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Final Problem Set

Infinite Games

Deadline: July 18th, 2016

Exercise 13.1 - Emptiness Game (4 Points)

Complete the proof of Theorem 5.2 in the lecture notes. To this end, show that if $q_I \in W_0(\mathcal{G}(\mathscr{A}))$, then $\mathcal{L}(\mathscr{A}) \neq \emptyset$.

Exercise 13.2 - Emptiness Game, Example (2 + 1 + 1 Points)

Consider the parity tree automaton $\mathscr{A} = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, q_0, \Delta, \Omega)$ where Δ and Ω are defined by $\Omega(q_3) = 1$, $\Omega(q_0) = \Omega(q_4) = 2$, $\Omega(q_1) = \Omega(q_2) = 3$, and

$$\Delta = \left\{ \begin{array}{c} q_0, a \\ \diagup \quad \diagdown \\ q_0 \qquad q_1 \end{array}, \begin{array}{c} q_0, b \\ \diagup \quad \diagdown \\ q_0 \qquad q_3 \end{array}, \begin{array}{c} q_1, a \\ \diagup \quad \diagdown \\ q_3 \qquad q_0 \end{array}, \begin{array}{c} q_1, b \\ \diagup \quad \diagdown \\ q_0 \qquad q_2 \end{array}, \begin{array}{c} q_2, a \\ \diagup \quad \diagdown \\ q_0 \qquad q_3 \end{array}, \begin{array}{c} q_2, b \\ \diagup \quad \diagdown \\ q_0 \qquad q_1 \end{array}, \begin{array}{c} q_2, b \\ \diagup \quad \diagdown \\ q_0 \qquad q_4 \end{array}, \begin{array}{c} q_3, a \\ \diagup \quad \diagdown \\ q_0 \qquad q_3 \end{array}, \begin{array}{c} q_3, b \\ \diagup \quad \diagdown \\ q_0 \qquad q_3 \end{array}, \begin{array}{c} q_4, a \\ \diagup \quad \diagdown \\ q_0 \qquad q_1 \end{array}, \begin{array}{c} q_4, b \\ \diagup \quad \diagdown \\ q_3 \qquad q_0 \end{array} \right\}.$$

a) Construct the emptiness game $\mathcal{G}(\mathscr{A})$ and determine the winner from (ε, q_0).

b) Give a tree $t \in \mathcal{L}(\mathscr{A})$ as function $t\colon \mathbb{B}^* \to \{a, b\}$.

c) Give a precise description of $\mathcal{L}(\mathscr{A})$ using natural language.

Exercise 13.3 - Regular Trees (2 + 2 Points)

A tree $t\colon \mathbb{B}^* \to \Sigma$ is regular if it has finitely many different sub-trees, i.e., if the set $\{t_w \mid w \in \mathbb{B}^*\}$ is finite.

Let $\mathscr{W} = (Q, \mathbb{B}, q_I, \delta, \lambda)$ be a deterministic finite word automaton (DFA) where we have replaced the set of accepting states by a labeling $\lambda\colon Q \to \Sigma$. We denote the unique state in which the run of $\mathscr{W}$ on $w \in \mathbb{B}^*$ is ending by $\delta^*(w)$. The automaton generates the tree $t_{\mathscr{W}} : \mathbb{B}^* \to \Sigma$ defined by $t_{\mathscr{W}}(w) = \lambda(\delta^*(w))$ for all $w \in \mathbb{B}^*$.

a) Show that a tree is regular if and only if it is generated by some DFA.

b) Show that every non-empty tree language recognized by a parity tree automaton contains a regular tree.

Exercise 13.4 - Deterministic Parity Tree Automata (2 + 2 Points)

A parity tree automaton $(Q, \Sigma, q_I, \Delta, \Omega)$ is deterministic, if for every state q and every letter a there is exactly one pair (q_0, q_1) of states such that $(q, a, q_0, a_1) \in \Delta$. Thus, a deterministic automaton has a unique run on every tree.

Let L be the language of trees over that alphabet $\{a, b\}$ that contain at least one a.

1. Show that there is a parity tree automaton that recognizes L.

2. Show that there is no deterministic parity tree automaton that recognizes L.