Verification

Lecture 8

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Plan for today

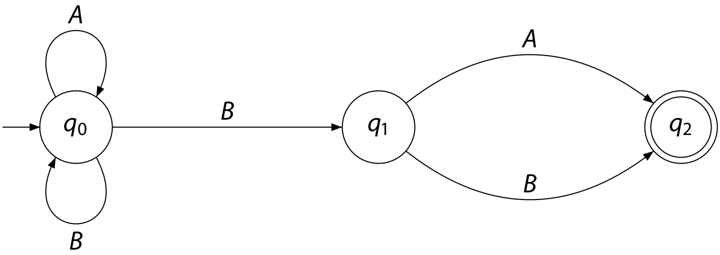
- Regular properties
  - Finite automata
  - Checking regular safety properties
  - Büchi automata

Regular properties

Review: Finite automata

A nondeterministic finite automaton (NFA) $\mathcal{A}$ is a tuple $(Q, \Sigma, \delta, Q_0, F)$ where:

- Q is a finite set of states
- Σ is an alphabet
- $\delta : Q \times \Sigma \to 2^Q$ is a transition function
- $Q_0 \subseteq Q$ a set of initial states
- $F \subseteq Q$ is a set of accept (or: final) states



Review: Accepted language revisited

Extend the transition function δ to $\delta^* : Q \times \Sigma^* \to 2^Q$ by:

$$\delta^*(q, \varepsilon) = \{\, q \,\} \quad \text{and} \quad \delta^*(q, A) = \delta(q, A)$$

$$\delta^*(q, A_1A_2 \ldots A_n) = \bigcup_{p \in \delta(q, A_1)} \delta^*(p, A_2 \ldots A_n)$$

$\delta^*(q, w)$ = set of states reachable from q for the word w

Then: $\mathcal{L}(\mathcal{A}) = \left\{ w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \varnothing \text{ for some } q_0 \in Q_0 \right\}$

The class of languages accepted by NFA (over Σ)

= the class of regular languages (over Σ)

Intersection

- Let NFA $\mathcal{A}_i = (Q_i, \Sigma, \delta_i, Q_{0,i}, F_i)$, with i=1, 2
- The product automaton

$$\mathcal{A}_1 \otimes \mathcal{A}_2 \;=\; (Q_1 \times Q_2, \Sigma, \delta, Q_{0,1} \times Q_{0,2}, F_1 \times F_2)$$

where δ is defined by:

$$\frac{q_1 \xrightarrow{A}_1 q_1' \;\wedge\; q_2 \xrightarrow{A}_2 q_2'}{(q_1, q_2) \xrightarrow{A} (q_1', q_2')}$$

- Well-known result: $\mathcal{L}(\mathcal{A}_1 \otimes \mathcal{A}_2) \;=\; \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$

Total NFA

Automaton $\mathcal{A}$ is called deterministic if

$$|Q_0| \leq 1 \quad \text{and} \quad |\delta(q, A)| \leq 1 \quad \text{for all } q \in Q \text{ and } A \in \Sigma$$

DFA $\mathcal{A}$ is called total if

$$|Q_0| = 1 \quad \text{and} \quad |\delta(q, A)| = 1 \quad \text{for all } q \in Q \text{ and } A \in \Sigma$$

any DFA can be turned into an equivalent total DFA

total DFA provide unique successor states, and thus, unique runs for each input word

Determinization

For NFA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ let $\mathcal{A}_{det} = (2^Q, \Sigma, \delta_{det}, Q_0, F_{det})$ with:

$$F_{det} = \left\{ Q' \subseteq Q \mid Q' \cap F \neq \varnothing \right\}$$

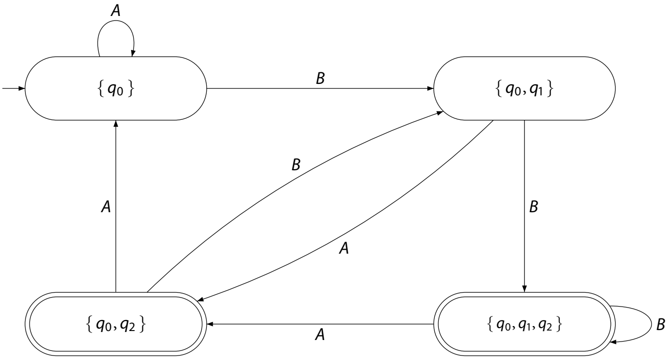
and the total transition function $\delta_{det} : 2^Q \times \Sigma \to 2^Q$ is defined by:

$$\delta_{det}(Q', A) = \bigcup_{q \in Q'} \delta(q, A)$$

$\mathcal{A}_{det}$ is a total DFA and, for all $w \in \Sigma^*$: $\delta^*_{det}(Q_0, w) = \bigcup_{q_0 \in Q_0} \delta^*(q_0, w)$

