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Problem Set 12

# Verification

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Please write the names of all group members on the solutions you hand in.

## Problem 1: Decision Procedures

(1) Apply the decision procedure for $T_{\mathsf{E}}$ to the following $\Sigma_{\mathsf{E}}$ formulae:

(a) $f(f(f(a))) = f(f(a)) \wedge f(f(f(f(a)))) = a \wedge f(a) \neq a$

(b) $f(g(x)) = g(f(x)) \wedge f(g(f(y))) = x \wedge f(y) = x \wedge g(f(x)) \neq x$

(2) Apply the DAG-based decision procedure for $T_{\mathsf{E}}$ to the $\Sigma_{\mathsf{E}}$-formulae (a) and (b).

(3) Apply the decision procedure for $T_{\mathsf{cons}}$ to the following $T_{\mathsf{cons}}$-formulae:

(c) $\mathsf{car}(x) = y \wedge \mathsf{cdr}(x) = z \wedge x \neq \mathsf{cons}(y, z)$

(d) $\neg\mathsf{atom}(x) \wedge \mathsf{car}(x) = y \wedge \mathsf{cdr}(x) = z \wedge x \neq \mathsf{cons}(y, z)$

## Problem 2: Theories / Nelson-Oppen

For each of the following formulae, identify the combination of theories in which it lies. To avoid ambiguity, prefer $T_{\mathbb{Z}}$ to $T_{\mathbb{Q}}$. Then apply the Nelson-Oppen method using the appropriate decision procedures. Use either the nondeterministic or the deterministic version.
*Hint: For theory $T_{\mathsf{A}}$, see chapter 9.5 for details.*

(a) $1 \leq x \wedge x \leq 2 \wedge \mathsf{cons}(1, y) \neq \mathsf{cons}(x, y) \wedge \mathsf{cons}(2, y) \neq \mathsf{cons}(x, y)$

(b) $a[i] \geq 1 \wedge a[i] + x \leq 2 \wedge x > 0 \wedge x = i \wedge a\langle x \triangleleft 2\rangle[i] \neq 1$