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Summer term 2015
Problem Set 12
Due: Tuesday, July 21^{st}, 2015

Recursion Theory

Problem 1: tt Degrees — 4 Points

Show that every tt degree is countably infinite.

Problem 2: tt Reduction — 4 Points

Show that

$$\{e \mid \mathrm{dom}(\varphi_e) \text{ is infinite}\} \leq_{\mathrm{tt}} \{e \mid \mathrm{dom}(\varphi_e) \text{ is recursive}\}.$$

Problem 3: Turing Cones — 4 Points

Let $B \subseteq \mathbb{N}$. The Turing cone of B is the set $\mathcal{C}_B = \{A \mid A \leq_T B\} \subseteq 2^{\mathbb{N}}$.

Show that $\mathcal{C}_B$ is a boolean algebra. To this end, do the following:

a) Show that $\mathcal{C}_B$ is closed under union, intersection, and complement.

b) Show that there are elements $Z, O \in \mathcal{C}_B$ such that for every $A \in \mathcal{C}_B$
  - $A \cup Z = A$ and $A \cap O = A$, and
  - $A \cup \overline{A} = O$ and $A \cap \overline{A} = Z$.

Problem 4: Extra Credit — 2 Points

Close the gap in the construction of the simple set S presented in last week's lecture.

To this end, show: for every partial recursive function $f\colon \mathbb{N} \to \mathbb{N}$ there is a total recursive function $g\colon \mathbb{N} \to \mathbb{N}$ such that $f(\mathbb{N}) = g(\mathbb{N})$.

Remark: The points of this exercise do not count towards the total points achievable. §