Thus: $\mathcal{L}(\mathcal{A}_{det}) = \mathcal{L}(\mathcal{A})$

Determinization



a deterministic finite automaton accepting $\mathcal{L}((A + B)^* B(A + B))$

Facts about finite automata

- They are as expressive as regular languages
- They are closed under $\cap$ and complementation
  - NFA $\mathcal{A} \otimes B$ (= cross product) accepts $\mathcal{L}(A) \cap \mathcal{L}(B)$
  - Total DFA $\overline{\mathcal{A}}$ (= swap all accept and normal states) accepts $\overline{\mathcal{L}(A)} = \Sigma^* \setminus \mathcal{L}(\mathcal{A})$
- They are closed under determinization (= removal of choice)
  - although at an exponential cost.....
- $\mathcal{L}(\mathcal{A}) = \emptyset$? = check for reachable accept state in $\mathcal{A}$
  - this can be done using a simple depth-first search
- For regular language $\mathcal{L}$ there is a unique minimal DFA accepting $\mathcal{L}$

Peterson's banking system

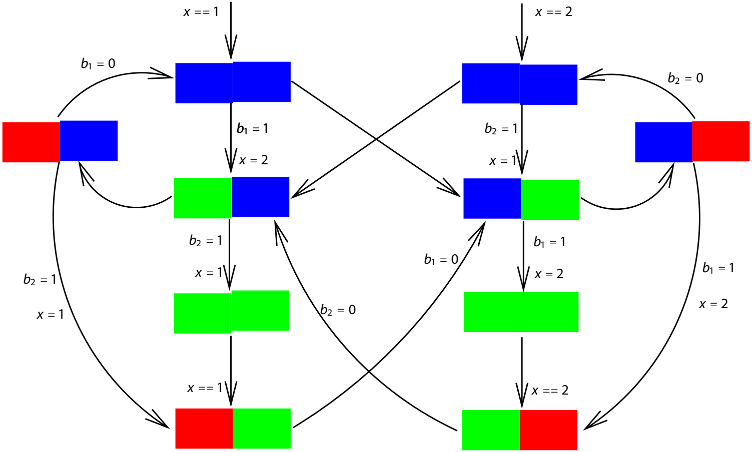
Person Left behaves as follows:

$$
\begin{array}{ll}
 & \textbf{while}\ \text{true}\ \ \{ \\
 & \quad \ldots\ldots \\
\mathit{rq}: & \quad b_1, x = \text{true}, 2; \\
\mathit{wt}: & \quad \textbf{wait until}(x == 1 \,||\, \neg b_2)\ \{ \\
\mathit{cs}: & \quad\quad \ldots @\text{account}_L \ldots \} \\
 & \quad b_1 = \text{false}; \\
 & \quad \ldots\ldots \\
 & \}
\end{array}
$$

Person Right behaves as follows:

$$
\begin{array}{ll}
 & \textbf{while}\ \text{true}\ \ \{ \\
 & \quad \ldots\ldots \\
\mathit{rq}: & \quad b_2, x = \text{true}, 1; \\
\mathit{wt}: & \quad \textbf{wait until}(x == 2 \,||\, \neg b_1)\ \{ \\
\mathit{cs}: & \quad\quad \ldots @\text{account}_R \ldots \} \\
 & \quad b_2 = \text{false}; \\
 & \quad \ldots\ldots \\
 & \}
\end{array}
$$

Is the banking system safe?



Can we guarantee that only one person at a time has access to the bank account?

"always $\neg$ ($@account_L \; \wedge \; @account_R$)"

Is the banking system safe?

- Safe = at most one person may have access to the account
- Unsafe: two have access to the account simultaneously
  - unsafe behaviour can be characterized by bad prefix
  - alternatively (in this case) by the finite automaton:

Regular safety properties

> Safety property P_{safe} over AP is regular
>
> if its set of bad prefixes is a regular language over 2^{AP}

every invariant is regular

Problem statement

Let

- P_{safe} be a regular safety property over AP
- $\mathcal{A}$ an NFA recognizing the bad prefixes of P_{safe}
  - assume that $\varepsilon \notin \mathcal{L}(\mathcal{A})$

  $\Rightarrow$ otherwise all finite words over 2^{AP} are bad prefixes
- TS a finite transition system (over AP) without terminal states

How to establish whether $TS \models P_{safe}$?

