Dr. Martin Zimmermann
Leander Tentrup, M.Sc.

Summer term 2015
Problem Set 5
Due: Tuesday, June 2nd, 2015

Recursion Theory

Problem 1: Ackermann Function — 2 + 6 Points

Recall the definition of the Ackermann function A:

$$A(0, y) = y + 1$$
$$A(x + 1, 0) = A(x, 1)$$
$$A(x + 1, y + 1) = A(x, A(x + 1, y))$$

For $n \in \mathbb{N}$ we define $f_n\colon \mathbb{N} \to \mathbb{N}$ by $f_n(y) = A(n, y)$.

1. Show that f_n is primitive recursive for every n.
2. Show that A is μ-recursive.

Hint: For 2.), retrace the evaluation of $A(2, 1)$.

Problem 2: Function Iteration — 2 Points

Given a function $f\colon \mathbb{N} \to \mathbb{N}$ we define the i-th iterate $f^i\colon \mathbb{N} \to \mathbb{N}$ of f inductively via $f^0(x) = x$ and $f^{i+1}(x) = f(f^i(x))$.

Prove: if f is primitive recursive, then so is $g\colon \mathbb{N}^2 \to \mathbb{N}$ defined by $g(x, i) = f^i(x)$.

Problem 3: Tongue-in-Cheek — 2 Points

Some natural numbers are "interesting", e.g.,

- 0 is interesting, because it is the additive identity,
- 1 is interesting, because it is the multiplicative identity,
- 2 is interesting, because it is the only even prime number, etc.

Show that every natural number is interesting.