Prof. Bernd Finkbeiner, Ph.D.
Peter Faymonville, M.Sc.
Michael Gerke, B.Sc.

Winter term 2011/2012
Problem Set 6

Verification

Please write the names of all group members on the solutions you hand in.

Problem 1: CTL warm-up

Express the following properties as CTL formulas over $AP = \{a, b, c\}$ and provide a justification. For more complicated formulas, also comment on their subformulas!

1. There exists a path on which the following holds for every state s: there exists a path which starts in s, and on which eventually a holds and in the next state, $\neg a$ holds.
2. There exists a reachable state s for which the following holds: a is true and on all paths starting from s, c holds as long as b does not hold.
3. On every path the following holds for every state: a is valid if and only if b is valid and in the previous state, c is valid.

Problem 2: CTL Semantics

Prove or disprove the following implications:

1. Let $\Phi_1 = \mathsf{AF}\, a \vee \mathsf{AF}\, b$ and $\Phi_2 = \mathsf{AF}\,(a \vee b)$.
   Prove or disprove $\Phi_1 \rightarrow \Phi_2$ and $\Phi_2 \rightarrow \Phi_1$.
2. Now consider $\Psi_1 = \mathsf{E}\,(a\, \mathsf{U}\, \mathsf{E}\,(b\, \mathsf{U}\, c))$ and $\Psi_2 = \mathsf{E}\,(\mathsf{E}\,(a\, \mathsf{U}\, b)\, \mathsf{U}\, c)$.
   Again, prove or disprove $\Psi_1 \rightarrow \Psi_2$ and $\Psi_2 \rightarrow \Psi_1$.

Problem 3: LTL vs. CTL

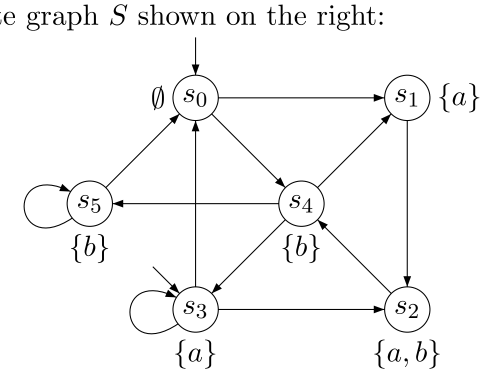
Prove that there does not exist an equivalent LTL-formula for the CTL-formula

$$\Phi = \mathsf{AF}\,(a \wedge \mathsf{EX}\, a).$$

Problem 4: CTL Model Checking

Consider the following CTL formulas and the state graph S shown on the right:

$$\Phi_1 = \mathsf{EG}\,\mathsf{AF}\,\neg b$$
$$\Phi_2 = \mathsf{E}\,(\mathsf{EX}\, a\, \mathsf{U}\, \neg a)$$
$$\Phi_3 = \mathsf{AX}\,(\mathsf{EG}\,\neg a \;\vee\; \mathsf{EG}\, b)$$

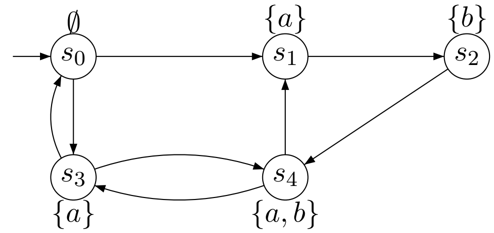


Give the satisfaction sets $Sat(\Phi_i)$ and decide whether $S \models \Phi_i$ holds ($1 \leq i \leq 3$).

Problem 5: CTL Model Checking with Fairness

Consider the CTL-formula $\Phi = \mathsf{AG}\big(a \to \mathsf{AF}\,(b \wedge \neg a)\big)$ together with the following CTL fairness assumption

$$fair \quad = \quad \Box\Diamond\,\mathsf{AX}\,(a \wedge \neg b) \to \Box\Diamond\,\mathsf{AX}\,(b \wedge \neg a)$$
$$\wedge \Diamond\Box\,\mathsf{EF}\,b \to \Box\Diamond\,b.$$



For the state graph $\mathcal{S}$ on the right, prove that $\mathcal{S} \models_{fair} \Phi$.