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Summer term 2015
Problem Set 11
Due: Tuesday, July 14th, 2015

Recursion Theory

Problem 1: Creative and Productive Sets — 4 Points

a) Show that $A = \{e \in \mathbb{N} \mid \varphi_e \text{ is not surjective}\}$ is productive.

b) Show that $B = \{e \in \mathbb{N} \mid \text{dom}(\varphi_e) \neq \emptyset\}$ is creative.

Remark: Since $\overline{A}$ is not enumerable, a) shows that there are productive sets whose complement is not creative.

Problem 2: Simple Sets — 4 Points

Show that there are two simple sets A and B such that $A \cup B = \mathbb{N}$.

Hint: Recall the construction of S in the lecture and spread it out even more, then distribute $\mathbb{N}$ between two copies of this set to construct A and B.

Problem 3: Closure Properties — 4 Points

Let $A \subseteq \mathbb{N}$ be productive and let $B \subseteq \mathbb{N}$ be simple.

Show that $A \cap B$ is productive.