

Recursion Theory

Problem 1: Equivalence

9 Points

Show that every μ -recursive function is URM-computable.

Hint: find the right level of abstraction to give a concise and complete proof that is worth nine points, but not more.

Problem 2: Domains

1.5 + 1.5 Points

Let $f, g: \mathbb{N} \rightarrow \mathbb{N}$ be μ -recursive functions. Let $h_1, h_2: \mathbb{N} \rightarrow \mathbb{N}$ be defined via

$$h_1(x) = \begin{cases} 1 & \text{if } x \in \text{dom}(f) \text{ or } x \in \text{dom}(g), \\ \perp & \text{otherwise,} \end{cases}$$

and

$$h_2(x) = \begin{cases} 1 & \text{if } x \in \text{dom}(f) \text{ and } x \in \text{dom}(g), \\ \perp & \text{otherwise.} \end{cases}$$

Show that h_1 and h_2 are μ -recursive.