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Summer term 2015
Problem Set 1
Due: Tuesday, 28th April 2015

Recursion Theory

Please indicate your **name** and your **matriculation number** on your solution and hand it in before the tutorial. We encourage you to collaborate in **groups** of up to **two** students. Only one submission per group is necessary.

Problem 1: Enumerability vs. Computability — 2 Points

Show: If $A \subseteq \mathbb{N}$ is enumerable, then there exists a computable function $f\colon \mathbb{N} \to \mathbb{N}$ with $\mathrm{dom}(f) = A$.

A proof sketch using a flow chart as in the lecture is sufficient, if you argue that it is correct.

Problem 2: Monotone Enumerability — 4 Points

An algorithm $\mathcal{A}$ enumerates a set $A \subseteq \mathbb{N}$ monotonically iff $\mathcal{A}$ started (without input) returns one by one (**in the natural order on** $\mathbb{N}$) exactly the elements of $\mathcal{A}$.

Prove: A is monotonically enumerable $\Leftrightarrow$ A is decidable.

A proof sketch using a flow chart as in the lecture is sufficient, if you argue that it is correct.

Problem 3: URMs — 2 + 2 + 2 Points

Give URMs that compute the following functions:

a) $\mathrm{sgn}\colon \mathbb{N} \to \mathbb{N}$ defined by $\mathrm{sgn}(x) = \begin{cases} 0 & \text{if } x = 0, \\ 1 & \text{if } x > 0. \end{cases}$

b) $\mathrm{grtr}\colon \mathbb{N}^2 \to \mathbb{N}$ defined by $\mathrm{grtr}(x, y) = \begin{cases} 0 & \text{if } x \leq y, \\ 1 & \text{if } x > y. \end{cases}$

c) $\mathrm{div3}\colon \mathbb{N} \to \mathbb{N}$ defined by $\mathrm{div3}(x) = \begin{cases} \frac{x}{3} & \text{if } x \text{ is a multiple of 3}, \\ \bot & \text{otherwise.} \end{cases}$

Here, $\bot$ denotes that the function is undefined.