Closure of Parity Tree Automata (2 + 2 Points)

Show that languages recognized by parity tree automata are closed under union and projection.

a) Given two parity tree automata $\mathscr{A}_1$ and $\mathscr{A}_2$ over the same alphabet construct a parity tree automaton $\mathscr{A}$ such that $\mathcal{L}(\mathscr{A}) = \mathcal{L}(\mathscr{A}_1) \cup \mathcal{L}(\mathscr{A}_2)$. Show formally that your automaton recognizes $\mathcal{L}(\mathscr{A}_1) \cup \mathcal{L}(\mathscr{A}_2)$.

b) Given a tree $t \colon \mathbb{B}^* \to \Sigma \times \Gamma$ we define its projection $p_\Sigma(t) \colon \mathbb{B}^* \to \Sigma$ to its first component by $p_\Sigma(t)(w) = a$ for every $w \in \mathbb{B}^*$ with $t(w) = (a, b)$.

Given a parity tree automaton $\mathscr{A}_e$ over the alphabet $\Sigma \times \Gamma$ construct a parity tree automaton $\mathscr{A}$ such that $\mathcal{L}(\mathscr{A}) = \{p_\Sigma(t) \mid t \in \mathcal{L}(\mathscr{A}_e)\}$. Show formally that your automaton recognizes this language.

Exercise 12.3 - Finitely Many a (2 + 2 Points)

Let L be the language of trees over $\Sigma = \{a, b\}$ containing only finitely many a-labeled vertices.

a) Give an S2S formula defining L.

b) Give a parity tree automaton recognizing L.

In both cases, explain your construction and argue informally that your solution is correct.

Exercise 12.4 - Acceptance Game (4 Points)

Complete the proof of Lemma 5.1 in the lecture notes. To this end, show that if $t \in \mathcal{L}(\mathscr{A})$, then $(\varepsilon, q_I) \in W_0(\mathcal{G}(\mathscr{A}, t))$.