Basic idea of the algorithm

$$TS \models P_{safe} \quad \text{if and only if} \quad \mathit{Traces}_{fin}(TS) \cap \mathit{BadPref}(P_{safe}) = \varnothing$$

$$\text{if and only if} \quad \mathit{Traces}_{fin}(TS) \cap \mathcal{L}(\mathcal{A}) = \varnothing$$

$$\text{if and only if} \quad TS \otimes \mathcal{A} \models \text{“always”}\ \Phi \quad \text{to be proven}$$

But . . . . . . this amounts to invariant checking on $TS \otimes \mathcal{A}$

$\Rightarrow$ checking regular safety properties can be done by depth-first search!

Synchronous product (revisited)

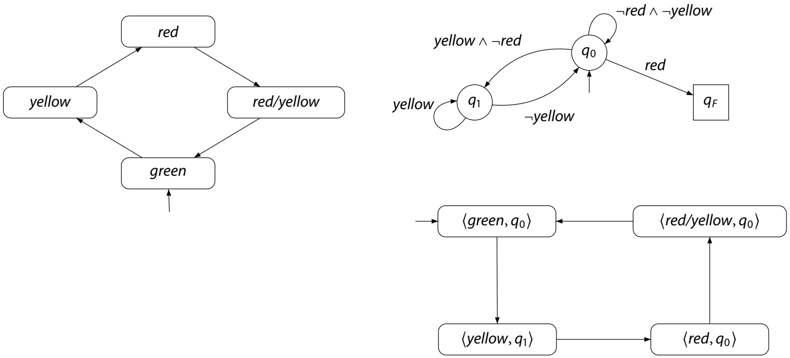
For transition system $TS = (S, Act, \to, I, AP, L)$ without terminal states and $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ an NFA with $\Sigma = 2^{AP}$ and $Q_0 \cap F = \varnothing$, let:

$$TS \otimes \mathcal{A} \; = \; (S', Act, \to', I', AP', L') \qquad \text{where}$$

- $S' = S \times Q$, $AP' = Q$ and $L'(\langle s, q \rangle) \; = \; \{ \, q \, \}$
- $\to'$ is the smallest relation defined by: $\dfrac{s \xrightarrow{\alpha} t \;\wedge\; q \xrightarrow{L(t)} p}{\langle s, q \rangle \xrightarrow{\alpha}' \langle t, p \rangle}$
- $I' = \{ \, \langle s_0, q \rangle \mid s_0 \in I \;\wedge\; \exists q_0 \in Q_0. \, q_0 \xrightarrow{L(s_0)} q \, \}$

without loss of generality it may be assumed that $TS \otimes \mathcal{A}$ has no terminal states

Example product



Verification of regular safety properties

Let TS over AP and NFA $\mathcal{A}$ with alphabet 2^{AP} as before, regular safety property P_{safe} over AP such that $\mathcal{L}(\mathcal{A})$ is the set of bad prefixes of P_{safe}.

The following statements are equivalent:

(a) $TS \vDash P_{safe}$

(b) $Traces_{fin}(TS) \cap \mathcal{L}(\mathcal{A}) = \varnothing$

(c) $TS \otimes \mathcal{A} \vDash P_{inv(A)}$

where $P_{inv(A)} = \bigwedge_{q \in F} \neg q$

Counterexamples

For each initial path fragment $\langle s_0, q_1 \rangle \ldots \langle s_n, q_{n+1} \rangle$ of $TS \otimes \mathcal{A}$:

$$q_1, \ldots, q_n \notin F \text{ and } q_{n+1} \in F \quad \Rightarrow \quad \underbrace{trace(s_0\, s_1 \ldots s_n)}_{\text{bad prefix for } P_{safe}} \in \mathcal{L}(\mathcal{A})$$

Verification algorithm

Require: finite transition system TS and regular safety property P_{safe}
Ensure: true if $TS \models P_{safe}$. Otherwise false plus a counterexample for P_{safe}.

