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Summer term 2015
Problem Set 9
Due: Tuesday, June 30th, 2015

Recursion Theory

Problem 1: Sum of Sets — 2 + 2 Points

The (recursion-theoretic) sum of two sets $A, B \subseteq \mathbb{N}$ is defined as

$$A \oplus B = \{2x \mid x \in A\} \cup \{2x + 1 \mid x \in B\}.$$

Show that $A \oplus B$ is the least upper bound of A and B with respect to $\leq_m$, i.e., show

1. $A \leq_m A \oplus B$ and $B \leq_m A \oplus B$, and
2. if $A \leq_m C$ and $B \leq_m C$, then also $A \oplus B \leq_m C$.

Problem 2: Reductions — 1 + 2 + 2 + 2 Points

Let $\mathcal{P}$ be the set of prime numbers, let $E = \{e \mid \mathrm{dom}(\varphi_e) = \emptyset\}$, $\mathrm{Tot} = \{e \mid \mathrm{dom}(\varphi_e) = \mathbb{N}\}$, and $\mathrm{Inf} = \{e \mid \mathrm{dom}(\varphi_e) \text{ infinite}\}$.

Show $\mathcal{P} \leq_m E \leq_m \mathrm{Tot} \equiv_m \mathrm{Inf}$.

Problem 3: Tongue-in-Cheek — 1 Point

Some natural numbers can be defined by sentences (in English) with at most twenty-five words, e.g.,

- 0: the number of wings the majority of people has.
- 1: the number of noses the majority of people has.
- 2: the number of ears the majority of people has.

Let n_0 be the smallest natural number that cannot uniquely be defined by sentences (in English) with at most twenty-five words.

Is n_0 well-defined?