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Summer term 2015
Problem Set 8
Due: Tuesday, June 23^{th}, 2015

Recursion Theory

Problem 1: Back to the Basics — 2 + 2 Points

1. Let $A \subseteq \mathbb{N}$ and $\overline{A} = \mathbb{N} \setminus A$ be μ-recursively enumerable. Show that A is μ-recursive.
2. Let $A \subseteq \mathbb{N}$ be μ-recursive. Show that A and $\overline{A}$ are μ-recursively enumerable.

Problem 2: K and Tot — 2 + 1 + 1 Points

Let $K = \{x \mid \varphi_x(x) \downarrow\}$, let $\overline{K} = \mathbb{N} \setminus K$, and let $\text{Tot} = \{x \mid \varphi_x \text{ is total}\}$. In the lecture, we proved $K \leq_m \text{Tot}$.

1. Show that $\overline{K} \leq_m \text{Tot}$ holds as well.
2. Show that neither $\text{Tot} \leq_m K$ nor $\text{Tot} \leq_m \overline{K}$ hold.
3. Show that neither Tot or $\overline{\text{Tot}}$ are enumerable.

Problem 3: Enumerable vs. Recursive — 4 Points

Show that every infinite μ-recursively enumerable set contains an infinite μ-recursive set.