Let NFA $\mathcal{A}$ (with accept states F) be such that $\mathcal{L}(\mathcal{A}) = BadPref(P_{safe})$;
Construct the product transition system $TS \otimes \mathcal{A}$;
Check the invariant $P_{inv(\mathcal{A})}$ with proposition $\neg F = \bigwedge_{q \in F} \neg q$ on $TS \otimes \mathcal{A}$

if $TS \otimes \mathcal{A} \models P_{inv(\mathcal{A})}$ **then**
    return true
else
    Determine initial path fragment $\langle s_0, q_1 \rangle \ldots \langle s_n, q_{n+1} \rangle$ of $TS \otimes \mathcal{A}$ with $q_{n+1} \in F$
    return $(\text{false}, s_0\, s_1 \ldots s_n)$
end if

Time complexity

The time and space complexity of checking a regular safety property P_{safe} against transition system TS is in:

$$\mathcal{O}(|TS| \cdot |\mathcal{A}|)$$

where $\mathcal{A}$ is an NFA recognizing the bad prefixes of P_{safe}

Büchi Automata

Peterson's banking system

Person Left behaves as follows:

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 & \textbf{while } \text{true} \;\; \{ \\
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rq: & \quad b_1, x = \text{true}, 2; \\
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cs: & \qquad \ldots @\text{account}_L \ldots \} \\
 & \quad b_1 = \text{false}; \\
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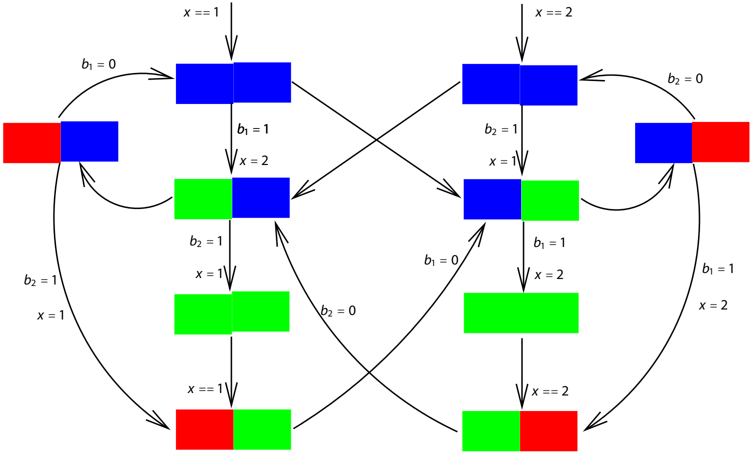
Person Right behaves as follows:

$$
\begin{array}{ll}
 & \textbf{while } \text{true} \;\; \{ \\
 & \quad \ldots\ldots \\
rq: & \quad b_2, x = \text{true}, 1; \\
wt: & \quad \textbf{wait until}(x == 2 \,||\, \neg b_1)\; \{ \\
cs: & \qquad \ldots @\text{account}_R \ldots \} \\
 & \quad b_2 = \text{false}; \\
 & \quad \ldots\ldots \\
 & \}
\end{array}
$$

Is the banking system live?



If someone wants to update the account, does (s)he ever get the opportunity to do so?

"always (req_L $\Rightarrow$ eventually @account$_L$) $\wedge$ always (req_R $\Rightarrow$ eventually @account$_R$)"

ω-regular expressions

1. $\underline{\varnothing}$ and $\underline{\varepsilon}$ are regular expressions over Σ
2. if $A \in \Sigma$ then $\underline{A}$ is a regular expression over Σ
3. if E, E_1 and E_2 are regular expressions over Σ
   then so are $E_1 + E_2$, $E_1.E_2$ and E^*

E^+ is an abbreviation for the regular expression $E.E^*$

An <u>ω-regular expression</u> G over the alphabet Σ has the form:

$$G = E_1.F_1^{\omega} + \ldots + E_n.F_n^{\omega} \quad \text{for } n > 0$$

where E_i, F_i are regular expressions over Σ such that $\varepsilon \notin \mathcal{L}(F_i)$, for all $0 < i \leq n$

Semantics of ω-regular expressions

- The semantics of regular expression E is a language $\mathcal{L}(\mathsf{E}) \subseteq \Sigma^*$:

$$\mathcal{L}(\underline{\varnothing}) = \varnothing, \quad \mathcal{L}(\underline{\varepsilon}) = \{\, \varepsilon \,\}, \quad \mathcal{L}(\underline{A}) = \{\, A \,\}$$

$$\mathcal{L}(\mathsf{E}+\mathsf{E}') = \mathcal{L}(\mathsf{E}) \cup \mathcal{L}(\mathsf{E}') \qquad \mathcal{L}(\mathsf{E}.\mathsf{E}') = \mathcal{L}(\mathsf{E}).\mathcal{L}(\mathsf{E}') \qquad \mathcal{L}(\mathsf{E}^*) = \mathcal{L}(\mathsf{E})^*$$

- The semantics of ω-regular expression G is a language $\mathcal{L}(\mathsf{G}) \subseteq \Sigma^\omega$:

$$\mathcal{L}_\omega(\mathsf{G}) \;=\; \mathcal{L}(\mathsf{E}_1).\mathcal{L}(\mathsf{F}_1)^\omega \cup \ldots \cup \mathcal{L}(\mathsf{E}_n).\mathcal{L}(\mathsf{F}_n)^\omega$$

  where $L^\omega = \{ w_0 w_1 w_2 \cdots \mid w_i \in L \text{ for all } i \}$ (for $L \subseteq \Sigma^*$).
- G_1 and G_2 are equivalent, denoted $\mathsf{G}_1 \equiv \mathsf{G}_2$, if $\mathcal{L}_\omega(\mathsf{G}_1) = \mathcal{L}_\omega(\mathsf{G}_2)$

ω-regular languages and properties

- $\mathcal{L} \subseteq \Sigma^\omega$ is ω-regular if $\mathcal{L} = \mathcal{L}_\omega(\mathrm{G})$ for some ω-regular expression G (over Σ)
- ω-regular languages possess several closure properties
  - they are closed under union, intersection, and complementation
  - complementation is not treated here; we use a trick to avoid it
- LT property P over AP is called ω-regular

  if P is an ω-regular language over the alphabet 2^{AP}

all invariants and regular safety properties are ω-regular!

Büchi automata

- NFA (and DFA) are incapable of accepting infinite words
- Automata on infinite words
  - suited for accepting ω-regular languages
  - we consider nondeterministic Büchi automata (NBA)
- Accepting runs have to “check” the entire input word $\Rightarrow$ are infinite

  $\Rightarrow$ acceptance criteria for infinite runs are needed
- NBA are like NFA, but have a distinct acceptance criterion
  - one of the accept states must be visited infinitely often

Büchi automata

A nondeterministic Büchi automaton (NBA) $\mathcal{A}$ is a tuple $(Q, \Sigma, \delta, Q_0, F)$ where:

- Q is a finite set of states with $Q_0 \subseteq Q$ a set of initial states
- Σ is an alphabet
- $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function
- $F \subseteq Q$ is a set of accept (or: final) states

The size of $\mathcal{A}$, denoted $|\mathcal{A}|$, is the number of states and transitions in $\mathcal{A}$:

$$|\mathcal{A}| \ = \ |Q| + \sum_{q \in Q} \sum_{A \in \Sigma} |\, \delta(q, A) \,|$$

Language of an NBA

- NBA $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ and word $\sigma = A_1 A_2 A_3 \ldots \in \Sigma^\omega$
- A *run* for σ in $\mathcal{A}$ is an infinite sequence $q_0\, q_1\, q_2 \ldots$ such that:
  - $q_0 \in Q_0$ and $q_i \xrightarrow{A_{i+1}} q_{i+1}$ for all $0 \le i$
- Run $q_0\, q_1\, q_2 \ldots$ is accepting if $q_i \in F$ for infinitely i
- $\sigma \in \Sigma^\omega$ is *accepted* by $\mathcal{A}$ if there exists an accepting run for σ
- The accepted language of $\mathcal{A}$:

$$\mathcal{L}_\omega(\mathcal{A}) = \left\{ \sigma \in \Sigma^\omega \;\middle|\; \text{there exists an accepting run for } \sigma \text{ in } \mathcal{A} \right\}$$

- NBA $\mathcal{A}$ and $\mathcal{A}'$ are equivalent if $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{A}')$

Deterministic BA

Büchi automaton $\mathcal{A}$ is called deterministic if

$$|Q_0| \leq 1 \quad \text{and} \quad |\delta(q, A)| \leq 1 \quad \text{for all } q \in Q \text{ and } A \in \Sigma$$

DBA $\mathcal{A}$ is called total if

$$|Q_0| = 1 \quad \text{and} \quad |\delta(q, A)| = 1 \quad \text{for all } q \in Q \text{ and } A \in \Sigma$$

total DBA provide unique runs for each input word